NAME: _____

CLASS: _____

Final mark: _____/150

Percentage: _____

Instructions

- Work neatly and accurately.
- SHOW WORKING OUT WHERE NECESSARY!
- Calculators may be used unless otherwise stated.
- Round off your answers to ONE decimal place where applicable unless stated otherwise.

LEARNING OUTCOME	QUESTION	MAXIMUM MARKS	MARKS OBTAINED
LO 1 and LO 2	**1**	**22**	
LO 1	**2**	**11**	
	3	**10**	
	4	**13**	
LO 2	**5**	**13**	
	6	**6**	
	7	**7**	
LO 1	**8**	**18**	
LO 2	**9**	**17**	
	10	**14**	
	11	**19**	

QUESTION 1

1.1 Solve for x:

1.1.1 $2.3^x = 7$ (3)

1.1.2 $\frac{5x}{2x-1} \leq 1$ (5)

1.1.3 $bx + k = x + \pi$ (3)

1.1.4 $x\log 5 = \log\frac{3}{5} + x\log 3$ (without the use of a calculator) (4)

1.2 The volume of a rectangular prism (box) is given by $V(x) = x^3 - 6x^2 + 9x - 4$.
If the length and width of the base are $(x-1)$ and $(x-4)$ respectively, find an expression for the height in terms of x. (3)

1.3 Given: $(x^2 - 2)(x-1)(2x+1) = 0$

Solve for x if:

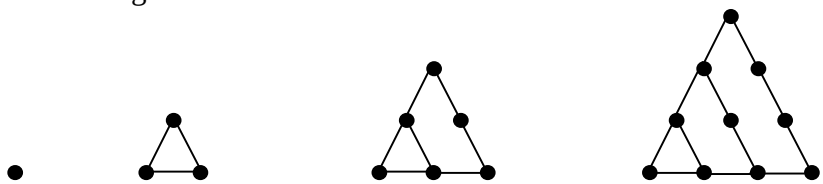
1.3.1 $x \in Z$ (1)

1.3.2 $x \in R$ (3)

[22]

QUESTION 2

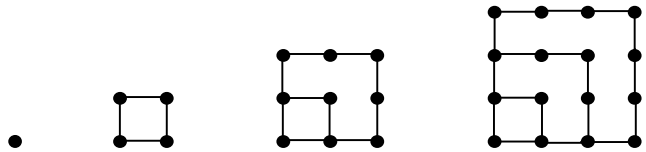
Consider the triangular numbers shown below:



1 3 6 10 and so on

and so on

Now consider the square numbers:



1 4 9 16 and so on

and so on

2.1 Show that both the triangular and square numbers form a quadratic sequence. (3)

2.2 Determine the general term for the triangular numbers. (3)

2.3 Is 102 a triangular number? Show a calculation to support your answer. (3)

2.4 Using the above patterns as hints, write down the 3rd pentagonal number. (2)

[11]

QUESTION 3

3.1 Write down the first 3 multiples of 3 larger than 100. (3)

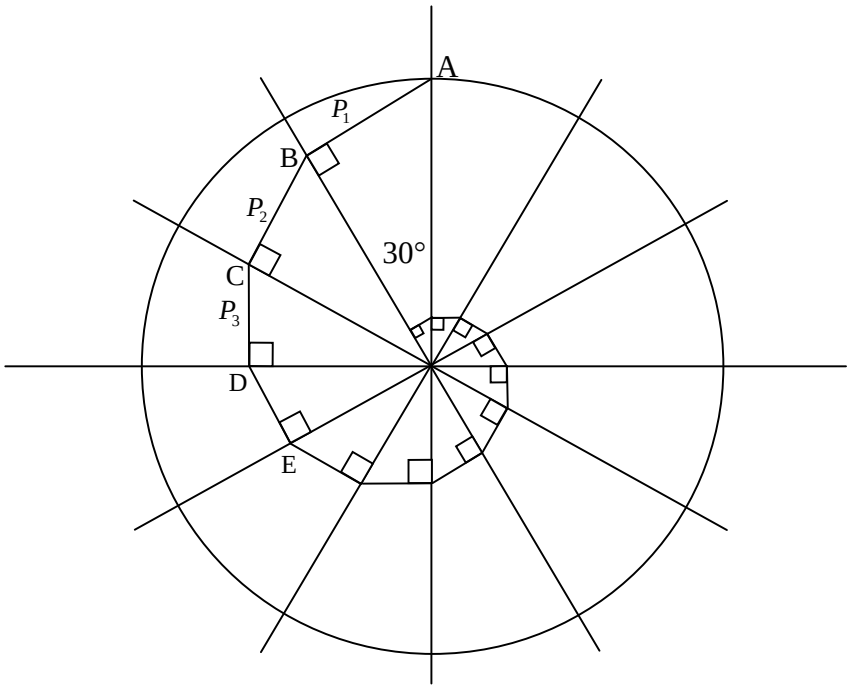
3.2 Determine the sum of the multiples of 3 between 100 and 200. (4)

3.3 Represent the sum of the multiples of 3 between 100 and 200 in sigma notation. (3)

[10]

QUESTION 4

The figure below shows a circle centre O of radius 2 units that has been divided into $30°$ sectors. Perpendiculars, P_1 (AB), P_2 (BC), P_3 (CD) etc. have been constructed as indicated in the diagram.



4.1 Use trigonometry to show that $P_1 = 1$, $P_2 = \frac{\sqrt{3}}{2}$ and $P_3 = \frac{3}{4}$. (8)

4.2 Prove that P_1 ; P_2 ; P_3 form a geometric sequence. (2)

4.3 If the spiral continues infinitely, show that the total length of the spiral does not exceed $4 + 2\sqrt{3}$ (or $\frac{2}{2-\sqrt{3}}$). (3)

[13]

QUESTION 5

5.1 Given that $f(x) = \left(\frac{1}{3}\right)^x$

5.1.1 Determine f^{-1} writing your answer in the form $f^{-1}(x) = ...$ (2)

5.1.2 State the domain and range of f^{-1}. (2)

5.1.3 Give the equation of the line about which f and f^{-1} are symmetric. (1)

5.1.4 Determine *g*, the reflection of f in the *y* axis. (1)

5.2 Given $p(x) = \frac{-2}{x+3} + 2$

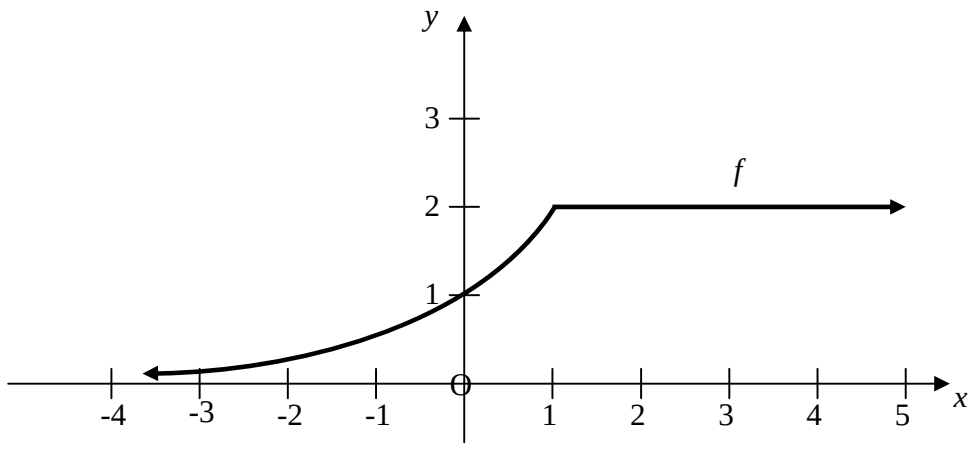
5.2.1 Sketch p, clearly indicating the equations of asymptotes and intercepts with the axes. (5)

5.2.2 Give the equation of g, which is the result of translating p 2 units up and 1 unit left. (2)

[13]

QUESTION 6

Use the sketch of f in the diagram to answer the following questions:



6.1 Is f a one-to-one function? Explain. (2)

6.2 State the range of *f*. (2)

6.3 Maintaining the range, give a restriction on the domain of f that would ensure that f^{-1} is a function. (2)

[6]

QUESTION 7

7.1 If the sides of a rectangle are $(x-1)$ and $(x+5)$, write down an expression for $f(x)$, the area of the rectangle. (2)

7.2 Sketch the graph of f showing clearly the intercepts with the axes and the coordinates of the turning point. (3)

7.3 For which values of x does f actually represent the area of a rectangle? Explain. (2)

[7]

QUESTION 8

8.1 Michal buys a house for R1,4 million. She pays a deposit of 10% and takes out a home loan for the balance.

8.1.1 Calculate the value of Michal's home loan. (1)

8.1.2 Michal chooses to repay the loan over 20 years at an interest rate of 10,25% p.a. compounded monthly. She makes monthly payments starting 1 month after the loan was granted. Calculate Michal's monthly instalments. (4)

8.1.3 Directly after the 120th payment, Michal wins "*Strictly Come Dancing*". She becomes an instant celebrity earning tons of money. She decides to pay off the balance on her home loan. How much must she pay the bank to settle her entire debt directly after her 120th payment? (3)

8.2 Tali (also a very successful dancer) decides to start saving for her retirement on her 21st birthday. She does so by paying R6000 per month into a pension fund starting immediately. The fund pays 8,3% p.a. compounded monthly. How much money will she have one month after she turns 40, if she makes the last payment on her 40th birthday? (4)

8.3 Michael B is an incredibly famous drummer for a band called

$$U(\log 2 + \log 5) \times [\sin^2 x + \cos^2 x + (drum)^0]$$

8.3.1 Give the abbreviated name of Michael's band. (1)

8.3.2 Michael's drumsticks used in his debut "**Winner**" are auctioned in aid of charity when he reaches the age of 32. The auctioneer at Sotheby's calculates that the value of the drumsticks had increased by 220% per annum since Michael's debut performance at 16. He sets this value as the reserve price. Calculate the reserve price if the drumsticks were worth R500 when Michael was 16. (3)

8.3.3 Mick Jagger buys this priceless artefact at 11% above the reserve price. How much did he pay for the drumsticks in pounds sterling if £1 = R11,34 at the time of purchase? (2)

[18]

QUESTION 9

9.1 If $f(x)=\frac{2}{x}$, find $f'(x)$ using first principles. (5)

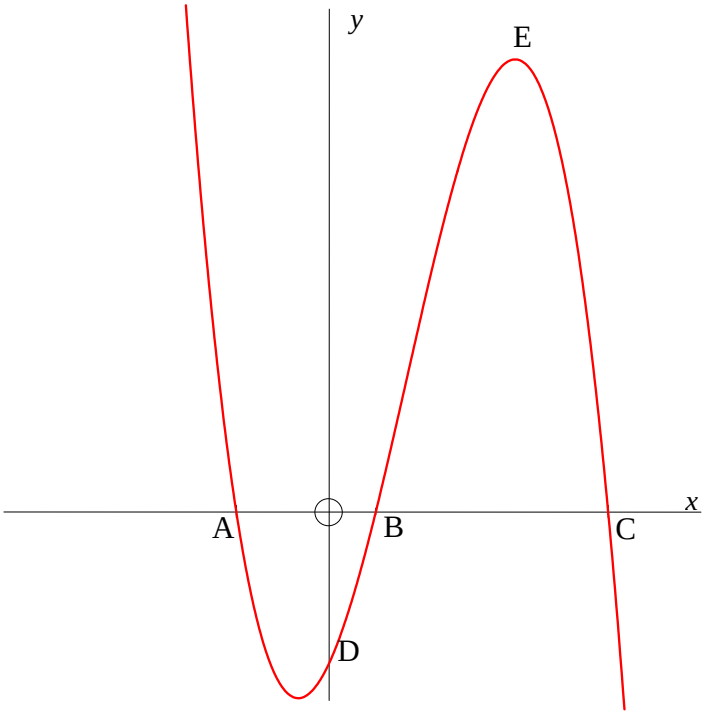
9.2 Find $f'(x)$ if $f(x)=\left(\sqrt{x^3}-3\right)^2$ (4)

9.3 $f(x)=2^x+2^{x+3}-144$ and $g(x)=\frac{16}{\sqrt{x}}$. Find the value of $g'(x)$ if $f(x)=0$. (8)

[17]

QUESTION 10

The sketch represents the graph of $f(x)=-x^3+5x^2+8x-12$.



10.1 Show all your working and determine the lengths of OA, OB, OC and OD. (6)

10.2 Calculate the coordinates of the turning point E. (4)

10.3 Determine the equation of the tangent to *f* at D. (4)

[14]

QUESTION 11

In ΔABC, $\hat{A}$ and $\hat{B}$ are $x°$ and $y°$ respectively.

11.1 Express $\hat{C}$ in terms of x and y. (1)

11.2 No angle in the triangle is greater than $\hat{A}$ and no angle is smaller than $\hat{C}$. The triangle is not an obtuse angled triangle and no angle is smaller than $20°$.
Provide brief reasons why each of the following inequalities is true:

11.2.1 $y \leq x$ (1)

11.2.2 $x \leq 90$ (1)

11.2.3 $x + y \leq 160$ (2)

11.2.4 $x + 2y \geq 180$ (2)

11.3 Represent all these constraints on the sheet of graph paper provided to you.
Indicate the feasible region clearly on your graph. (6)

11.4 Use your graph to determine the magnitude of <u>all three angles</u> in each of the following cases:

11.4.1 if $\hat{A}$ is as small as possible. (2)

11.4.2 if $\hat{B}$ is as large as possible. (2)

11.4.3 if the difference between $\hat{A}$ and $\hat{B}$ is as large as possible. (2)

[19]

QUESTION 11.3