UNIVERSITY OF KWAZULU-NATAL
JUNE 2008 TEST
COURSE AND CODE: FOUNDATION MATHEMATICS (MATH 099)
07 JUNE 2008

DURATION: 3 HOURS **MARKS: 140**

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PLEASE NOTE:

1. This paper consists of 15 pages, including this cover page. Check that you have them all.
2. Answer all questions and show working details.
3. Write your answers in the space provided. If the provided space is not enough, you can use the reverse side of the previous page, ie. opposite where you are working.
4. Write legibly in dark blue or black pen.
5. **Write your student number in the space provided below.**

STUDENT NUMBER:...

STUDENT SIGNATURE:.....................................

SEAT NUMBER:...

Question Number	1	2	3	4	5	6	7	8	Total
Total	18	25	15	14	15	21	10	22	140
Internal Marks	LW	LW	LW	LW					
External Marks									

Question 1: (18 marks)

Circle only the letter that is next to the correct answer for each of the following:

1.1 Given $x - 2 \leq 4 + 4x$ then:

A $x \leq 2$

B $x \geq 2$

C $x \leq -2$

D $x \geq -2$

1.2 Given the sequence (number pattern) : 1 ; 3 ; 6 ; 10 ; 15 ;... The next number is

A 19

B 20

C 21

D 22

1.3 If 3 a = 5 b = 4 c and a, b and c are natural numbers , then

A a > b > c.

B b > c > a.

C c > b > a.

D a > c > b.

1.4 Which statement is false?

A A prime number has only two factors.

B The sum of odd numbers is even

C All irrational numbers are real numbers

D $\sqrt{2}\sqrt{18}$ is rational

1.5 If P = { a, b, c, d }, then the number of sub-sets is :

A 4 B 8

C 16 D 32

1.6 If the line $g(x) = 3 - 2x$ is reflected about the y axis then the new equation is:

A $x = 2y - 3$ B $y = 2x + 3$

C $y = 2x - 3$ D $x = 2y - 3$

Question 2: (25 marks)

2.1 (a) Define a rational number. (2)

"Z divided by 2" 1 mark.

A rational number can be expressed in the form $\frac{p}{q}$, $\underbrace{p, q \in \mathbb{Z}, q \neq 0}$

need this!

(b) Show that 1, 2 57575757 … is a rational number. (4)

Let $x = 1.2575757\ldots$ $\ldots ①$

$100x = 125.757575\ldots$ $\ldots ②$

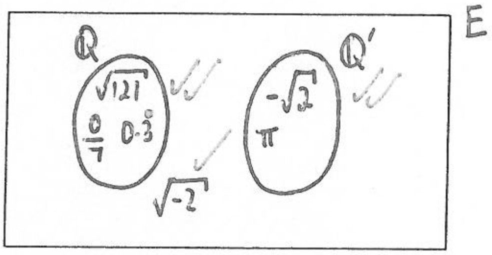
② - ①: $99x = 124.5$

$x = \frac{1245}{990}$ which is of the form of a rational number

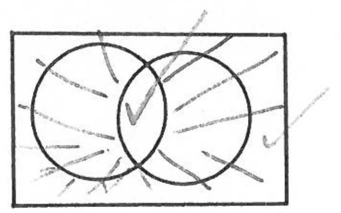
$\therefore$ 1.25757... is rational.

2.2 Given : $\sqrt{121}$ $\sqrt{-2}$ $-\sqrt{2}$ π $\frac{0}{7}$ $0.\dot{3}$

Draw a Venn- Diagram and show the above numbers in their correct places using the sets of **Rational numbers and Irrational numbers.** (5)



2.3 Use the diagram below and illustrate $(A \cap B)'$ (2)



2.4 Show that the sum of three odd natural numbers is odd. (4)

Let the 3 odd natural numbers be

$2a+1$, $2b+1$ and $2c+1$, $a, b, c \in \mathbb{N}_0$

$2a+1 + 2b+1 + 2c+1$ ½

$= 2a+2b+2c+3$

$= \underbrace{2(a+b+c+1)}+1$ which is of the form of an odd number.

$\therefore$ The sum of 3 odd natural numbers is odd. ½

If $\mathbb{N}$ or ... instead of $\mathbb{N}_0$?

$2(a+b+c)+3$ 1 mark.

If $2x+1$, $2x+3$, $2x+5$ Then ½ mark.

If x, $x+2$, $x+4$ then 0.

2.5 Study the data in the table below and calculate the values of p and v. (4)

12	p	0,15	100
0,6	2,4	48	v

$12 \times \frac{6}{10} = 7.2$ ½

$\frac{15}{100} \times \frac{48}{1} = \frac{15 \times 12}{25} = \frac{36}{5} = 7.2$ ½ ∴ Inverse proportion

$p \times \frac{24}{10} = \frac{72}{10}$ and $100v = 7.2$

$p = 3$ $v = 0.072$

2.6 A lift can safely carry 12 adults each with an average mass of 95 kg. How many teenagers with an average mass 65 kg can it safely carry? (4)

as mass ↓ , no. of people ↑ (inverse proportion)

$12 \times 95 = 65 \times x$ inverse proportion

Let the no. of teenagers of average mass 65kg be x.

$x = \frac{12 \times 5 \times 19}{5 \times 13}$

If $x = 17$ at this stage, then max 3.

$x = 17.538...$

∴ we choose a max. of 17 people (teenagers) to safely be on the lift.

Question 3:(15 marks)

3.1 Solve for t and illustrate your solution graphically.

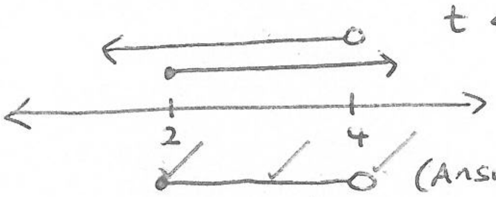
$$t + 2 \leq 4(t-1) < 12$$

(5)

$t+2 \leq 4t-4$ and ~~$t+2 \leq$~~ $t-1 < 3$

$6 \leq 3t$ $\qquad t < 4$

$2 \leq t$



(Answer)

$2 \leq t < 4, \quad t \in \mathbb{R}$

~~If diagram missing 3~~

$10 \times \frac{1}{2} = 5$

3.2 Solve for d and illustrate your solution using interval notation.

$$\frac{d}{d+4} > \frac{1}{d-2}$$

(7)

$$\frac{d(d-2) - (d+4)}{(d+4)(d-2)} > 0$$

If only to this step, then max ½ + int. not.
If mult by denom. then 0.

$$\frac{d^2 - 3d - 4}{(d+4)(d-2)} > 0$$

$$\frac{(d-4)(d+1)}{(d+4)(d-2)} > 0$$

Critical values: $\pm 4, -1, 2$

Restriction: $d \neq -4$ & 2

order

		-4		-1		2		4	
$d-4$	$-$		$-$		$-$		$-$	o	$+$
$d+1$	$-$		$-$	o	$+$		$+$		$+$
$d+4$	$-$	o	$+$		$+$		$+$		$+$
$d-2$	$-$		$-$		$-$	o	$+$		$+$
$\frac{(d-4)(d+1)}{(d+4)(d-2)}$	$+$	u	$-$	o	$+$	o	$-$	o	$+$

Solution: $d \in (-\infty, -4) \cup (-1, 2) \cup (4, \infty)$

$14 \times \frac{1}{2} = 7$

3.3 If $px + 2 < qx$ determine x. (3)

$px - qx < -2$

$x(p-q) < -2$

if $p - q > 0$ then $x < \frac{-2}{p-q}$

$6 \times \frac{1}{2} = 3$

or if $p-q < 0$ then $x > \frac{-2}{p-q}$

Question 4 : (14 marks)

4.1 Determine the centre and the radius of the circle given the equation:

$y^2 + 2y + x^2 - 4x = 11.$ (4)

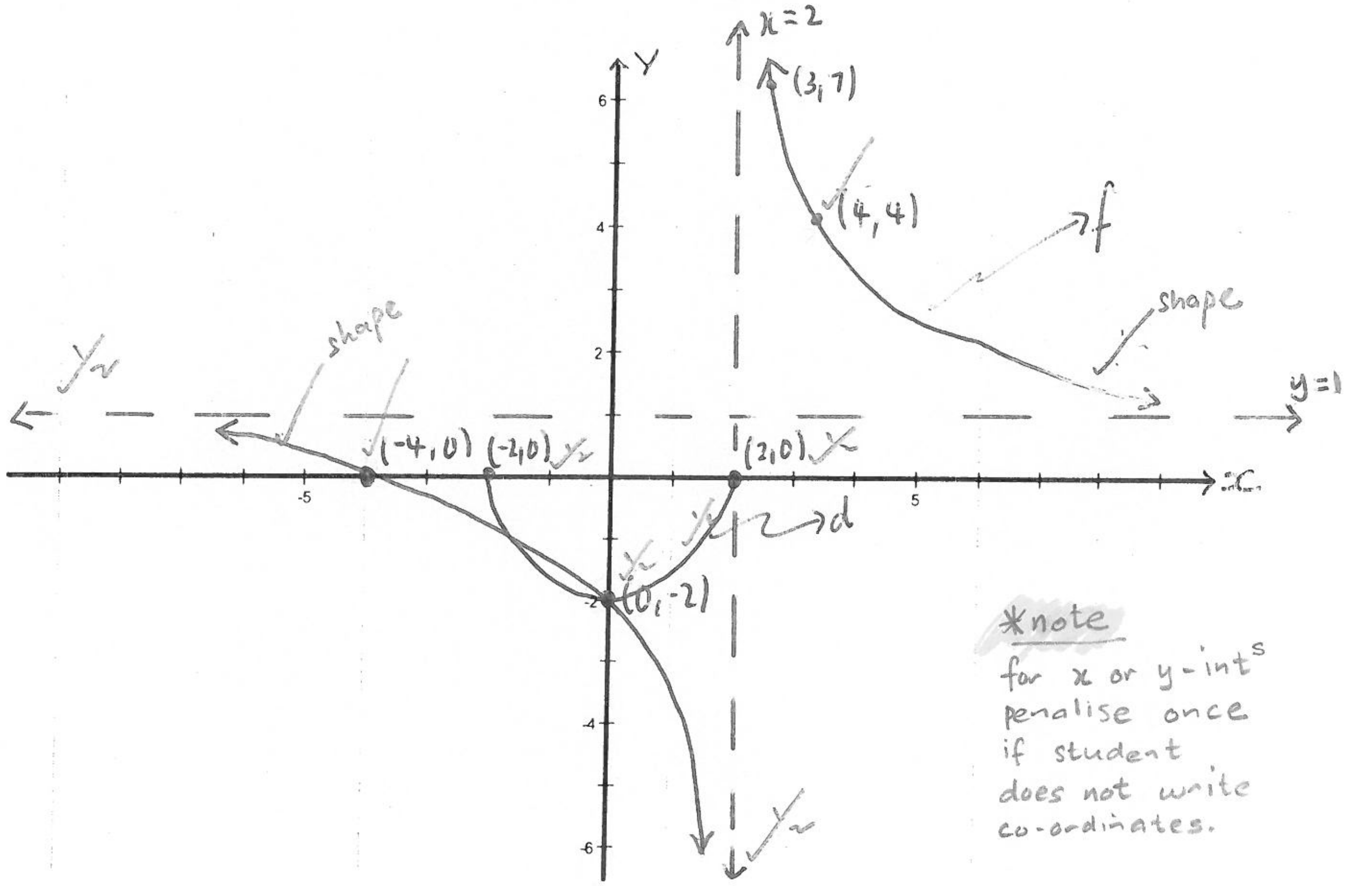
$x^2 - 4x + 4 + y^2 + 2y + 1 = 11 + 5$

$(x-2)^2 + (y+1)^2 = 16$

$\odot(2, -1)$ and radius = 4 units

4.2 a) Sketch the following : $f(x) = \dfrac{6}{x-2} + 1$ and $d(x) = -\sqrt{4-x^2}$.

(Make sure that you clearly show all intercepts and asymptotes.) (7)



*note
for x or y-ints penalise once if student does not write co-ordinates.

4.2 b) Determine the inverse of $f(x) = \dfrac{6}{x-2} + 1$ i.e $f^{-1}(x)$.

Reflect $y = \dfrac{6}{x-2} + 1$ about $y = x$ (3)

to get $x = \dfrac{6}{y-2} + 1$ ✓

$$x - 1 = \frac{6}{y-2}$$

$$y - 2 = \frac{6}{x-1}$$

$$y = \frac{6}{x-1} + 2 \checkmark$$

$$f^{-1}(x) = \frac{6}{x-1} + 2 \checkmark$$

QUESTION 5: (15 marks)

5.1 The table below represents a stage in a Gauss reduction calculation.

x	y	z	1
2	3	1	4
0	4	2	6
0	0	-1	5

Determine the value(s) of x, y and z. (4)

$$-z = 5$$
$$z = -5 \checkmark$$
$$4y + 2(-5) = 6$$
$$4y = 16$$
$$y = 4 \checkmark$$
$$2x + 12 - 5 = 4$$
$$2x = -3$$
$$x = -1\tfrac{1}{2} \text{ OR } \tfrac{-3}{2} \checkmark\tfrac{1}{2}\ \checkmark\tfrac{1}{2}$$

Solution $= \{(x, y, z) \in \mathbb{R}^3 : (-1\tfrac{1}{2}, 4, -5)\}$ $\checkmark\tfrac{1}{2}$

$\checkmark\tfrac{1}{2}$

5.2 Solve for p if

$$2p + q + 2r = 4$$
$$q + r = 1$$

(6)

p	q	r	1
2	1	2	4
0	1	1	1
0	0	0	0

Parametric Solution

Let $r = t$, $t \in \mathbb{R}$

$q = 1 - t$

$2p + 1 - t + 2t = 4$

$2p = 3 - t$

$p = \frac{3-t}{2}$

Solution $= \{ (p, q, r) \in \mathbb{R}^3,\ t \in \mathbb{R} : \left(\frac{3-t}{2}, 1-t, t\right) \}$

5.3 A parabola $h(x)$, has range $(-\infty, 1]$ and line of symmetry $x = -2$. If the parabola cuts the y – axis at -3 , determine the equation of the parabola. (5)

Turning pt. $(-2, 1)$

$y = a(x+2)^2 + 1$

Subst. $(0, -3)$

$-3 = a(0+2)^2 + 1$

$a = -1$

$\therefore y = -(x+2)^2 + 1$ is the required equation

Question 6 : (21 marks)

Given : $t(x) = \dfrac{-2}{\sqrt{6-4x}}$, $f(x) = -2x^2 + 5x + 3$.

and $g(x) = 6 - 2x$.

6.1 Determine the domain of t. (2)

$6 - 4x > 0$

$6 > 4x$

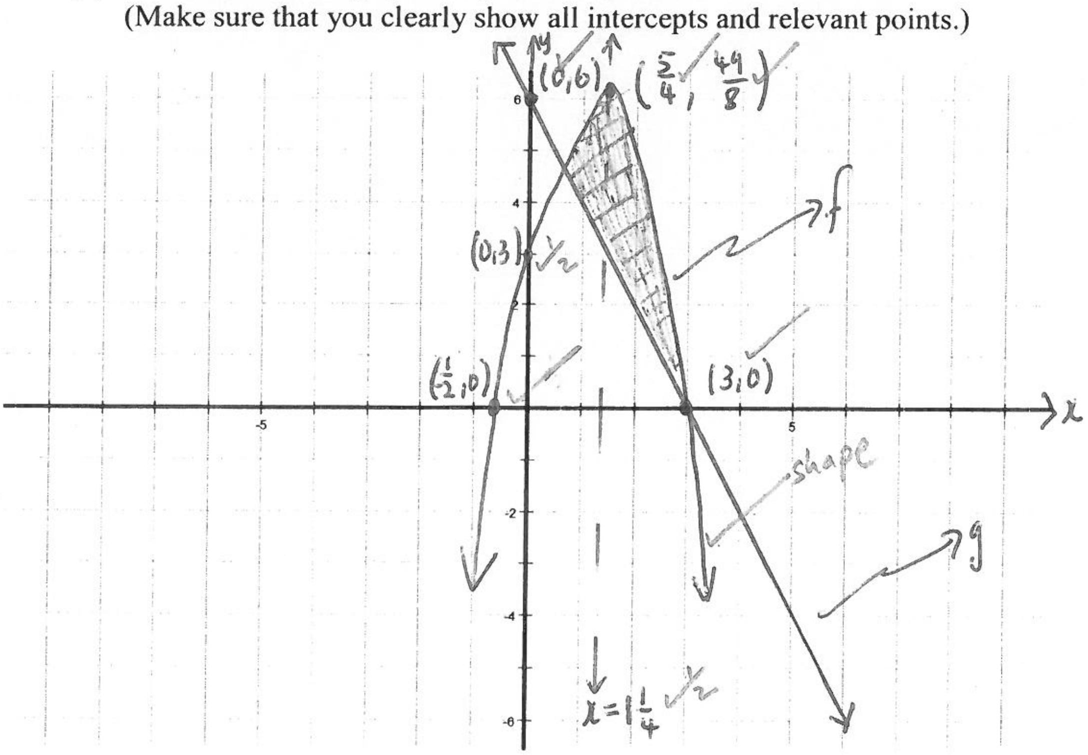
$1\frac{1}{2} > x$, $x \in \mathbb{R}$

6.2 Calculate : $(t \circ g)(2.5)$. (4)

$g(2.5) = 6 - 2(2.5) = 6 - 5 = 1$

$(t o g)(2.5) = t(g(2.5)) = t(1) = \dfrac{-2}{\sqrt{6-4(1)}} = \dfrac{-2}{\sqrt{2}} = \dfrac{-2\sqrt{2}}{2} = -\sqrt{2}$.

6.3 (a) Make neat sketch graphs of f and g below: (7)
(Make sure that you clearly show all intercepts and relevant points.)



$x = \dfrac{-5}{-4} = 1\frac{1}{4}$ and $f(\frac{5}{4}) = \dfrac{-2}{1} \times \dfrac{25}{16} + \dfrac{25}{4} + 3$

$= \dfrac{-25 + 50 + 24}{8}$

$= \dfrac{49}{8}$

$= 6\frac{1}{8}$

$2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$

(b) Write down the range of f. (2)

$y \leq \frac{49}{8}$; $y \in \mathbb{R}$

(c) Explain how you would use the graphs to solve the equation : (4)

$2x^2 + 3 = 7x.$

$-2x^2 - 3 = -7x$

$-2x^2 + 5x - 3 = -2x$

$-2x^2 + 5x + 3 = -2x + 6$

$f(x) = g(x)$

The x-co-ordinate of the point of intersection of the graphs of f and g represent the solution to $2x^2 + 3 = 7x$.

(d) Shade, on the graphs you sketched above, the region
$\{(x, y) \in R^2 : y \leq f(x)\} \cap \{(x, y) \in R^2 : y \geq g(x)\}$. see graph. (2)

QUESTION 7 : (10 marks)

7.1 Jabu is 8 years younger than Nonhlanhla. Two years ago, Jabu's age was half of Nonhlanhla's age. Determine, by constructing suitable equation(s), Jabu's present age. (4)

Let Jabu's present age be x years ½

$(x - 2) = \frac{1}{2}(x + 8 - 2)$

$2x - 4 = x + 6$

$x = 10$

Jabu is 10 years old.

7.2 The distance between town A and town B is 800 km. Jabu leaves town A to go to town B, whilst at the same time, Cyril leaves town B to go town A. Jabu and Cyril travel along the same road and meet after 5 hours. If Jabu was travelling twice as fast as Cyril, determine, by constructing suitable equation(s), how fast each of them were travelling? (6)

Let Cyril's average speed be x km/h ✓½

Jabu

speed = $2x$ km/h
time = 5 h
distance = $10x$ km

Cyril

speed = x km/h
time = 5 h
distance = $5x$ km

Total distance = 800

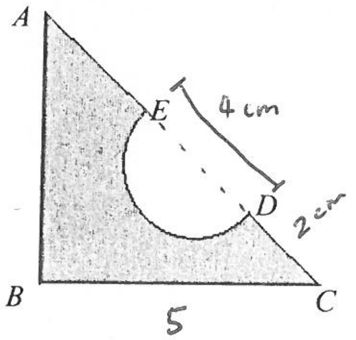
$10x + 5x = 800$ ✓ ✓ ½ ½

$15x = 800$ ½ $\therefore$ $x = 53\frac{1}{3}$ ✓

$\therefore$ Cyril travelled at $53\frac{1}{3}$ km/h and Jabu at $106\frac{2}{3}$ km/h. ✓

QUESTION 8 : (22 marks)

8.1 In the diagram below, triangle ABC is right angled at B. DE is the diameter of the semi-circle. AC = 13 cm, BC = 5 cm, DE = 4 cm and DC = 2 cm.
Determine the perimeter of the shaded region. (5)
(Leave your answer in terms of π.)



$AE = 13 - 6 = 7$ cm

$\overset{\frown}{DE} = \frac{1}{2}(2\pi r) = \pi(2) = 2\pi$ cm ½ ½ ½

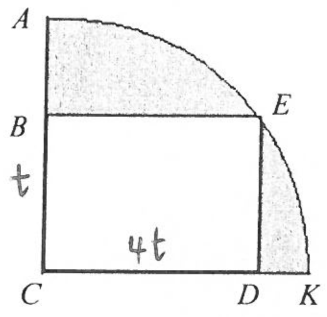
$AB = 12$ cm (Pythagoras) ½

Perimeter of shaded region

$= (7 + 2\pi + 2 + 5 + 12)$ cm ½ ½ ½

$= (26 + 2\pi)$ cm ½ ½ ½

8.2 ACK represents a quarter circle with centre C. $BCDE$ is a rectangle with $CD = 4t$ mm and BC = t mm. $BC : CK = 1 : 4$. If the area of the shaded region equals $(16\pi - 16)$ mm^2 then determine t. (5)



$$\frac{BC}{CK} = \frac{1}{4}$$

$CK = 4t$ mm

area of shaded region

$$= \frac{1}{4}(\pi(4t)^2) - t \times 4t$$

$$= 4\pi t^2 - 4t^2$$

$$\therefore \quad 4(\pi t^2 - t^2) = 16(\pi - 1)$$

$$t^2 = \frac{16(\pi - 1)}{4(\pi - 1)} = 4$$

$$t = 2 \quad (t > 0)$$

8.3 A cylinder has radius 2 cm and height h cm. A cube, with edge 4 cm, has volume equal to 25% of the volume of the cylinder. Determine h. (5)

Volume of cube = 25% of volume of cylinder

$$64 = \frac{1}{4} \times \pi(2)^2 h$$

$$64 = \pi h$$

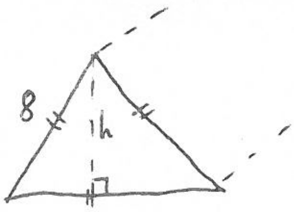
$$h = \frac{64}{\pi} \text{ cm}$$

8.4 A rectangular prism has length 5 cm, width 4 cm and height 6 cm. A triangular prism has an equilateral triangle as a base. The sides of the equilateral triangle are 8 cm.

a) Determine the total surface area of the rectangular prism. (2)

$$\text{TSA of rect. prism} = 2(5\times 4) + 2(5\times 6) + 2(4\times 6)\ \text{cm}^2$$
$$= (40 + 60 + 48)\ \text{cm}^2$$
$$= 148\ \text{cm}^2$$

b) If the triangular prism has the same surface area as the rectangular prism then determine the height of the triangular prism. (5)



$$h = \sqrt{8^2 - 4^2} \quad \text{(Pythagoras)}$$
$$h = \sqrt{48}\ \text{cm}$$

$$\text{TSA of } \Delta \text{ prism} = 2\left(\tfrac{1}{2} \times 8 \times \sqrt{48}\right) + 3(8 \times \text{height})$$
$$148 = 8\sqrt{48} + 24 \times \text{height}$$
$$\text{height} = \frac{148 - 8\sqrt{48}}{24}\ \text{cm}$$
$$\text{height} = \frac{37 - 2\sqrt{48}}{6}\ \text{cm}$$