

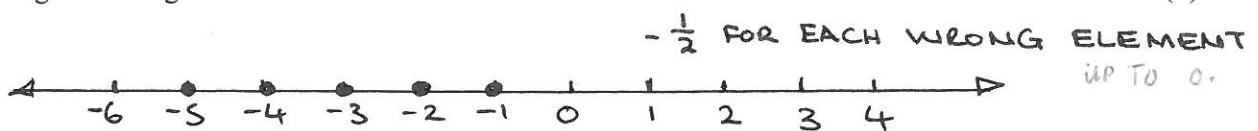
**UNIVERSITY OF KWAZULU-NATAL**  
**JUNE 2007 TEST**  
**COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )**

**QUESTION ONE [ 17 marks ]**

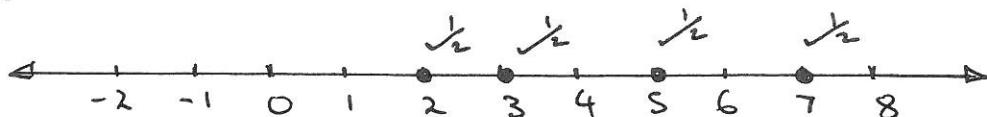
- (a) True or False? All natural numbers are rational numbers. (*WITH REASON*) (1)  
 TRUE ✓ SINCE  $\mathbb{N} \subset \mathbb{Q}$  ✓

- (b) Let  $M$  be a set of real numbers between  $-6$  and  $10$ . Illustrate on separate number lines, the set of all elements in  $M$  that are:

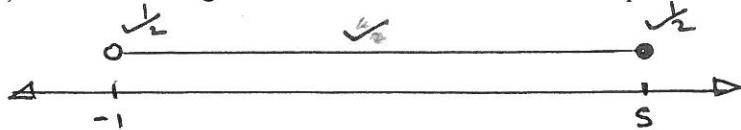
- (i) negative integers. (2)



- (ii) prime numbers. (2)



- (iii) real numbers greater than  $-1$  but less than or equal to  $5$ . (2)



- (c) Define a rational number and hence, show that  $0.\overline{583}$  is a rational number. (5)

A RATIONAL NUMBER IS A NUMBER WHICH CAN BE WRITTEN IN THE FORM  $\frac{a}{b}$ ,  $a, b \in \mathbb{Z}$ ,  $b \neq 0$ . ✓

LET  $x = 0.\overline{58333\dots}$  (1) ✓

THEN  $\underline{10x} = \underline{5.83333\dots}$  (2) ✓

$$(2) - (1) \quad 9x = 5.25 \quad \checkmark$$

$$\therefore x = \frac{5.25}{9} = \frac{525}{900} = \frac{21}{36} \quad \checkmark \text{ WHICH IS}$$

OF THE FORM OF A RATIONAL NUMBER.

HENCE  $0.\overline{583}$  IS RATIONAL. ✓

**UNIVERSITY OF KWAZULU-NATAL**  
**JUNE 2007 TEST**  
**COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )**

---

- (d) Prove that the product of two odd numbers is an odd number. (5)

LET  $a, b \in \mathbb{Z}$  ✓  
 THEN  $a = 2m + 1$  AND  $b = 2n + 1$  ARE ODD,  $m, n \in \mathbb{Z}$

NOW,

$$\begin{aligned}
 ab &= (2m+1)(2n+1) \checkmark \\
 &= 4mn + 2m + 2n + 1 \\
 &= 2(2mn + m + n) + 1 \checkmark \\
 &= 2k + 1 \checkmark \quad \text{WHERE } 2mn + m + n = k, k \in \mathbb{Z} \checkmark
 \end{aligned}$$

HENCE, THE PRODUCT OF TWO ODD NUMBERS IS AN ODD NUMBER.

**QUESTION TWO [ 18 marks ]**

- (a) Let  $B$  be any non-empty set, and  $A$  be a subset of  $B$ .  
 True or False? If false, give the correct answer:

(i)  $A \cup A' = \{\}$  (2)

FALSE, ✓ SINCE  $A \cup A' = B$  ✓

(ii)  $A \cap \{\} = B$  (2)

FALSE. ✓

$$A \cap \{\} = \{\} \checkmark$$

**UNIVERSITY OF KWAZULU-NATAL**  
**JUNE 2007 TEST**  
**COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )**

---

- (b) Consider the following sets of different make of motor vehicles which were recently auctioned by the Department of Transport, where  $E$  is the universal set.

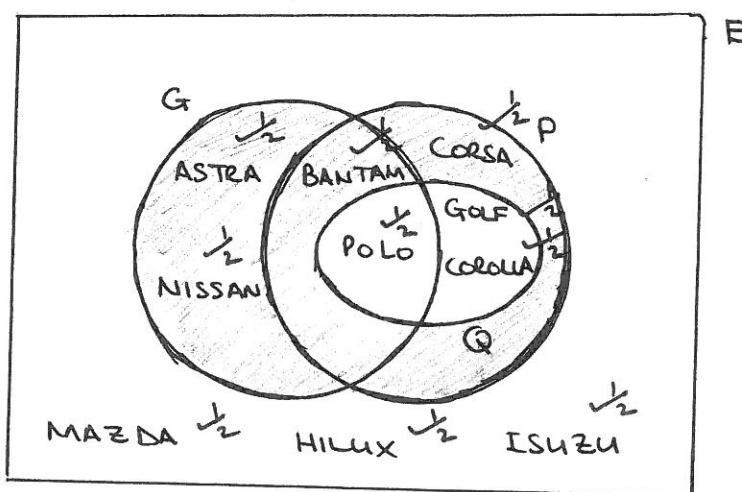
$$E = \{ \text{Corsa, Polo, Corolla, Mazda, Astra, Golf, Hilux, Nissan, Isuzu, Bantam} \}$$

$$G = \{ \text{Astra, Polo, Nissan, Bantam} \}$$

$$P = \{ \text{Bantam, Corolla, Polo, Golf, Corsa} \}$$

$$Q = \{ \text{Golf, Polo, Corolla} \}$$

- (i) Draw a Venn diagram, showing all the sets given above, and put the elements in their correct places. (5)



- (ii) Determine:

$$(Q \cap P)' \cap G \quad (3)$$

$$= \{ \text{ASTRA} ; \text{NISSAN} ; \text{BANTAM} \} \quad \frac{1}{2} \text{ FOR INCORRECT BRACKETS}$$

- (iii) Determine:

$$n(G \cup G) \quad (3)$$

$$= n(\{ \text{MAZDA, HILUX, ISUZU, CORSA, GOLF, POLO, BANTAM, COROLLA} \}) = 10$$

- (iv) On your Venn diagram in Question b(i) above, shade  $Q' \cap (G \cup P)$ . (3)

SEE ABOVE

SHADED :  $G$  ✓  
 $P$  ✓  
 $Q'$  ✓

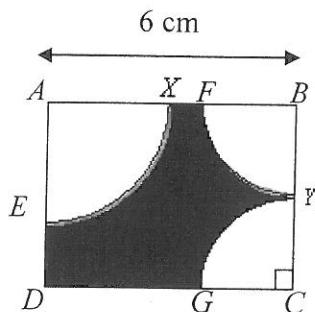
# UNIVERSITY OF KWAZULU-NATAL

JUNE 2007 TEST

**COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )**

### QUESTION THREE [ 28 marks ]

- (a) In the diagram below, all curved lines are arcs of circles.  $ABCD$  is a rectangle with  $AB = 6 \text{ cm}$ ,  $AX = XB$  and  $BY = YC = 2 \text{ cm}$ . Determine the perimeter of the shaded region. (6)



$$AX = XB = \frac{6}{2} = 3 \text{ cm} \quad \checkmark_2$$

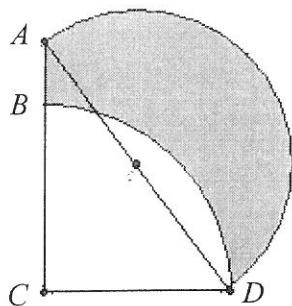
$$BY = YC = 2 \text{ cm} \quad \therefore BC = AD = 4 \text{ cm} \quad \checkmark_2$$

$$\begin{aligned}
 & \text{PERIMETER OF SHADED REGION} = \overbrace{\widehat{EX} + \widehat{XF} + \widehat{FY} + \widehat{YD} + \widehat{GD}}^{\checkmark_2} + \overbrace{DI}^{\checkmark_2} \\
 & = \frac{1}{4}(2\pi \times 3) + (6 - 3 - 2) + \frac{1}{4}(2\pi \times 2) + \frac{1}{4}(2\pi \times 2) + (6 - 2) + (4 - 3) \\
 & = \frac{3\pi}{2} + 1 + \pi + \pi + 4 + 1 \\
 & = \left( \frac{7\pi}{2} + 6 \right) \text{ cm } \checkmark_2 \text{ UNITS}
 \end{aligned}$$

**UNIVERSITY OF KWAZULU-NATAL**  
**JUNE 2007 TEST**  
**COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )**

---

(b).



In the diagram alongside,

- $ACD$  is a right-angled triangle.
- $AC = 28 \text{ cm}$  and  $BC = 21 \text{ cm}$ .
- A semicircle is drawn using  $AD$  as diameter.
- Sector  $BCD$  has  $CD$  as radius.

Determine the area of the shaded region.

Leave your answer in terms of  $\pi$ .

(7)

$\cancel{x}$        $\checkmark_2$        $\cancel{x}$

$$\text{AREA OF SHADED REGION} = \text{AREA } \triangle ACD + \text{AREA } \odot AD - \text{SECTOR } BCD$$

$$= \left( \frac{1}{2} \times 28 \times 21 \right) + \frac{1}{2} \pi \left( \frac{AD}{2} \right)^2 - \frac{1}{4} (\pi \times [21]^2)$$

$$= (294 + \frac{\pi}{8} \times (AD)^2 - \frac{441\pi}{4}) \text{ cm}^2$$

$$\begin{aligned} \text{BUT } (AD)^2 &= (AC)^2 + (CD)^2 && (\text{PYTHAGORAS}) \\ &= (28)^2 + (21)^2 \\ &= 1225 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{AREA OF SHADED REGION} &= (294 + \frac{\pi}{8} \times 1225 - \frac{441\pi}{4}) \text{ cm}^2 \\ &= \left( 294 + \frac{343\pi}{8} \right) \text{ cm}^2 \text{ UNITS} \end{aligned}$$

$42 \frac{7}{8}$

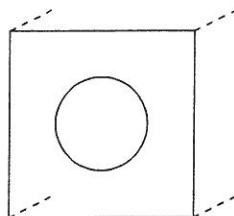
# UNIVERSITY OF KWAZULU-NATAL

JUNE 2007 TEST

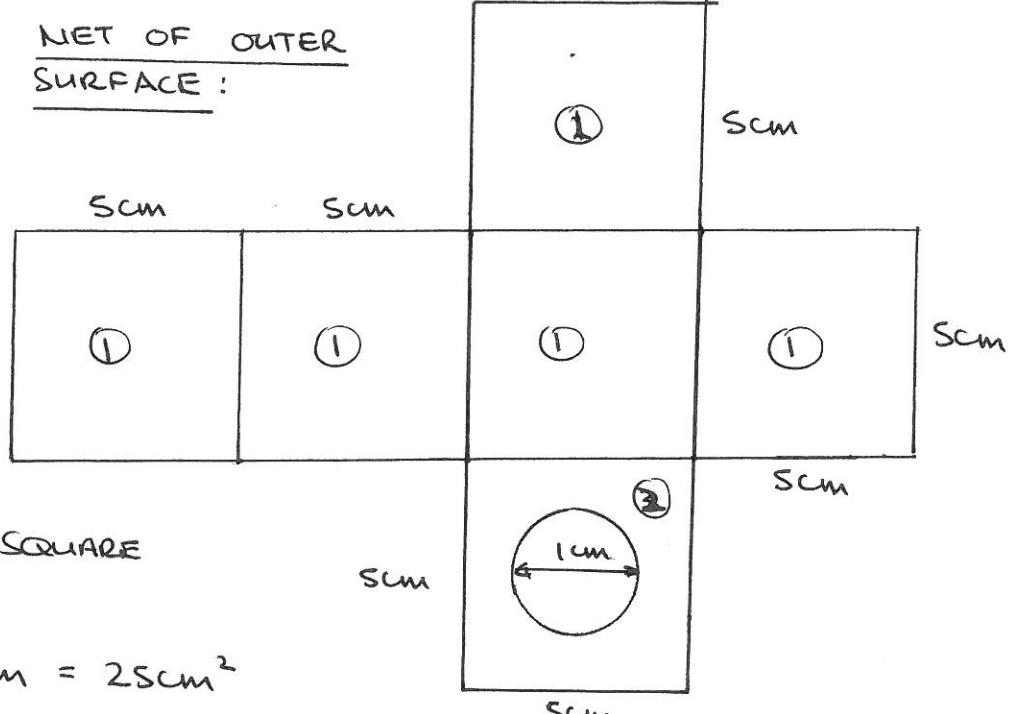
COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )

- (c). A solid wooden cube has side of length 5 cm. A circular hole of diameter 1 cm is drilled through one of the faces of the wooden cube as shown below. The hole is 2 cm deep.  
Determine the total surface area of the wooden cube.

(8)



NET OF OUTER SURFACE :



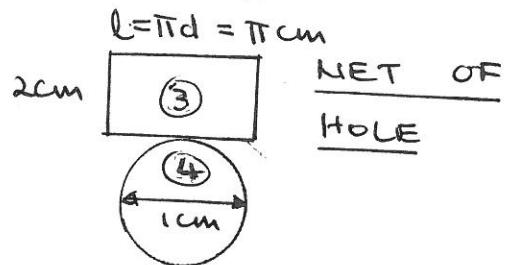
$$\begin{aligned} \text{AREA 1} &= \text{AREA OF SQUARE} \\ &= s \times s \\ &= 5\text{cm} \times 5\text{cm} = 25\text{cm}^2 \end{aligned}$$

$$\begin{aligned} \text{AREA 2} &= \text{AREA OF SQUARE} \\ &\quad - \text{AREA OF CIRCLE} \end{aligned}$$

$$\begin{aligned} &= (s \times s) - \pi r^2 \\ &= (s \times s) - \pi (\frac{1}{2})^2 = (25 - \frac{\pi}{4}) \text{cm}^2 \end{aligned}$$

$$\text{AREA 3} = \text{AREA OF RECTANGLE} = \text{LENGTH} \times \text{BREADTH}$$

$$\text{AREA 4} = \text{AREA OF CIRCLE} = \pi r^2 = \pi (\frac{1}{2})^2 = \frac{\pi}{4} \text{cm}^2$$



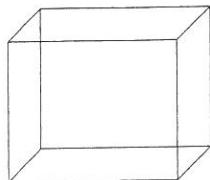
$$\begin{aligned} \text{TOTAL SURFACE AREA} &= (5 \times \text{AREA 1}) + \text{AREA 2} + \text{AREA 3} + \text{AREA 4} \\ &= (5 \times 25) + (25 - \frac{\pi}{4}) + 2\pi + \frac{\pi}{4} \end{aligned}$$

$$\begin{aligned} &= 125 + 25 + 2\pi \\ &= (150 + 2\pi) \text{cm}^2 \end{aligned}$$

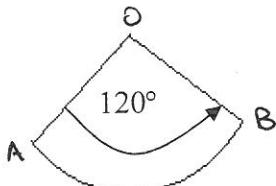
$$\frac{\text{Ratio Det } y:b}{x:y = ab}$$

**UNIVERSITY OF KWAZULU-NATAL**  
**JUNE 2007 TEST**  
**COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )**

(d).



Prism 1



Cross Section of Prism 2

Prism 1, which is a rectangular prism, has a length of  $2x$  mm, a breadth of  $x$  mm and a height of  $h_1$  mm.

Prism 2, whose cross section is a sector with radius  $x$  mm, has angle  $\angle AOB = 120^\circ$ .

Prism 1 is completely full of water. All the water from prism 1 is then poured into prism 2. The water level in prism 2 is  $h_2$  mm.

Determine  $h_2 : h_1$ . (7)

$$\begin{aligned} \text{VOLUME OF PRISM 1} &= \text{AREA OF BASE} \times \text{HEIGHT OF PRISM} \\ &= (\text{LENGTH} \times \text{BREADTH}) \times \text{HEIGHT} \\ &= (2x^2 \times x) \times h_1 = 2x^2 h_1 \text{ mm}^3 \quad (1) \end{aligned}$$

$$\begin{aligned} \text{VOLUME OF PRISM 2} &= \text{AREA OF BASE} \times \text{HEIGHT OF PRISM} \\ &= \text{AREA OF SECTOR} \times \text{HEIGHT} \\ &= \frac{120}{360} \times \pi x^2 \times h_2 = \frac{\pi x^2}{3} h_2 \text{ mm}^3 \quad (2) \end{aligned}$$

BUT THE VOLUMES ARE EQUAL

$$\therefore \frac{\pi x^2}{3} h_2 = 2x^2 h_1 \quad (1)$$

$$\frac{h_2}{h_1} = \frac{2x^2}{1} \times \frac{3}{\pi x^2} = \frac{6}{\pi} \quad (x \neq 0)$$

$$\therefore h_2 : h_1 = 6 : \pi$$

**UNIVERSITY OF KWAZULU-NATAL**  
**JUNE 2007 TEST**  
**COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )**

**QUESTION FOUR [ 16 marks ]**

(a). State the type of proportion between  $x$  and  $y$ , in each of the following (give a reason in each case) :

(i)

$x$	2	4	- 8
$y$	3	6	- 12

$$\frac{2}{3} = \frac{4}{6} = \frac{-8}{-12} \checkmark$$

(2)

∴ DIRECT PROPORTION ✓

(ii)

$x$	1	3	5
$y$	2	4	6

$$\frac{1}{2} \neq \frac{3}{4} \neq \frac{5}{6} \checkmark \text{ AND } 1 \times 2 \neq 3 \times 4 \neq 5 \times 6 \checkmark$$

(2)

∴ NO PROPORTION ✓

(iii)

$x$	- 1	- 4	- 6
$y$	12	3	2

$$-1 \times 12 = -4 \times 3 = -6 \times 2 \checkmark$$

(2)

∴ INVERSE PROPORTION ✓

(b). Jane is 25 years younger than her mother. In 12 years time she will be half her mothers' age.  
Calculate Jane's age, by making use of suitable equations:

(5)

LET JANE'S AGE BE  $x$  ∴

$$\begin{array}{|c|c|} \hline \text{JANE'S AGE} & \text{MOTHERS AGE} \\ \hline x & x+25 \checkmark \\ \hline \end{array}$$

IN TWELVE YEARS

$$\begin{array}{|c|c|} \hline x+12 & x+37 \checkmark \\ \hline \end{array}$$

$$x+12 = \frac{1}{2}(x+37) \checkmark$$

$$\therefore 2x+24 = x+37 \checkmark$$

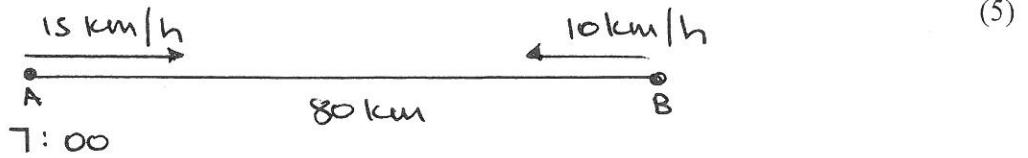
$$x = 13 \quad \therefore \text{JANE IS } 13 \text{ YEARS OLD} \checkmark$$

# UNIVERSITY OF KWAZULU-NATAL

JUNE 2007 TEST

COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )

- (c). Two cyclists are 80 km apart and travel towards each other. They start at 7:00 and cycle at a constant speed of 15 km/h and 10 km/h. By using suitable equations determine at what time the cyclists will meet.



SPEED =  $\frac{\text{DISTANCE}}{\text{TIME}}$  ✓ . IN  $t$  HOURS THE DISTANCE TRAVELED

$$\text{IS : } 15t + 10t = 80 \text{ km} \checkmark$$

$$25t = 80$$

$$t = 3\frac{1}{5} \text{ hr} = 3h12 \text{ min} \checkmark$$

$$\therefore \text{TIME OF MEETING} = 7 \text{ hours} + 3 \text{ hours } 12 \text{ min} = 10:12 \checkmark$$

**QUESTION FIVE [ 6 marks ]**

- (a). Determine the solution of the following system of equations :

$$\begin{aligned} x + 2y - z &= 1 \\ 5x - y + 2z &= 5 \\ 2x + 4y - 2z &= 2 \end{aligned} \quad (6)$$

$$\left[ \begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 5 & -1 & 2 & 5 \\ 2 & 4 & -2 & 2 \end{array} \right] \xrightarrow{\substack{(A) \\ (B) \\ (C)}} \left[ \begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 11 & -7 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \begin{array}{l} (5 \times A) - B \\ (2 \times A) - C \end{array}$$

$$\text{LET } z = t \checkmark, \quad t \in \mathbb{R} \checkmark$$

$$\therefore 11y - 7z = 0$$

$$11y - 7t = 0 \quad \therefore y = \frac{7t}{11} \checkmark$$

$$x + 2y - z = 1$$

$$x + 2\left(\frac{7t}{11}\right) - t = 1 \quad \therefore x = 1 - \frac{14t}{11} + t$$

$$\text{Solution} = \left\{ (x, y, z) \in \mathbb{R}^3, t \in \mathbb{R} : \left(1 - \frac{14t}{11}, \frac{7t}{11}, t\right) \right\} \checkmark$$

# UNIVERSITY OF KWAZULU-NATAL

JUNE 2007 TEST

COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )

## QUESTION SIX [ 12 marks ]

- (a) Solve for  $x$  and illustrate your solution graphically:

$$\begin{aligned}
 -x+1 &\leq \frac{3+2x}{2} < 5 & (5) \\
 -x+1 &\leq \frac{3+2x}{2} \quad \text{AND} \quad \frac{3+2x}{2} < 5 \\
 -2x+2 &\leq 3+2x \quad \text{AND} \quad 3+2x < 10 \\
 -4x &\leq 1 \quad \text{AND} \quad 2x < 7 \\
 x &\geq -\frac{1}{4} \quad \text{AND} \quad x < \frac{7}{2} \\
 \end{aligned}$$

**ANSWER**  $\frac{1}{2}$

- (b) Solve for  $x$  and write your solution in interval notation:

$$\frac{1}{x-3} \leq \frac{9}{4x+3}$$

$$\frac{1}{x-3} \leq \frac{9}{4x+3}$$

$$\frac{1}{x-3} - \frac{9}{4x+3} \leq 0 \quad \checkmark$$

$$\frac{4x+3 - 9(x-3)}{(x-3)(4x+3)} \leq 0$$

$$\frac{4x+3 - 9x + 27}{(x-3)(4x+3)} \leq 0$$

$$\frac{-5x + 30}{(x-3)(4x+3)} \leq 0$$

$$\frac{-5(x-6)}{(x-3)(4x+3)} \leq 0$$

$$\frac{-5(x-6)}{(x-3)(4x+3)} \leq 0 \quad \text{--- A}$$

	$x < -\frac{3}{4}$	$-\frac{3}{4} < x < 3$	$x > 3$	
$-5$	-	-	-	-
$(x-6)$	-	-	-	+
$(x-3)$	-	-	0	++
$(4x+3)$	-	0	+	++++
$-5(x-6)$	+	0	-	-
$(x-3)(4x+3)$	+	0	-	-

WE NEED TO FIND VALUES FOR  $x$  FOR WHICH THE FRACTION A IS  $\leq 0$

$$\therefore x \in \left(-\frac{3}{4}; 3\right) \cup \left[6; \infty\right)$$

CRITICAL VALUES  $\frac{1}{2}; \frac{1}{2}$

$\frac{1}{2}; \frac{1}{2}$

$\frac{1}{2}; \frac{1}{2}$

RESTRICTIONS :  $x \neq 3; x \neq -\frac{3}{4}$

# UNIVERSITY OF KWAZULU-NATAL

JUNE 2007 TEST

COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )

**QUESTION SEVEN [ 14 marks ]**

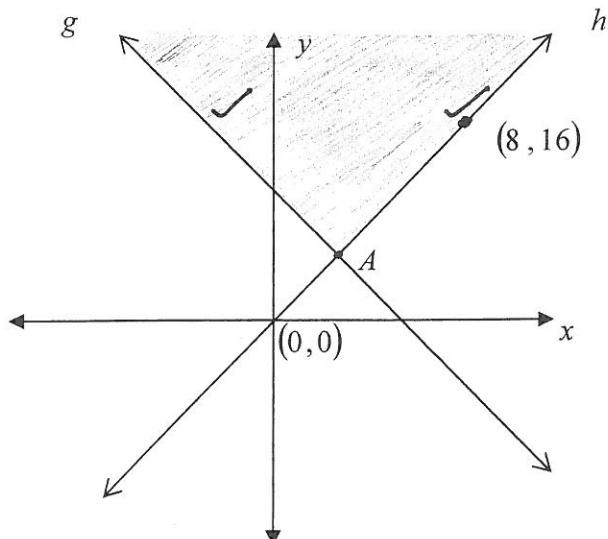
- (a) Find the domain, in set-builder notation, of the following function:

$$f(x) = \frac{1}{\sqrt{2-x}} \quad (3)$$

$$\Delta_f : \quad 2-x > 0 \quad \checkmark \\ x < 2 \quad \checkmark$$

$$\text{DOMAIN} = \{x \in \mathbb{R} \mid x < 2\}$$

- (c) The graphs of  $g(x) = -x + 6$  and  $h(x)$  are illustrated on the diagram below:



- (i) Determine the equation of  $h$ . (3)

GRADIENT / SLOPE OF  $h$ :

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16 - 0}{8 - 0} = 2$$

POINT  $(0,0)$  GIVES :

$$0 = 2(0) + c$$

$$\therefore c = 0 \quad \checkmark$$

$$\therefore h = mx + c$$

$$\therefore h = 2x + c$$

# UNIVERSITY OF KWAZULU-NATAL

JUNE 2007 TEST

COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )

- (ii) Determine the co-ordinates of point A, ie. the point of intersection of  $g$  and  $h$ . (2)

AT THE POINT OF INTERSECTION

$$h(x) = g(x) \quad \checkmark$$

$$\therefore 2x = -x + 6$$

$$3x = 6$$

$$\therefore x = 2 \quad \checkmark$$

SUBSTITUTE  $x = 2$  INTO  $h$   
(OR  $g$ )

$$h(x) = 2x \quad \therefore h(2) = 2(2) = 4 \quad \checkmark$$

$\therefore$  Pt. A is  $(2; 4)$



- (iii) Shade the region represented by  $\{y \geq g\} \cap \{y \geq h\}$  (2)

SEE DIAGRAM

- (iv) Show that  $(g^{-1} \circ g)(x) = (g \circ g^{-1})(x) = x$  (4)

$$g(x) = -x + 6$$

$$g^{-1}(x) = -x + 6 \quad \checkmark$$

$$\begin{aligned} LHS: (g^{-1} \circ g)x &= g^{-1}(g(x)) = g^{-1}(-x+6) \\ &= -(-x+6) + 6 \\ &= x - 6 + 6 \end{aligned}$$

$$= x \quad \checkmark$$

$$\begin{aligned} RHS: (g \circ g^{-1})(x) &= g(g^{-1}(x)) \quad \checkmark \\ &= g(-x+6) \quad \checkmark \\ &= -(-x+6) + 6 \\ &= x - 6 + 6 \\ &= x \quad \checkmark \\ &= LHS \end{aligned}$$

$$\therefore (g^{-1} \circ g)(x) = (g \circ g^{-1})(x) = x$$

**UNIVERSITY OF KWAZULU-NATAL**  
**JUNE 2007 TEST**  
**COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )**

---

**QUESTION EIGHT [ 12 marks ]**

Given the following quadratic equation:

$$f(x) = 3x^2 + 2x - 1$$

- (a) Write  $f(x)$  in the form  $y = a(x - p)^2 + q$ . (3)

$$\begin{aligned} f(x) &= 3x^2 + 2x - 1 \\ &= 3\left[x^2 + \frac{2}{3}x - \frac{1}{3}\right] \checkmark_2 \\ &= 3\left[x^2 + \frac{2}{3}x + \frac{1}{9} - \frac{1}{9} - \frac{1}{3}\right] \\ &= 3\left[\left(x + \frac{1}{3}\right)^2 - \frac{4}{9}\right] \checkmark_2 \\ \therefore f(x) &= 3\left(x + \frac{1}{3}\right)^2 - \frac{4}{3} \checkmark_2 \end{aligned}$$

- (b) Find the equation of the inverse of  $f(x)$ . (3)

$$f(x) = 3x^2 + 2x - 1 = 3\left(x + \frac{1}{3}\right)^2 - \frac{4}{3}$$

CAN BE WRITTEN AS  $y = 3\left(x + \frac{1}{3}\right)^2 - \frac{4}{3}$

INTERCHANGING  $x$  AND  $y$  gives

$$x = 3\left(y + \frac{1}{3}\right)^2 - \frac{4}{3}$$

$$x + \frac{4}{3} = 3\left(y + \frac{1}{3}\right)^2$$

$$\frac{x}{3} + \frac{4}{9} = \left(y + \frac{1}{3}\right)^2 \checkmark_2$$

$$\therefore y + \frac{1}{3} = \pm \sqrt{\frac{x}{3} + \frac{4}{9}}$$

$$y = -\frac{1}{3} \pm \sqrt{\frac{x}{3} + \frac{4}{9}} \checkmark_2$$

**UNIVERSITY OF KWAZULU-NATAL**  
**JUNE 2007 TEST**  
**COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )**

---

(c) Write down the greatest restriction of  $f(x)$ , called  $g(x)$ , so that the inverse of  $g(x)$ , is a function. (2)

$$f(x) = 3(x + \frac{1}{3})^2 - \frac{4}{3}$$

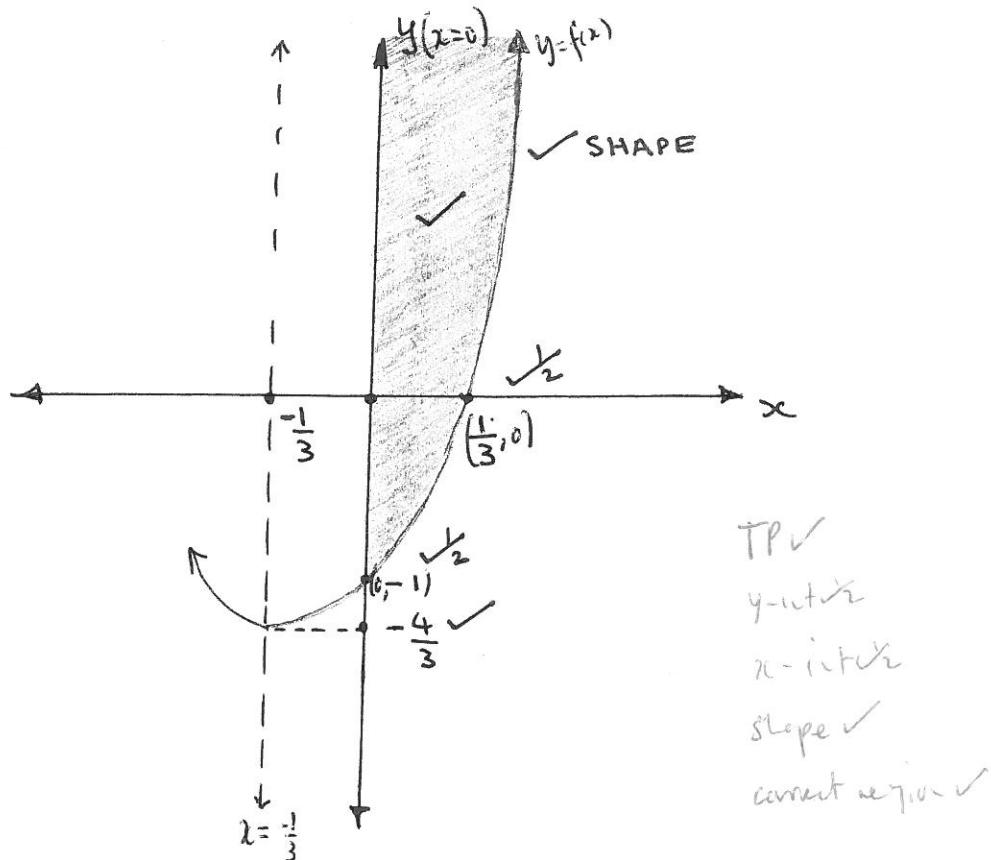
∴ Axis of symmetry is  $x = -\frac{1}{3}$

$$\therefore g(x) = 3x^2 + 2x - 1, x \leq -\frac{1}{3} \quad \checkmark$$

$$\underline{\text{OR}} \quad g(x) = 3x^2 + 2x - 1, x > -\frac{1}{3}$$

(d) Sketch the region defined by:

$$A = \{(x; y) \in R^2 \mid x \geq 0\} \cap \{(x; y) \in R^2 \mid y \geq f(x)\} \quad (4)$$



**UNIVERSITY OF KWAZULU-NATAL**  
**JUNE 2007 TEST**  
**COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )**

---

**QUESTION NINE [ 27 marks ]**

- (a) State if the following are **TRUE** or **FALSE**.

(i) A circle is a function.

(1)

FALSE ✓

- (ii) The hyperbola of the form  $y = \frac{k}{x}$  must be restricted for its inverse to be a function.

(1)

FALSE ✓

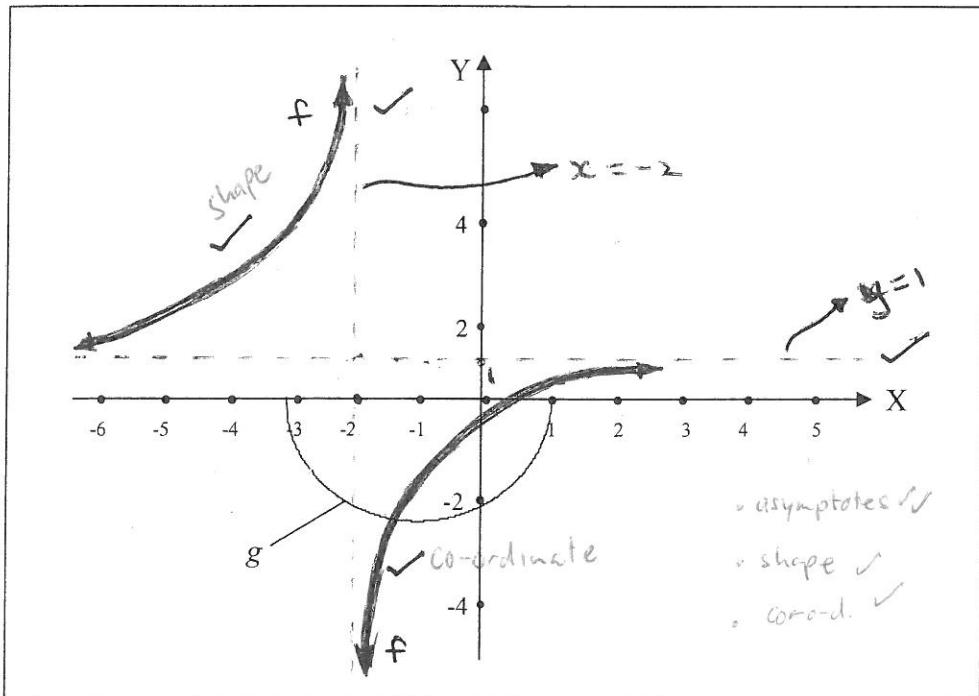
- (b) The general equation of a hyperbola is  $y = \frac{k}{x}$ . Explain what effect the constant  $k$ , has on the shape of graph of a hyperbola.

(2)

THE LARGER ✓ THE VALUE OF  $k$ , THE  
FURTHER ✓ AWAY FROM THE ORIGIN THE  
GRAPH IS.

**UNIVERSITY OF KWAZULU-NATAL**  
**JUNE 2007 TEST**  
**COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )**

(c) Study the graph sketched below carefully and answer the questions that follow.



(i) Write down the equation of the graph  $g$ , sketched above. (4)

$$g(x) = -\sqrt{-((x+1)^2 + 4)}$$

(ii) Write down the domain of the inverse of  $g$ . (2)

$$D(g^{-1}) = \{ x \in \mathbb{R} \mid -2 \leq x \leq 0 \}$$

or  $[-2; 0]$

(iii) If the graph of  $g$  is reflected about the line  $y = -x$ , determine the new equation of  $g$ . (3)

$$y = -\sqrt{-(x+1)^2 + 4}$$

$$\therefore -x = -\sqrt{-(y+1)^2 + 4}$$

$$x = \sqrt{-(y+1)^2 + 4}$$

$$x^2 = -(y+1)^2 + 4$$

$$(-y+1)^2 = 4-x^2$$

$$-y+1 = \pm \sqrt{4-x^2}$$

$$y = 1 \pm \sqrt{4-x^2}, 0 \leq x \leq 2$$

**UNIVERSITY OF KWAZULU-NATAL**  
**JUNE 2007 TEST**  
**COURSE AND CODE: FOUNDATION MATHEMATICS ( MATH 099/199 )**

(d) On the axis above (on which the graph of  $g$  is sketched) :

- (i) Sketch the graph of the equation  $(x+2)(y-1) = -3$ . Call this new graph  $f$ .  
 SEE DIAGRAM

(4)

(e) A circle with equation  $x^2 + y^2 + 6x - 4y = k$  has a radius of 4 units.

- (i) Calculate the coordinates of the centre of the circle.

(3)

$$\begin{aligned} (x^2 + 6x) + (y^2 - 4y) &= k \\ (x^2 + 6x + 3^2 - 3^2) + (y^2 - 4y + (-2)^2 - (-2)^2) &= k \\ (x^2 + 6x + 9) + (y^2 - 4y + 4) &= k + 9 + 4 \\ (x+3)^2 + (y-2)^2 &= k+9+4 \end{aligned}$$

CIRCLE CENTRE IS  $(-3; 2)$  ✓

(ii) Hence, determine  $k$ .

$$\begin{aligned} k+9+4 &= (\text{RADIUS})^2 = (4)^2 = 16 \quad \checkmark \\ \therefore k &= 16 - 9 - 4 = 3 \text{ UNITS}^2 \end{aligned}$$

(2)

(f) The graph of  $f(x) = ax^2 + bx + c$ , where  $a \neq 0$ , has the following properties:

- Passes through the point  $(0, 4)$ .
- Has axis of symmetry  $x = -2$ .
- Has a maximum value of 10.

$$y = a(x+2)^2 + 10$$

$$a = -\frac{6}{4} \quad \checkmark$$

Determine  $a$ ,  $b$  and  $c$ .

$$f(x) = ax^2 + bx + c$$

PASSES THROUGH  $(0; 4)$

$$\therefore 4 = a(0)^2 + b(0) + c$$

$$\underline{c = 4} \quad \checkmark$$

AXIS OF SYMMETRY

$$x = -2 = -\frac{b}{2a}$$

$$\therefore -b = -4a$$

$$\underline{b = 4a} \quad \checkmark$$

MAXIMUM VALUE OF 10

(5)

$\therefore$  TURNING POINT IS  $(-2; 10)$

$$10 = a(-2)^2 + b(-2) + 4$$

$$10 = 4a - 2b + 4$$

$$10 = 4a - 2(4a) + 4 \quad (\text{SUB. } b=4a)$$

$$10 = 4a - 8a + 4$$

$$-4a = 6 \quad \therefore \quad a = -\frac{6}{4} \quad \checkmark$$

$$\text{Page 18 of 18} \quad \therefore b = 4a \text{ GIVES } b = 4\left(-\frac{6}{4}\right)$$

$$\underline{b = -6} \quad \checkmark$$