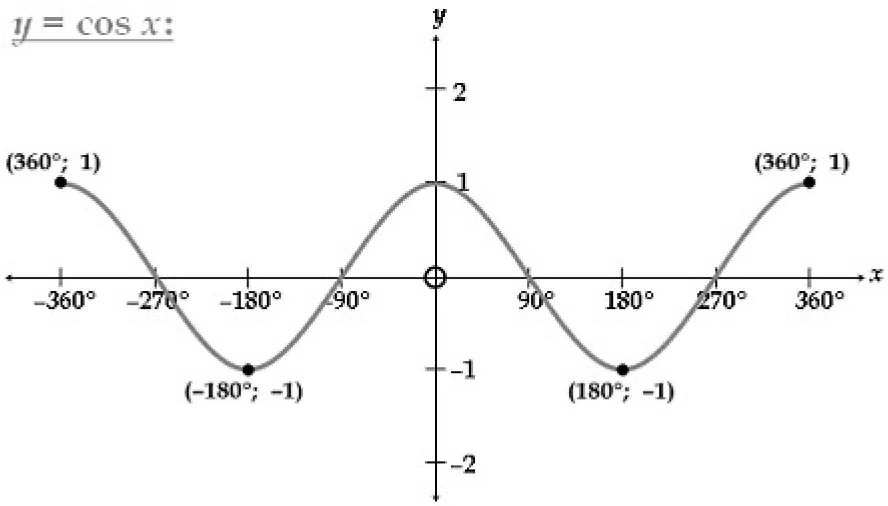
XT - MATHS Grade 11

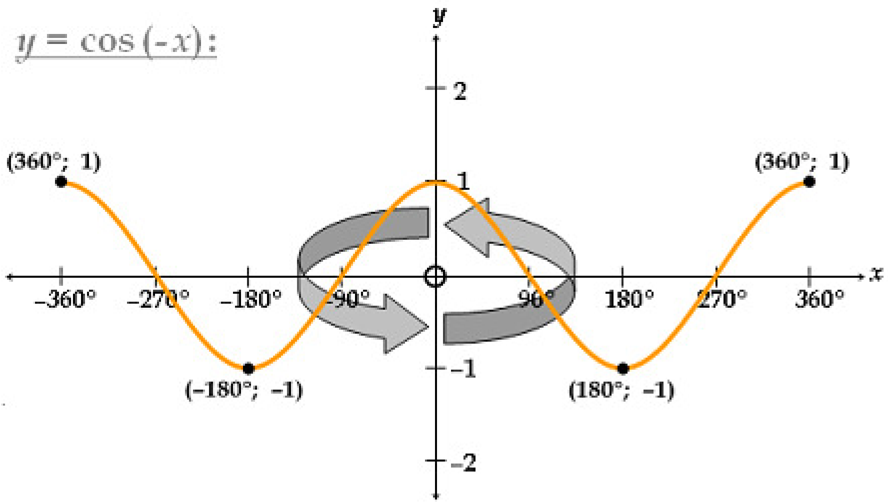
Subject: Trigonometry 2: Graphs **Date:** 2010/06/29

Total Marks: 41

1. TRUE 2

Explanation: A reflection in the y-axis means that x is replaced with $-x$.
Thus $y = \cos x$ becomes $y = \cos(-x)$, but $\cos x = \cos(-x)$ as $y = \cos x$ is symmetrical about the y-axis.





2. TRUE 2

Explanation: The period of both graphs is $180°$. This means that the same information is repeated every $180°$ earlier or after the part that has been drawn.
Thus the next point of intersection will be $180°$ after $28°$, that is $208°$.

3. A 4

Explanation: If the product of the two graphs is negative, the one graph must be positive and the other graph must be negative.

The values of x for which $\cos 2x = 0$:

$$2x = 90° \quad \text{or} \quad 2x = 270° \qquad \text{[for the given interval]}$$

$$x = 45° \qquad\qquad x = 135°$$

From the graph:

for $x = 0°$	:	$\tan x = 0$ and $\cos 2x$ positive
for $0° < x < 28°$	:	both $\tan x$ and $\cos 2x$ positive
for $x = 28°$	:	both $\tan x$ and $\cos 2x$ positive
for $28° < x < 45°$	:	both $\tan x$ and $\cos 2x$ positive
for $x = 45°$	:	$\tan x$ positive and $\cos 2x = 0$
for $45° < x < 90°$	:	$\tan x$ positive and $\cos 2x$ negative
for $x = 90°$	:	$\tan x$ does not exist
for $90° < x < 135°$	:	both $\tan x$ and $\cos 2x$ negative
for $x = 135°$	:	$\tan x$ negative and $\cos 2x = 0$
for $135° < x < 180°$	:	$\tan x$ negative and $\cos 2x$ positive
for $x = 180°$	:	$\tan x = 0$ and $\cos 2x$ positive

Then …

$\tan x . \cos 2x < 0$ for $45° < x < 90°$ and $135° < x < 180°$

$\therefore \tan x . \cos 2x < 0$ for $(45° ; 90°)$ and $(135° ; 180°)$

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4. FALSE 2

Explanation: If the asymptotes are at $x = -30°$ and $x = 30°$, then the period of this function will be $60°$.
There will therefore be 3 'repeats' of the graph between $0°$ and $180°$.
Therefore, the value of b will be 3.

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5. B 2

Explanation: $\mathbf{A}$ has the greatest amplitude, but this is not asked.
The first x-intercept of $\mathbf{A}$ is at $x = 90°$; this means the period is $360°$.

$\mathbf{B}$ and $\mathbf{C}$ have the same amplitudes, even though $\mathbf{C}$ is a reflection about the x-axis.
The first x-intercept of $\mathbf{B}$ is at $x = 135°$; this means the period is $540°$.

The first x-intercept of $\mathbf{C}$ is at $x = 45°$; this means the period is $180°$.

Thus the graph with the greatest period is $\mathbf{B}$.

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6. C 3

Explanation: The minimum value of f is -1, therefore the amplitude of this graph is 1.
As f represents a cosine graph and the graph is in the same form as a 'normal' cosine graph, the value of a will be equal to 1.
The graph of f has been moved $45°$ to the right [$\cos(-90°) = 0$, but in this graph $\cos(-45°) = 0$].
As the graph has been moved $45°$ to the right, the value of b will be equal to $-45°$.

The maximum value of g is 2, therefore the amplitude of this graph is 2.
As g represents a sine graph and the graph has been rotated around the x-axis, the value of c will be equal to -2.

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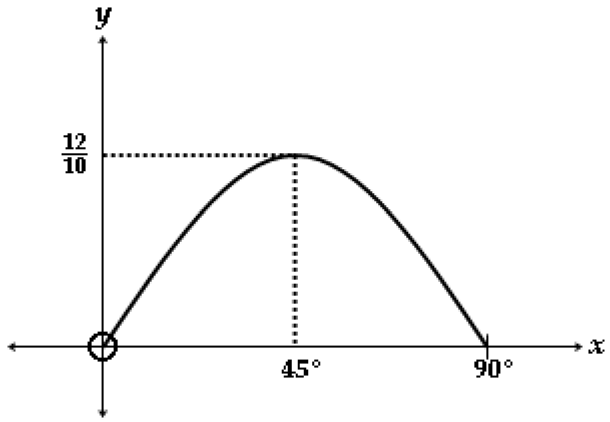
7. C 2

Explanation: The value $\frac{12}{10}$ has no impact on where the turning points are, but only on the value of the turning points.

The period of $y = \frac{12}{10}\sin 2x$ has been doubled, that is there is twice as much information recorded as the original graph.

The graph of the equation $y = \frac{12}{10}\sin 2x$ for $0° \leq x \leq 90°$ is :



Therefore, the maximum height of $\frac{12}{10}$ is achieved at $x = 45°$.

8. (60° ; 90°]
(60 ; 90] 2

Explanation: Both graphs must lie above (or both below) the x-axis at the same time.
The only interval on this graph where this occurs is in **(60° ; 90°]** .
Note: At **60°** the product of the graphs is zero; hence **60°** is excluded from the solution.

9. y = 2 cos x 2

Explanation: If the graph is moved **30°** to the right, then the y-axis moves **30°** to the left.
Therefore, all x-values will become **30°** larger, i.e. x will become $x + 30°$.
The new equation will then be …

$$y = 2\cos\big((x + 30°) - 30°\big)$$

$$y = 2\cos x$$

10. y = cos (x + 15°)
y = cos (x + 15) 2

Explanation: The graph moves **30°** to the right

∴ the y - axis moves **30°** to the left

∴ x - values changes to $x - 30°$

Therefore, the new equation will be ... $y = \cos\big((x - 30°) + 45°\big)$

$$y = \cos(x + 15°)$$

11. -30°; 150°
150°; -30°
-30; 150
150; -30 2

Explanation: The graph of $y = \cos x$ will cut the x-axis at **90°**, **270°**, **–90°** and **–270°**.
The graph of $y = \cos(x - 60°)$ has shifted **60°** to the right, and thus will cut the x-axis at **150°**, **330°** (which is outside the required interval), **–30°** and **–210°** (which is also outside the interval).

12. (1) **(180°; 0)** 4

(2) **(331°; –0,96)**

Explanation: The coordinates of $\mathbf{B}$ are $(180°;\ 0)$, the normal x-intercept for a sine function.

From symmetry, $\mathbf{A}$ is as far from the y-axis ($29°$) as $\mathbf{C}$ is from $360°$.
Thus the x-coordinate of $\mathbf{C}$ is $360° - 29° = 331°$.

From symmetry, $\mathbf{A}$ is as far above the x-axis ($0,96$) as $\mathbf{C}$ is below it.
Thus the y-coordinate of $\mathbf{C}$ is $-0,96$.

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13. (1) $(45°;\ 1)$ 4

(2) $(-75°;\ -0{,}5)$

Explanation: The coordinates of $\mathbf{B}$ are easily determined: $(45°;\ 1)$
This can be seen from either the fact that $(0°;\ 0)$ from a basic cosine function has been moved $45°$ to the right; or from the fact that the period of the sine function has been halved; thus its maximum point $(90°;\ 1)$ moves to $(45°;\ 1)$.

From symmetry, $\mathbf{A}$ and $\mathbf{C}$ are symmetrical to each other by reflection about the line ***x = 45⁰***. Thus $\mathbf{A}$ is as far to the right of the line $x = 45°$ [$165° - 45° = 120°$] as $\mathbf{C}$ is to its left.
Thus the x-coordinate of $\mathbf{C}$ is $45° - 120° = -75°$.

From symmetry, $\mathbf{A}$ and $\mathbf{C}$ are on the same horizontal line.
Therefore, the y-coordinate of $\mathbf{C}$ is also $-0{,}5$.

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14. (1) $y = 1$ 4

(2) $-0{,}71$

Explanation: At $\mathbf{B}$, a tangent will be horizontal. That means the gradient of the tangent will be zero. Therefore: $m = 0$
The tangent will cut the y-axis at $(0°;\ 1)$.
Therefore: $c = 1$
Thus the equation of the tangent will be $y = 1$.

To determine the y-coordinates of the endpoints of $g(x)$, substitute $180°$ or $-180°$ into $g(x)$:

$$g(180°) = \cos(180° - 45°) = -0,71 \qquad \text{OR} \qquad g(-180°) = \cos(-180° - 45°) = -0,71$$

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15. (1) $60°$ 4

(2) $0°$

(3) $60°$ to the left

Explanation: (1) The period of $y = \tan x$ is $180°$, therefore the period of $y = \tan 3x$ will be $180° \div 3 = 60°$.

(2) The maximum height of $y = \cos x$ is 1, which occurs at $x = 0°$.
Therefore, the maximum height of $y = \cos x - 2$ is $1 - 2 = -1$, which still occurs at $x = 0°$.

(3) $y = \sin \mathrm{A}$ can only become $y = \cos \mathrm{A}$ if $\sin \mathrm{A}$ is changed to either $\sin(90° - \mathrm{A})$ or $\sin(90° + \mathrm{A})$.
If $y = \sin(x + 30°)$ is changed to $y = \sin(x + 90°)$, the equation will change to $y = \cos x$.
Therefore ...

$$y = \sin(x + 30° + 60°)$$

This means that the y-axis must be moved $60°$ to the right which means that the graph must be moved $60°$ to the left.

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15 Questions, 4 Pages