UNIVERSITY OF KWAZULU-NATAL
JANUARY 2009 SUPPLEMENTARY EXAMINATION
COURSE AND CODE: FOUNDATION MATHEMATICS (MATH099/199)

DURATION: 3 HOURS **MARKS: 150**

INTERNAL EXAMINER: Mr S. Pillay & P. Latchmanan
EXTERNAL EXAMINER: Dr O.K. Narain

PLEASE NOTE:

1. This paper consists of 17 pages, including this cover page. Check that you have them all.
2. Answer all questions and show working details.
3. Write your answers in the space provided. If the provided space is not enough, you can use the reverse side of the previous page, ie. opposite where you are working.
4. Write legible in dark blue or black pen.
5. **Write your student number in the space provided below.**

STUDENT NUMBER:.......................................

STUDENT SIGNATURE:.................................

SEAT NUMBER:.......................

Question Number	1	2	3	4	5	6	7	8	Total
Total	18	18	15	25	10	26	30	8	150
Internal Marks									
External marks									

QUESTION ONE (18 marks)

Circle only the letter that is next to the correct answer for each of the following:
Nb: there will be a penalty for incorrect answers.

1.1 $(a-b)^3 = \ldots\ldots\ldots\ldots$

A. $a^3 - b^3$

B. $a^3 - 3a^2b - 3ab^2 - b^3$

C. $a^3 - 3a^2b + 3ab^2 - b^3$

D. $a^3 + 3a^2b + 3ab^2 - b^3$

1.2 $|3-7| - 2^0 = \ldots\ldots\ldots$

A. 1

B. 2

C. 3

D. -2

1.3 Given the statements: (i) 2 is a prime number.
(ii) 6 is a multiple of 18.
* ~~(iii)~~ → no statement (iii).

A. Both are true

B. Only (i) is true

C. Only (ii) is true

D. Both are false

1.4 $(12x)^2\left(-\frac{1}{2}\right)^3 = \ldots\ldots\ldots\ldots$

A. x^2

B. $-16x^2$

C. $16x^2$

D. $8x^2$

1.5 The total surface area of a closed box with dimensions height = 5 cm, length = 3 cm and width = 2 cm is:

A. 62 cm

B. 30 sq. cm

C. 10 cm

D. 62 sq. cm

1.6 If $f(x) = 2x - 6$ then $f^{-1}(x) = \ldots\ldots\ldots\ldots$

A. $6x - 2$

B. $6x + 4$

C. $0.5x + 3$

D. $0.5x - 3$

QUESTION TWO (18 marks)

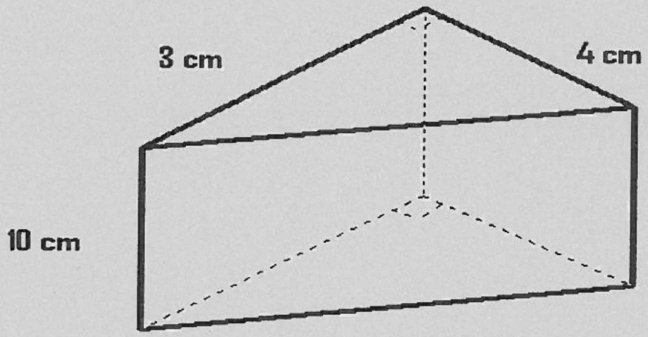


Fig. 1

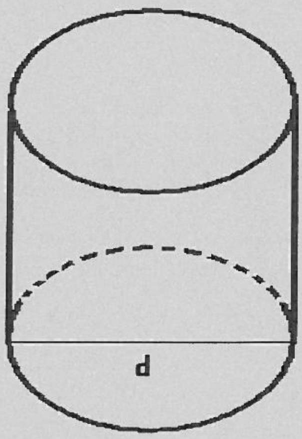


Fig. 2

2.1 Refer to Fig. I:

Calculate the total surface area of the right triangular prism. (5)

2.2 The right triangular prism is filled with water and the water is poured into the cylinder (fig. 2). If the diameter of the cylinder is 12 cm, determine the height of the water in the cylinder. (5)

2.3. Show that $1.2\dot{5}\dot{7}$ is a rational number. (4)

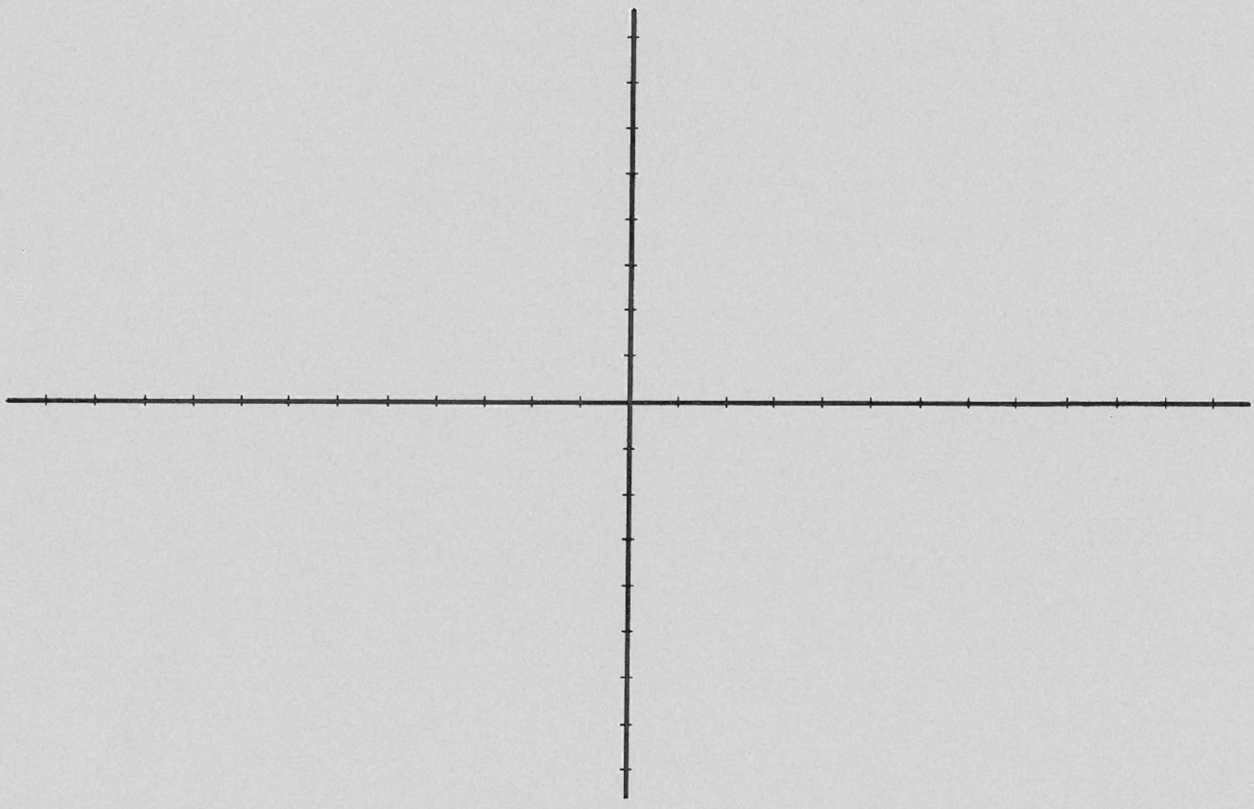
2.4 Show that the sum of three odd natural numbers is odd. (4)

QUESTION THREE ; (15 marks)

3.1 Consider:

$$f(x)=\begin{cases}3^x, & x<1\\ |x-2|+2, & 1\le x<4\end{cases}$$

3.1.1 Sketch the graph of f. (7)



3.1.2 Determine the domain of f in interval notation. (2)

3.2 Determine:

$(f \circ g)(3)$ if $g(x) = x - 1$. (3)

3.3 Graph the set $H = \{ (x, y) \in R^2 \mid y \leq f(x) \} \cap \{ (x, y) \in R^2 \mid y \geq 1 \}$ on the set of axes above. (3)

QUESTION FOUR : (25 marks)

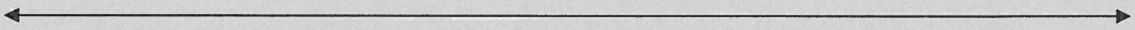
4.1 Solve for x and write your answer using set builder notation.

$5 \times 3^x - 9 \times 3^{x-2} \leq 12$ **and** $|2x - 4| \leq 4$ (6)

4.2 Consider:

$$3 < |2x+1| < 5$$

Solve for x and represent your solution on the number line provided below. (6)



4.3 Simplify:

$$\frac{5^{x+3}+5^{x+1}}{2^{x+7}+2^{x+1}}$$ (3)

4.4 Solve for x:

$$3\times 9^{x}-4\times 3^{x}+1=0 \qquad (5)$$

4.5 Solve for n:

$$\frac{2^{2n+3}\left[\,2^{4n-1}\,\right]^{n}}{16\left[\,2^{n+1}\,\right]^{n+1}}=1 \qquad (5)$$

QUESTION FIVE : (10 marks)

5.1 Write as a single log:

$$2\log x - 4\log(3x-2) + \frac{1}{2}\log(4x-1) \qquad (4)$$

5.2 Solve for x:

$$\log_7 x^3 + \log_7(2x+1) = \log_7 x^2 \qquad (6)$$

QUESTION SIX (26 marks)

6.1 Determine the following limits (if it exists).

6.1.1 $\lim_{a \to 2}\left[\dfrac{a^3 - 8}{a^2 - 4}\right]$ (3)

6.1.2 $\lim_{x \to \infty}\left[\dfrac{2x^2 - x}{x^3 + 1}\right]$ (3)

6.2 Determine $f'(x)$ from first principles.

$f(x) = \sqrt{x}$ (5)

6.3 Consider $f(x) = -x^3 + 3x^2 + 1$.

6.3.1 Determine the equation of the tangent to the curve at $x = 1$. (4)

6.3.2 Calculate the local maximum and minimum turning points. (5)

6.4 Differentiate with respect to the variable in the following:

6.4.1 $y = (\sqrt{x} + 3)^2$ (3)

6.4.2 $f(t) = \dfrac{6 - t^3}{2t}$

(3)

QUESTION SEVEN : (30 marks)

7.1 Given $\tan\theta = m$ and $\theta < 90°$ simplify (with the aid of a diagram):

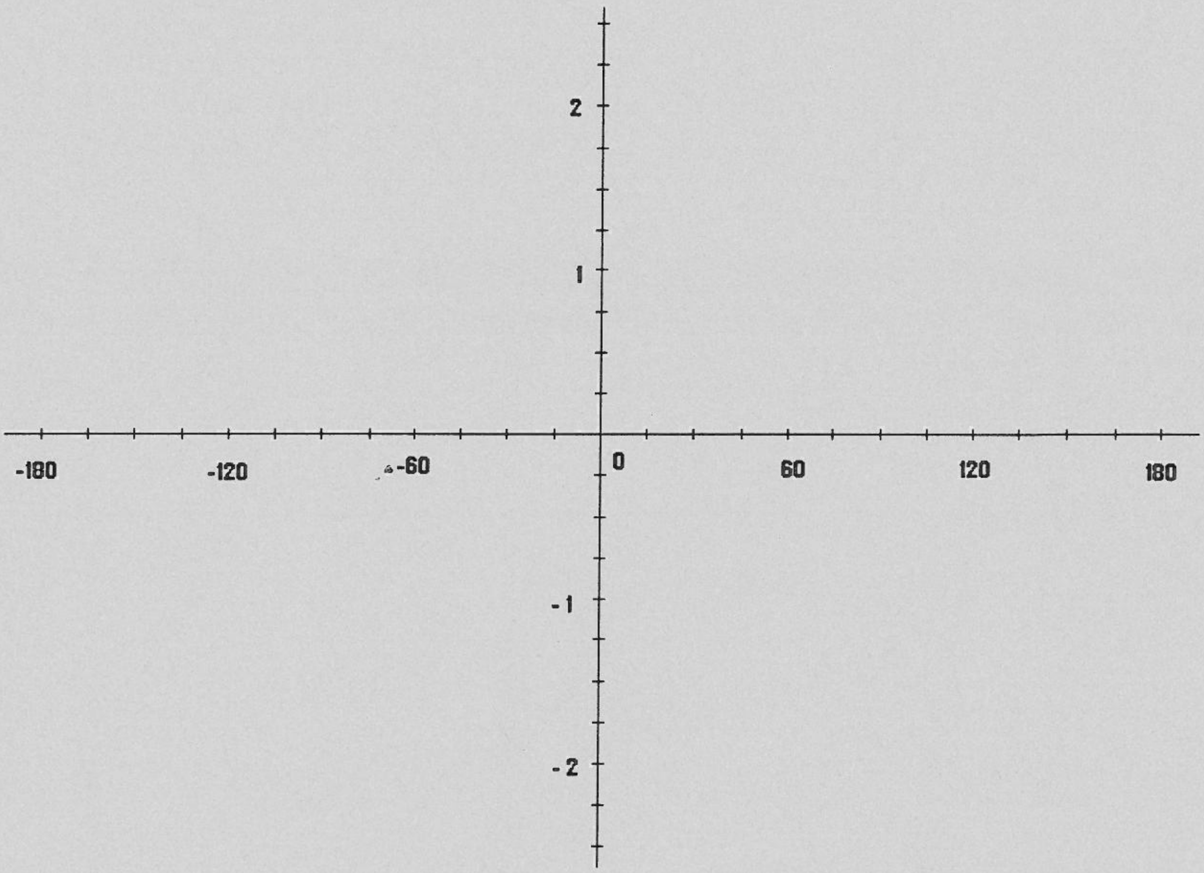
7.1.1 $\sin^2\theta - \tan\frac{\pi}{4}$ (4)

7.1.2 $\sec^2\theta - \cot\theta$ (3)

7.2.1 Sketch the following graphs on the set of axes provided.

$$f = \left\{ (x;y) \middle| \; y = \cos\left(x + 30^\circ\right) \; , -180^\circ \leq x \leq 180^\circ \right\}$$

$$g = \left\{ (x;y) \middle| \; y = \tan\frac{x}{2} \; , -180^\circ \leq x \leq 180^\circ \right\}$$ (6)



7.2.2 What is the amplitude of f? (1)

7.2.3 What is the period of g? (1)

7.3 Simplify: $\tan\frac{\pi}{6} - (\sin\frac{5\pi}{6})\ (\sec\frac{-5\pi}{6})$ (5)

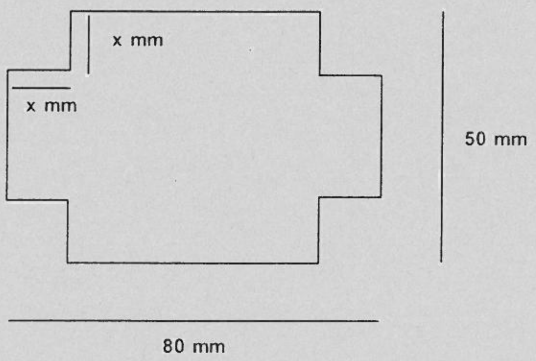
7.4 Solve :

$\sin\theta\cos 40° + \cos\theta\sin 40° = 0.5$ for $\theta \in\ -180° < \theta < 180°$ (6)

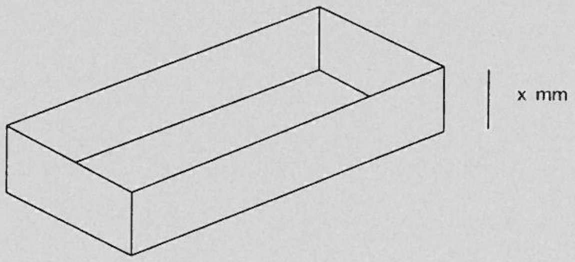
7.5 Prove: $$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$$ (4)

QUESTION EIGHT : (8 marks)

8. A piece of cardboard 80 mm by 50 mm has square corners of side x mm cut out as shown below.



The edges are then folded up so as to form a box without a lid, of depth x mm.



8.1 Calculate the volume of the box in terms of x.

8.2 Determine x so that the volume of the box is a minimum.