
Question One (10 marks)

Circle only the letter that is next to the correct answer for each of the following:

1.1 A natural number which is not prime and greater than one is called:

A	Even number	B	Odd number
C	composite number	D	undefined

1.2 The surface area of a closed cylinder (in terms of radius r, diameter d and height h) is given by the formula:

A	$\pi\ dh + 2\pi r^2$	B	$\pi\ dh + \pi r^2$
C	$2\pi\ dh + 2\pi r^2$	D	$2(\pi\ dh + 2\pi r^2)$

1.3 Simplify: $-(-3x^2)^3 =$

A	$-9x^5$	B	$-9x^6$
C	$-27x^5$	D	$27x^6$

1.4 If $P = \{0,1,2\}$, then the total number of sub-sets is:

A	4	B	6
C	8	D	{ }

1.5 Simplify: $|2-7| + 2^{-1}$

A	7.5	B	-7.5
C	5.5	D	-5.5

Question Two (26 marks)

2.1 Consider the following sets :

E, universal set of whole numbers less than 12;
A, set of integers between 0 and 10 that are multiples of 3;
B, set of prime numbers greater than 0 and less than 11;
C, set of odd numbers greater than 2 and less than or equal to 11.

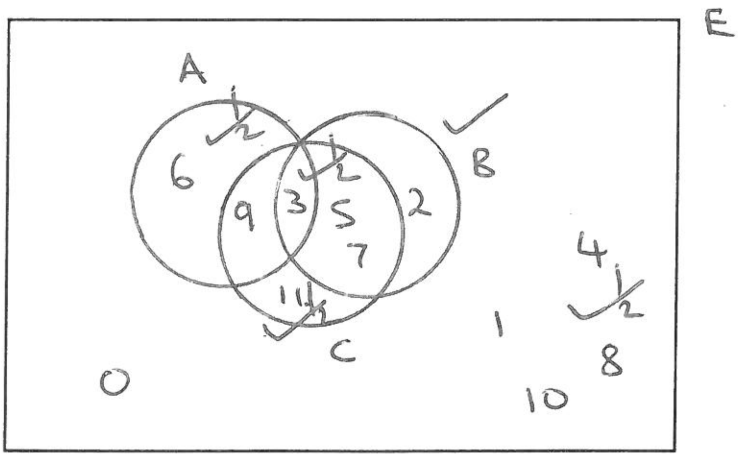
2.1.1 List the elements of sets A, B and C.

A = $\{3; 6, 9\}$

B = $\{2, 3, 5, 7\}$

C = $\{3, 5, 7, 9, 11\}$ (3)

2.1.2 Draw a Venn diagram, showing all the sets given above, and put the elements in their correct places



$-\frac{1}{2}$ FOR REGION WITH NO ELEMENTS

(3)

2.1.3 $n(B \cap C) = 3$ (1)

2.1.4 Determine $(B \cup C)'$ and illustrate your answer on the number line:

$(B \cup C)' = \{0, 1, 4, 6, 8, 10\}$ (3)

Full marks IF 0 NOT LISTED

2.2 Solve for x if :

$$\frac{n^x \cdot n^{\frac{3}{x}}}{n^{7-x}} = 1 \qquad (4)$$

$$n^{x+\frac{3}{x}} = n^{7-x} \quad ✓\tfrac{1}{2}$$

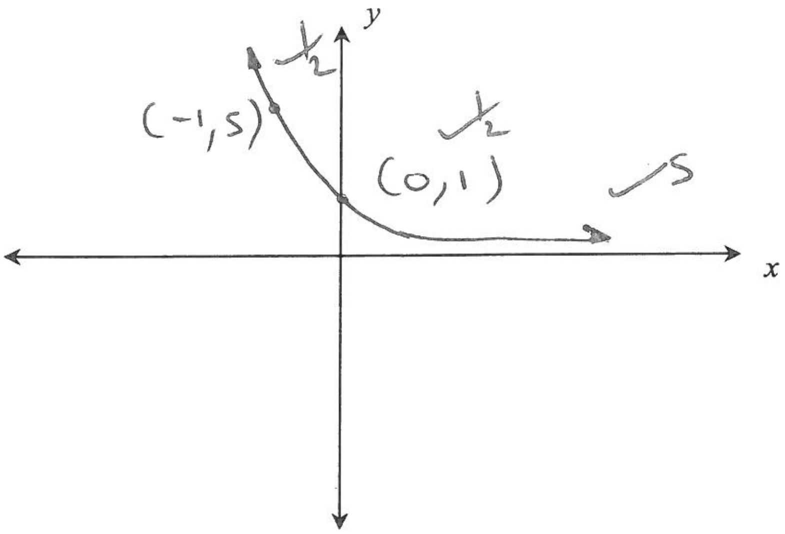
$$x + \frac{3}{x} = 7 - x \quad ✓$$

$$2x^2 - 7x + 3 = 0 \quad ✓\tfrac{1}{2}$$

$$(x-3)(2x-1) = 0 \quad ✓$$

$$x = 3 \ ✓\tfrac{1}{2} \quad \text{OR} \quad x = \tfrac{1}{2} \ ✓\tfrac{1}{2}$$

2.3.1 Sketch the graph of $f(x) = \left(\frac{1}{5}\right)^x$ on the set of axes below. (2)



2.3.2 If g is a reflection of f about the line $y = x$, determine the equation of g. (2)

LET $y = \left(\frac{1}{5}\right)^x$ THE INVERSE IS $f^{-1}(x)$

$$x = \left(\frac{1}{5}\right)^y \quad ✓$$

$$y = \log_{\frac{1}{5}} x \quad ✓\tfrac{1}{2}$$

ie. $g(x) = \log_{\frac{1}{5}} x$ ✓½

2.4 Prove that : $\log_b a = \dfrac{\log_c a}{\log_c b}$. (3)

LET $\log_b a = k$

$\therefore \quad a = b^k$ ✓

$\log_c a = k \log_c b$ ✓

$\therefore \quad k = \dfrac{\log_c a}{\log_c b}$ ✓½

ie. $\log_b a = \dfrac{\log_c a}{\log_c b}$ ✓½

2.5 Solve for m: $2\log_4 m + \log_m 4 = \log 1000$ (5)

LET $\log_4 m = k$

$\therefore \quad 2k + \dfrac{1}{k} = 3$

$2k^2 - 3k + 1 = 0$ ✓

$(2k-1)(k-1) = 0$ ✓

$k = \frac{1}{2}$ ✓½ OR $k = 1$ ✓½

ie $\log_4 m = \frac{1}{2}$ or $\log_4 m = 1$

$m = 2$ ✓ OR $m = 4$ ✓

Question Three (16 marks)

3.1 Solve the following system of equations:

$$\begin{aligned} x - 2y + z &= 2 \\ 2x + y + 3z &= -1 \\ 3x - 6y + 3z &= 6 \end{aligned}$$

(6)

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ 2 & 1 & 3 & -1 \\ 3 & -6 & 3 & 6 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ 0 & 5 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

LET $z = t$, $t \in \mathbb{R}$

$5y + t = -5$

$y = -\frac{t}{5} - 1$

$x + \left(\frac{t}{5} + 1\right) + t = 2$

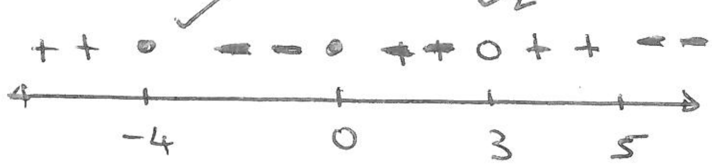
$x = -\frac{7t}{5}$

3.2 Solve for x: $\dfrac{x^2 + x - 12}{5 - x} < 0$

and write your answer in set builder notation.

(6)

$\dfrac{(x+4)(x-3)}{5-x} < 0$ CV's: $-4, 3, 5$



SOLUTION: $\{x \in \mathbb{R} \mid -4 < x < 3\} \cup \{x \in \mathbb{R} \mid x > 5\}$

3.3 The instruction for mixing a certain type of concrete is 1 part cement, 2 parts sand, and 3 parts gravel. If there are $4m^3$ of sand how much cement and gravel should you mix with this sand? (4)

LET THE RATIO OF CEMENT : SAND : GRAVEL BE

$C : S : G = 1 : 2 : 3$ ✓

BUT $C : 4 : G = 2 : 4 : 6$ ✓

ANS : $2m^3$ ✓ OF CEMENT AND $6m^3$ ✓ OF GRAVEL

Question Four: (17 marks)

4.1 Solve: $|\ 2x-3\ | > 5.$ (4)

$2x-3 > 5$ ✓ OR $2x-3 < -5$ ✓

$x > 4$ ✓ OR $x < -1$ ✓

4.2 Express $-2 < x < 3$ in the form $|ax + b| < c$. (3)

THE AVERAGE OF -2 AND 3 IS $\frac{1}{2}$ ✓

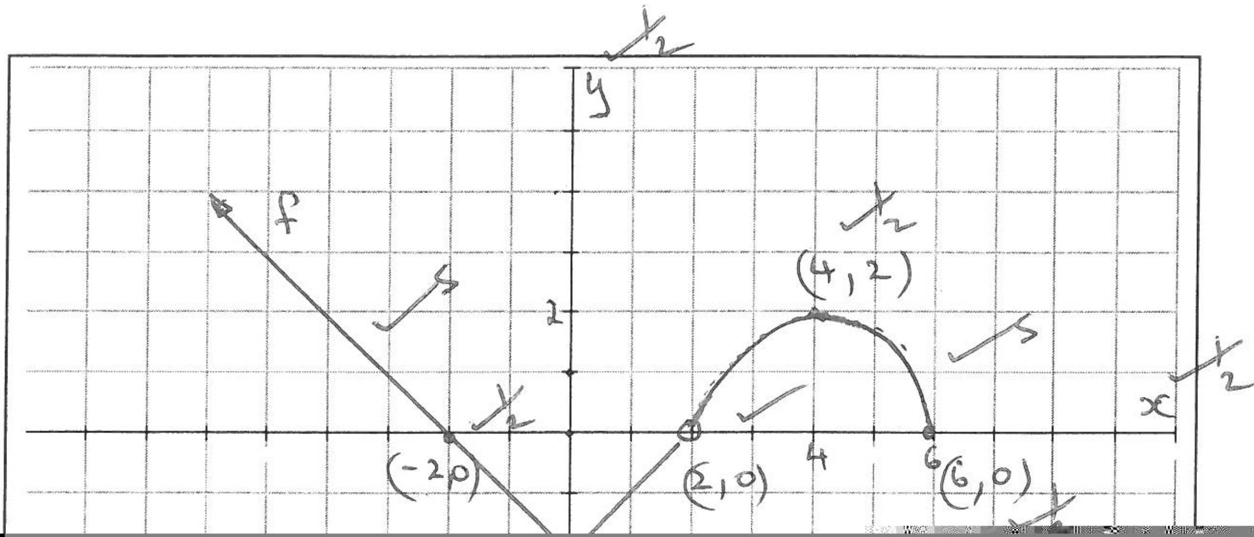
So $-2-\frac{1}{2} < x-\frac{1}{2} < 3-\frac{1}{2}$ ✓

$\therefore |x-\frac{1}{2}| < 2\frac{1}{2}$ ✓

4.3.1 Sketch the graph of f on the set of axes provided below.

$$f(x) = \begin{cases} |x| - 2, & x < 2, \\ \sqrt{4 - (x-4)^2}, & x > 2. \end{cases}$$

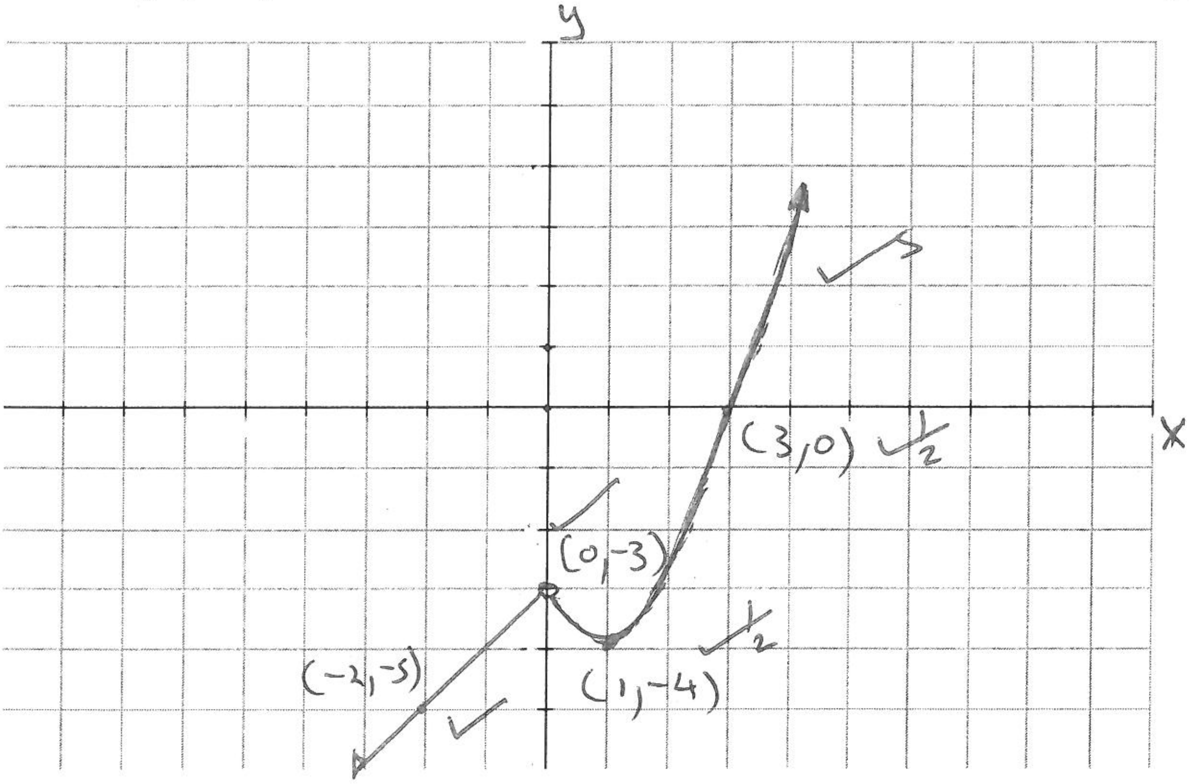
(6)



Question Five : (29 marks)

5.1 Given: $f(x)=\begin{cases}(x-1)^2-4, & x>0\\ x-3, & x<0\end{cases}$

5.1.1 Sketch the graph of f. (4)



5.1.2 Use the graph to find $\lim_{x\to 0} f(x)$. Justify your answer. (2)

$\lim_{x\to 0^+} f(x) = -3$ ✓½

$\lim_{x\to 0^-} f(x) = -3$ ✓½

$\therefore \lim_{x\to 0} f(x) = -3$ ✓

5.2 Given: $f(x) = -3x^2$ determine $f'(x)$ from first principles. (5)

$$f(x+h) = -3(x+h)^2 = -3x^2 - 6xh - 3h^2$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{-6xh - 3h^2}{h}$$

$$= \lim_{h \to 0} (-6x - 3h)$$

$$= -6x$$

5.3 Consider: $f(x) = x^8 - \dfrac{2}{\sqrt{x}}$ and $g(x) = \left(x^{12} - 3x^{-2}\right)\left(x^{12} + 3x^{-2}\right)$

Determine:

5.3.1 $f'(x)$ $= 8x^7 - 2\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}$ (3)

$$= 8x^7 + x^{-1\frac{1}{2}}$$

5.3.2 $D_x[g(x)]$ (3)

$$g(x) = x^{24} - 9x^{-4}$$

$$D_x[g(x)] = 24x^{23} + 36x^{-5}$$

5.3.3 Determine the equation of the tangent to the curve of f at $x = 1$. (4)

IF $x = 1$, $y = -1$ $(1, -1)$

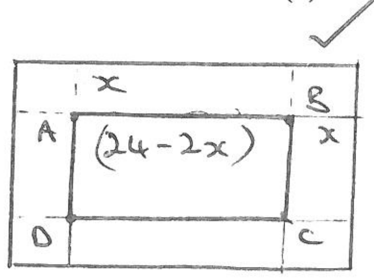
y' AT $x = 1$: $m = 8 + 1 = 9$

$\therefore \quad y + 1 = 9(x - 1)$

$y = 9x - 10$

5.4 A rectangular container which is open at the top, is to be made from a square metal sheet which is 24 m x 24 m in dimension. The container is made by cutting out equal squares from each corner of the sheet and turning up the sides. Find the volume of the largest container that can be made this way. (8)

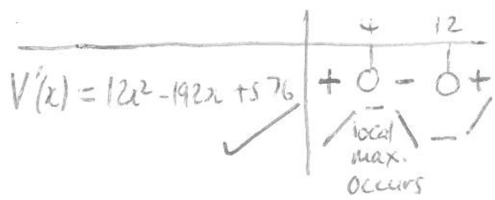
$V = (24 - 2x)^2 \cdot x$



$V(x) = 576x - 96x^2 + 4x^3$

$V'(x) = 576 - 192x + 12x^2$

FOR $V'(x) = 0$



$12x^2 - 192x + 576 = 0$

$x^2 - 16x + 48 = 0$

$(x - 4)(x - 12) = 0$

$x = 4$ OR $x = 12$ (N/A)

$\therefore$ max. volume $= 1024\ m^3$

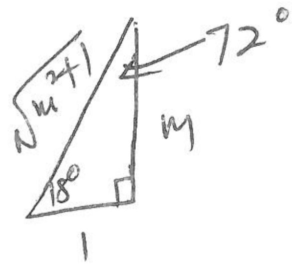
6.1.1

$$\cos 144^\circ = \cos(2\times 72^\circ)$$

$$= 2\cos^2 72^\circ - 1$$

$$= \frac{2(m)^2}{m^2+1} - 1$$

$$=$$



Q 6.1.2

$$\cot 36^\circ = \frac{1}{\tan 36^\circ}$$

$\frac{1}{2}$ $\frac{1}{2}$

$$= \left(\frac{2\tan 18^\circ}{1-\tan^2 18^\circ}\right)^{-1}$$

$$= \left(\frac{2m}{1-m^2}\right)^{-1}$$

$$= \frac{1-m^2}{2m}$$

6.2 Simplify without using a calculator:

$$\frac{(1-\cos^2 10°)\sin 810° \tan(-225°)}{\sec 240° \sin^2 190°}$$ (6)

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

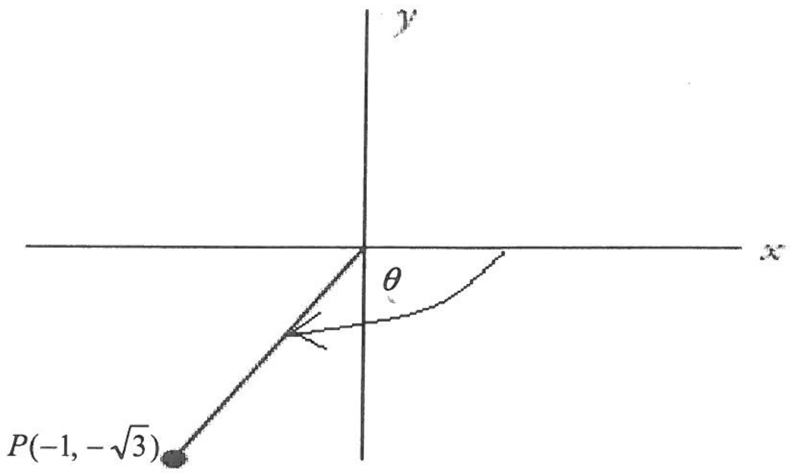
$$= \frac{\sin^2 10° \cdot \sin 90° (-\tan 45°)}{-\sec 60° (-\sin^2 10°)}$$

$\frac{1}{2}$ $\frac{1}{2}$

$$= \frac{1 \cdot (-1)}{2}$$ ✓ ✓

$$= \frac{1}{2}$$ ✓

6.3 In the diagram below, P is a point on the terminal arm of θ .



6.3.1 Evaluate $\tan\theta$. $= \sqrt{3}$ ✓ (1)

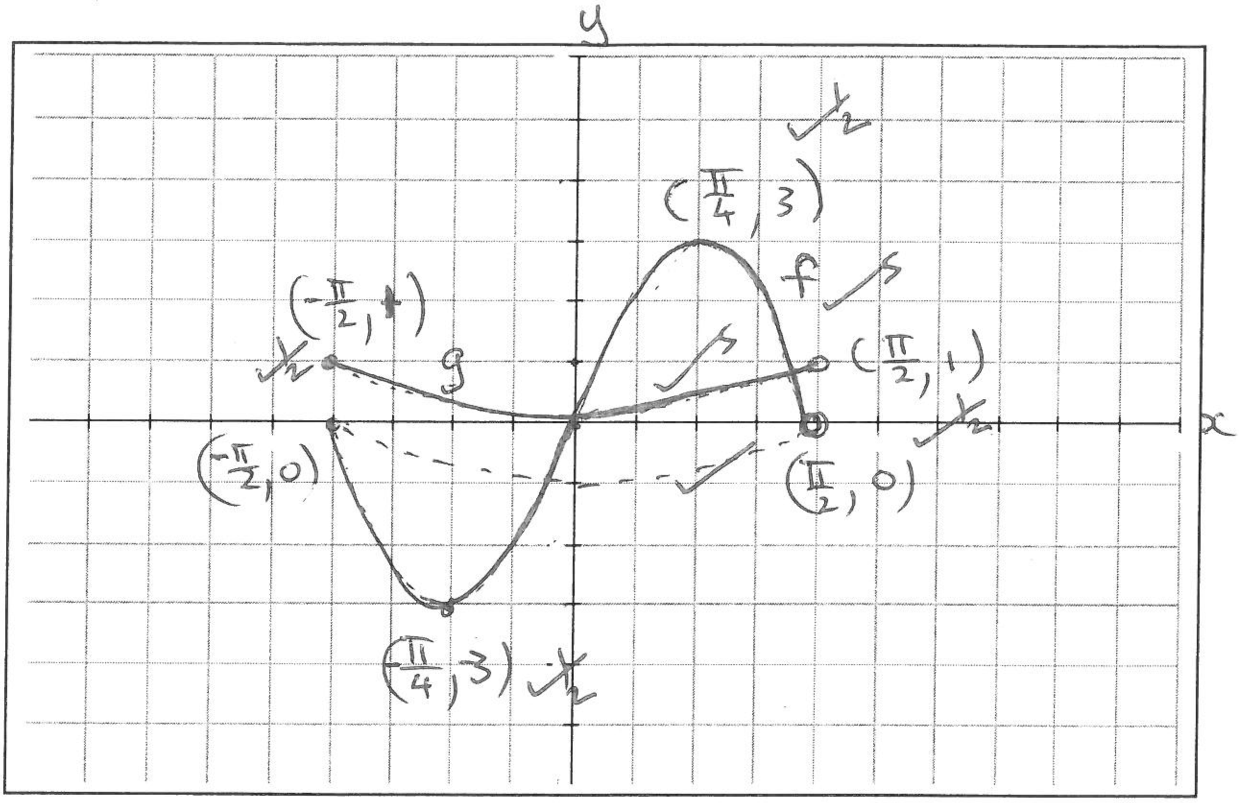
6.3.2 Hence, **without using a calculator**, determine
θ if $\theta \in R$. (3)

$$\theta = -\pi + \frac{\pi}{3}$$ ✓ ✓

$$= -\frac{2\pi}{3}$$ ✓

6.4 Consider $f(\theta)=3\sin 2\theta, -\frac{\pi}{2}\le\theta<\frac{\pi}{2}$ and $g(\theta)=1-\cos\theta, -\frac{\pi}{2}\le\theta<\frac{\pi}{2}$.

6.4.1 Sketch the graph of *f* and *g* on the set of axes provided below. (5)



6.4.2 Write down the range of $y=f\left(\theta+\frac{\pi}{8}\right)$ using interval notation. (2)

½ ½

$y \in [-3, 3]$

½ ½

6.5 Determine the general solution (in radian measure) of the equation :

$$2\sin^2\theta = -\sin 2\theta. \qquad (6)$$

$2\sin^2\theta + 2\sin\theta\cos\theta = 0$ ✓

$\sin\theta(\sin\theta + \cos\theta) = 0$

$\sin\theta = 0$ ✓½ OR $\tan\theta = -1$ ✓½

REF. $\angle = 0$ ✓½ REF. $\angle = \frac{\pi}{4}$ ✓½

$\theta = 0 + 2k\pi$ ✓½, $k \in \mathbb{Z}$ ✓½

$\theta = \pi + 2k\pi$ ✓½, $k \in \mathbb{Z}$ ✓½ OR $\theta = \frac{3\pi}{4}$ ✓½ $+ \pi k$, $k \in \mathbb{Z}$ ✓½

6.6 Prove the identity :

$$\frac{2\sin^2 x}{2\tan x - \sin 2x} = \cot x. \qquad (6)$$

$$LHS = \frac{2\sin^2 x}{\frac{2\sin x}{\cos x} - 2\sin x\cos x}$$ ✓½ ✓

$$= \frac{2\sin^2 x\cos x}{2\sin x - 2\sin x\cos^2 x}$$ ✓ ✓

$$= \frac{2\sin^2 x\cos x}{2\sin x(1-\cos^2 x)}$$ ✓

$$= \frac{\sin x\cos x}{\sin^2 x}$$ ✓½

$$= \frac{\cos x}{\sin x}$$ ✓½ $= \cot x = RHS$ ✓½

$$\therefore \frac{2\sin^2 x}{2\tan x - \sin 2x} = \cot x$$

Question Seven : (15 marks)

7.1 A number has two digits whose sum is 11. If the digits are reversed then the difference between the numbers is 9. Determine the original number. (5)

Let the number be $10x + y$. ✓

Reverse the digits, the no. is $10y + x$ ✓½

$x + y = 11$ ✓½ — ①

AND $9x - 9y = 9$ ✓ — ②

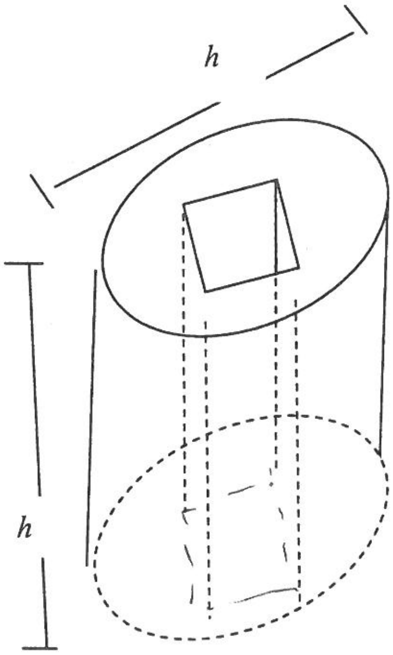
$\therefore$ $18y = 90$ ✓½

$y = 5$ AND $x = 6$ ✓½

THE ORIGINAL NO. IS 65 ✓

7.2 Study the diagram given alongside carefully.

- It is a circular wooden solid with square hole drilled down the length, h, as shown in the diagram.
- Each side of the square hole drilled is $\frac{h}{4}$.
- The length of the solid is equal to the diameter of the solid.



7.2.1 Show that the volume of the solid in terms of π and h can be written as:

$$\frac{h^3}{16}(4\pi - 1) \qquad (4)$$

VOLUME $= \pi r^2 h - s^2 h$ ✓

$= \frac{\pi h^3}{4}$ ✓ $- \frac{h^3}{16}$ ✓

$= \frac{h^3}{16}(4\pi - 1)$ ✓

$\left\{ \begin{array}{l} r = \frac{h}{2} \\ \text{AND } s = \frac{h}{4} \end{array} \right.$

7.2.2 Determine the total surface area of the solid in terms of π and h.
Hint: this means inside and out. (6)

① AREA OF OUTSIDE (LATERAL S.A)

$= 2\pi r h$ ✓½

$= \pi h^2$ ✓

② AREA OF BASE $= \pi\left(\frac{h}{2}\right)^2 - \left(\frac{h}{4}\right)^2$

$= \frac{\pi h^2}{4} - \frac{h^2}{16}$ ✓

②×2 $= \frac{\pi h^2}{2} - \frac{h^2}{8}$ ✓

③ AREA OF INSIDE

$= 4lb$

$= 4h\left(\frac{h}{4}\right)$ ✓½

$= h^2$ ✓

TOTAL AREA $= \pi h^2 + \frac{\pi h^2}{2} - \frac{h^2}{8} + h^2$

$= \left(\frac{3}{2}\pi h^2 + \frac{7}{8}h^2\right)$ ✓ units^2

The end