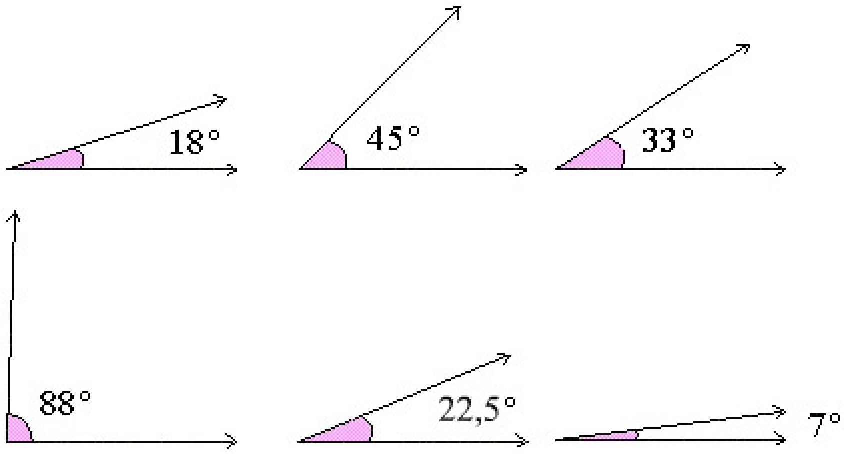
SHAPE, SPACE and MEASUREMENT

Types of Angles

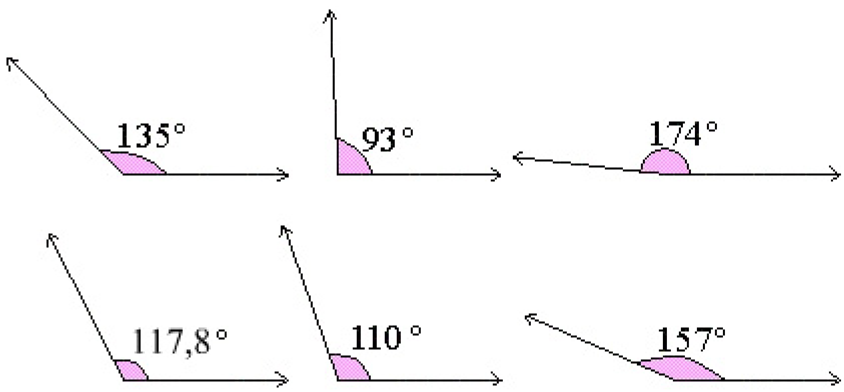
Acute angles are angles of less than ninety degrees.

For example: The angles below are acute angles.



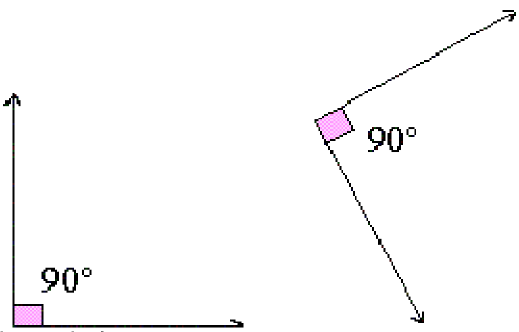
Obtuse angles are angles greater than 90° and less than 180°.

For example: The angles below are obtuse angles.

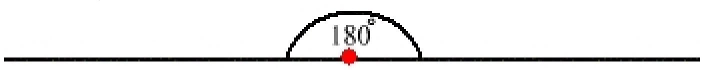


Right angles are angles of 90°.

For example: The angles below are right angles.

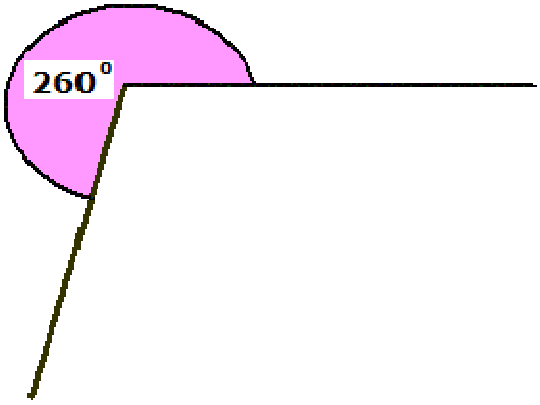


The straight line is an angle of 180°, as shown below.



Reflex angles are angles greater than 180° and less than 360°.

For example: The angle below is a reflex angle.



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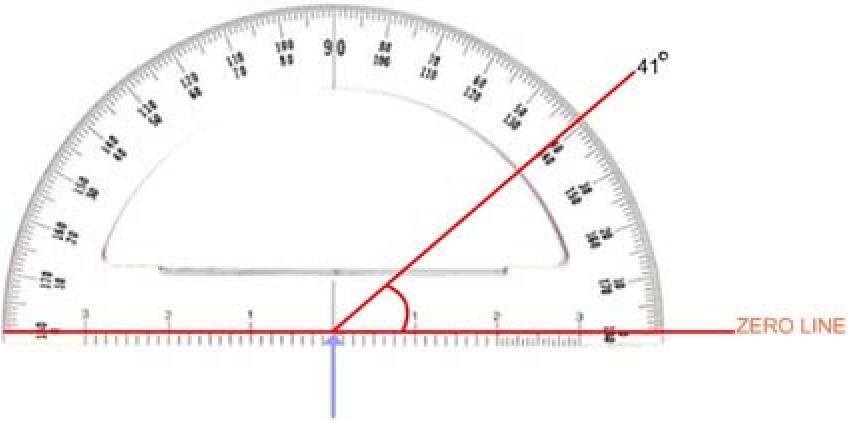
Measuring Angles

We use a protractor for measuring angles in degrees. Below is a picture of a protractor and how we use it. Place the centre of the zero line above the angle at A, the zero line should be covering one side of the angle. Starting at zero, follow the scale around to the other side of the angle and read off the number of degrees (41° in the angle below).

Note: we are using the inside scale because it is the one which starts at zero (The outside scale starts at 180°).

If we were measuring the angle on the other side, starting from the left, then we would use the outside scale and the result would be 139°.



Note: Place the centre of the zero line at the tip of the angle and start at zero on the scale.

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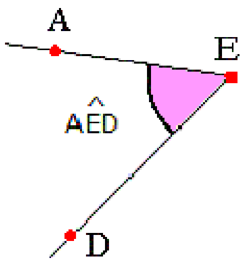
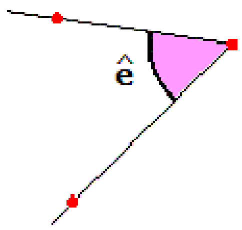
Written Angle Notation

Angles are written in the following ways:

1) Using three capital letters, with the angle sign over the middle letter.



2) Using a small letter inside the angle.

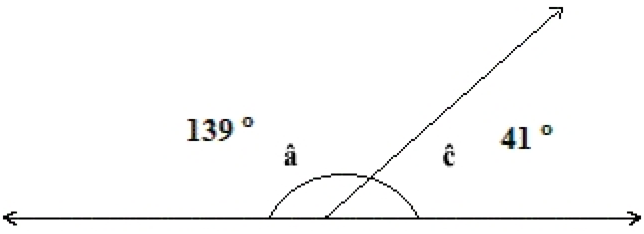


Either of these methods can be used in problems on angles.

Angle Properties

Angles On a Straight Line

Angles which are next to each other on a straight line add up to 180°. This is shown in the diagram below.



angle a + angle c = 180°
Using this property, we can solve problems involving missing angles.

For example: Given the diagram above, with only one of the angles given, calculate the missing angle. Angle a = 70°. Calculate angle c.

$70^\circ + c = 180^\circ$ (angles on a straight line)
$c = 180^\circ - 70^\circ = 110^\circ$

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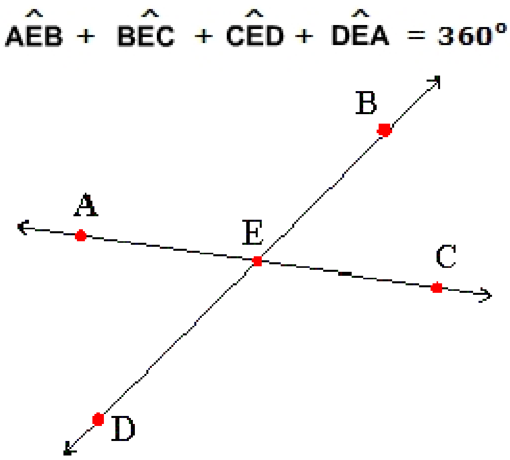
Angles at a Point

Angles meeting at a point add up to 360°.

In the diagram below:



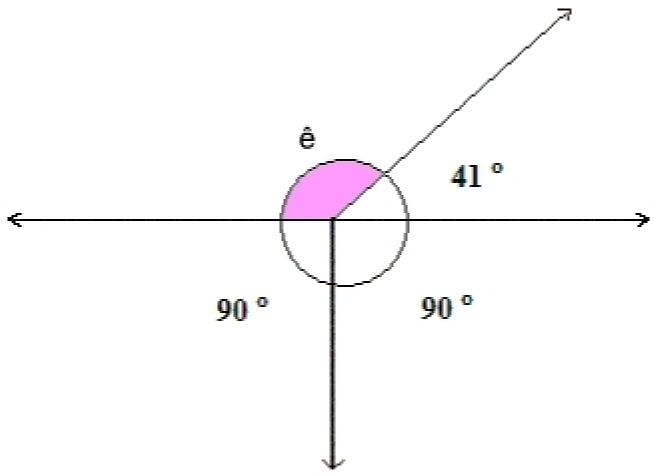
Again, this property can be used to solve problems.

Example 1: In the diagram above, if three of the angles are given, then the missing angle is found by adding the three angles together and subtracting from 360°.

Angles AÊD, BÊC, and AÊB are equal to 30°, 30° and 150° respectively. Calculate angle DÊC.

Angle DÊC = $360^\circ - (30^\circ + 30^\circ + 150^\circ)$
$= 360^\circ - 210^\circ$
$= 150^\circ$

Example 2: In the diagram below, the three angles given are 90°, 90 °and 41°. Calculate angle e.



Angle e = $360^\circ - (41^\circ + 90^\circ + 90^\circ)$

$= 139^\circ$

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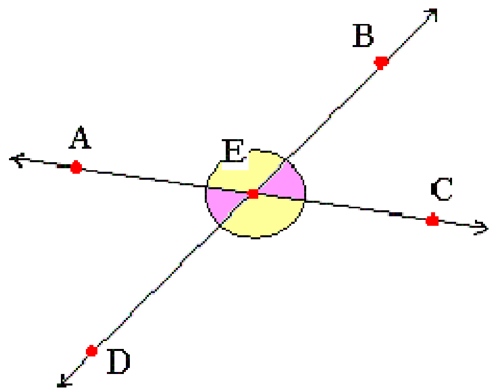
Vertically Opposite Angles

When two straight lines cut, forming an X shape, then the opposite angles are equal. In the diagram below:

$A\hat{E}B = D\hat{E}C$ (yellow angles)

$A\hat{E}D = B\hat{E}C$ (purple angles)



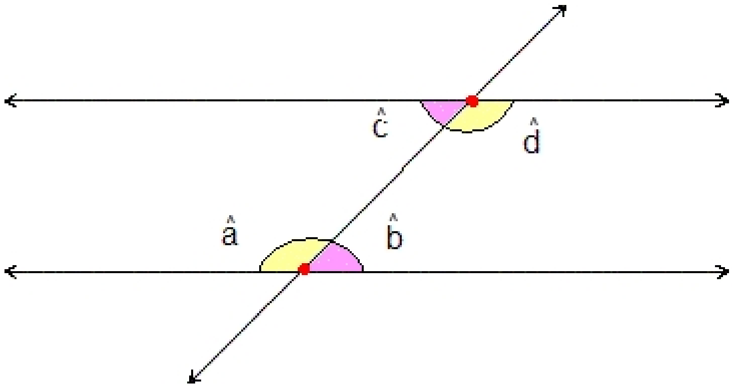
Angles on Parallel Lines

Alternate Angles

Alternate angles are equal and pairs of them are shown in the diagram below.
They form a Z shape.

angle c = angle b (alternate angles)

angle a = angle d (alternate angles)



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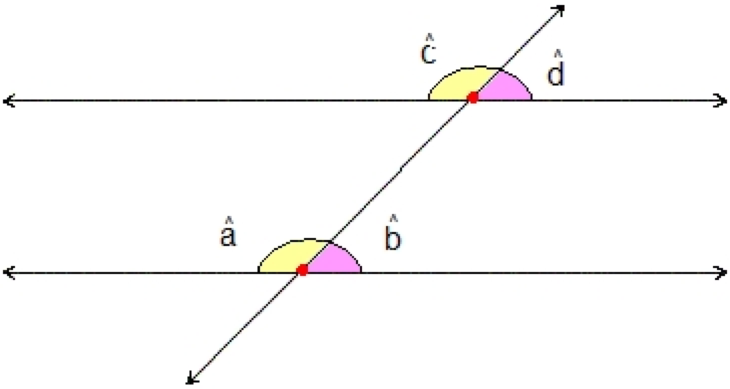
Corresponding Angles

Corresponding angles are equal and pairs of them are shown in the diagram below.



angle b = angle d (corresponding angles)

angle a = angle c (corresponding angles)

These are above the parallel lines and there are two more below, unlabelled. They form an F shape.

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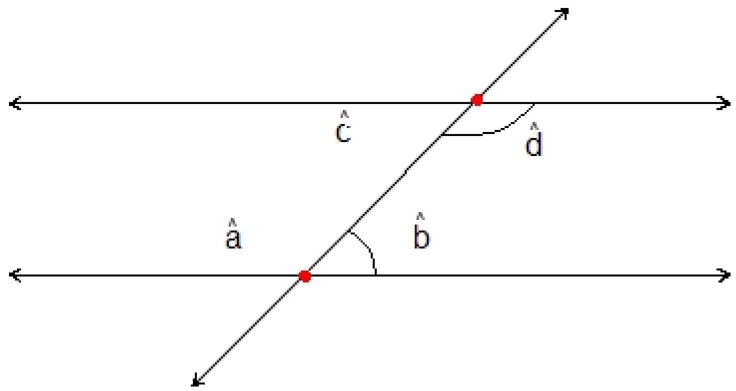
Interior Angles

These are on the inside of the parallel lines as shown below.



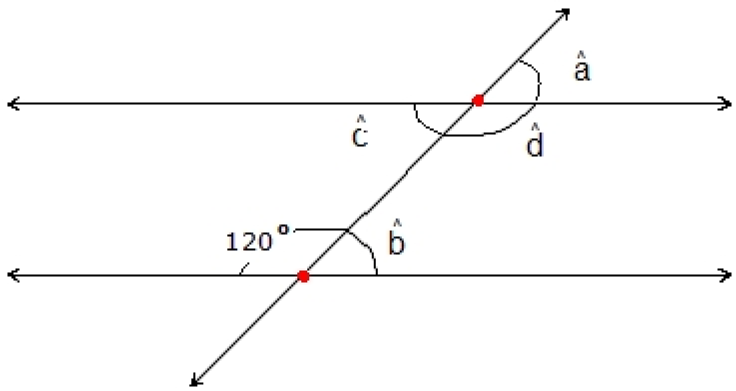
Interior angles add up to 180^{o} .

$$\hat{b} + \hat{d} = 180^{o}$$

$$\hat{a} + \hat{c} = 180^{o}$$

Again these properties can be used to solve problems.

For example: In the diagram below, calculate angles a, b, c and d.



$$\hat{c} + 120^{o} = 180^{o} \text{ (interior angles)}$$

$$\hat{c} = 180^{o} - 120^{o}$$

$$\hat{c} = 60^{o}$$

$$\hat{d} = 120^{o} \text{ (alternate angles)}$$

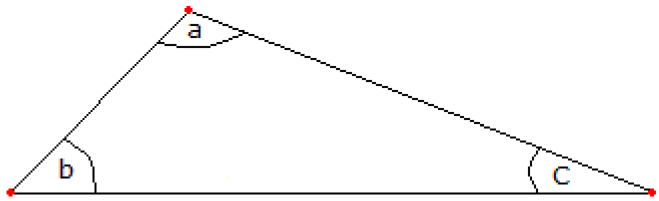
$$\hat{b} = 60^{o} \text{ (alternate angles)}$$

$$\hat{a} = 60^{o} \text{(corresponding angles)}$$

Angles in Shapes

Angles in a Triangle

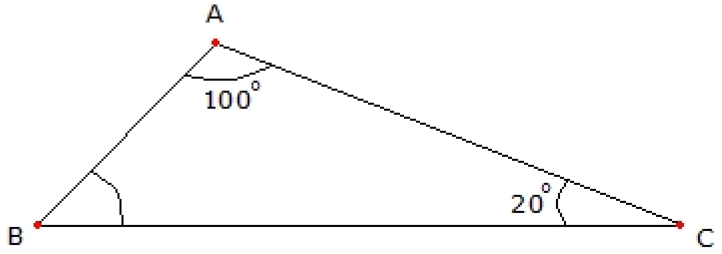
The sum of the angles in a triangle is 180°.



$$\hat{a} + \hat{b} + \hat{c} = 180^\circ$$

This property can be used to solve problems on angles.

Example 1: Given triangle ABC with angle $B\hat{A}C=100^\circ$ and angle $B\hat{C}A=20^\circ$, Calculate angle $A\hat{B}C$.

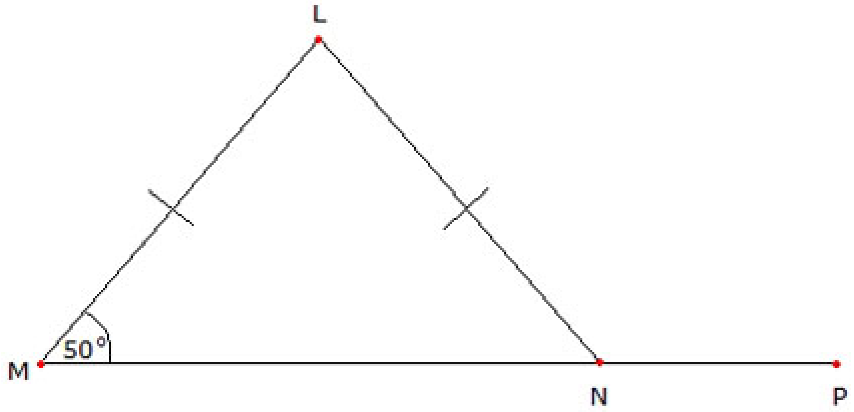


$$\begin{aligned} A\hat{B}C &= 180^\circ - (100^\circ + 20^\circ) \quad \text{(angles in a triangle)} \\ &= 180^\circ - 120^\circ \\ &= 60^\circ \end{aligned}$$

The method is to add the two angles given and subtract this total from 180° .

Note: Reasons are often asked for and should be given briefly, in brackets.

Example 2: Given triangle LMN with angle $L\hat{M}N$ =50° and LM= LN, calculate angle $L\hat{N}M$, angle $M\hat{L}N$ and angle $L\hat{N}P$.

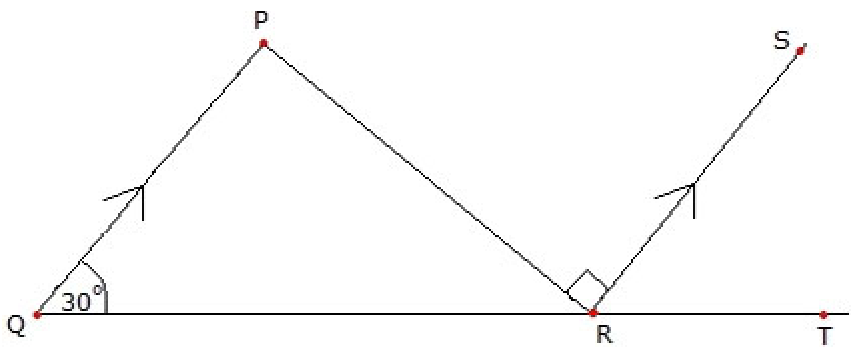


$L\hat{N}M = L\hat{M}N = 50^\circ$ (isosceles triangle)

$M\hat{L}N = 180 - 100$ (angles in a triangle)
$\quad = 80^\circ$

$L\hat{N}P = 180 - 80$ (angles on a straight line)
$\quad = 100^\circ$

Example 3: Given triangle PQR with $P\hat{Q}R$ =30°, the line RS is parallel to PQ and $P\hat{R}S$=90°. Calculate angles $Q\hat{P}R$, $Q\hat{R}P$and $S\hat{R}T$.



$Q\hat{P}R = 90^\circ$ (alternate angles)

$Q\hat{R}P = 180^\circ - 120^\circ$ (angles in a triangle)
$\quad = 60^\circ$

$S\hat{R}T = 180^\circ - (90^\circ + 60^\circ)$ (angles on a straight line)
$\quad = 30^\circ$

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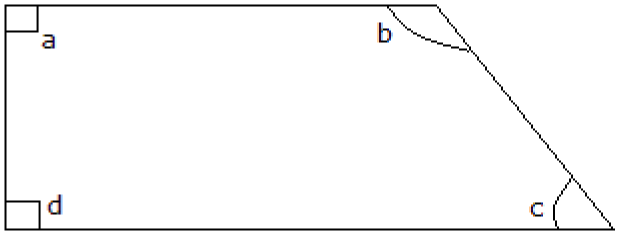
Angles in a Quadrilateral

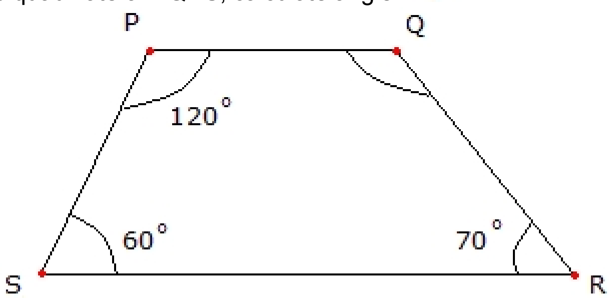
The sum of the angles in a quadrilateral is 360°.



$$\hat{a} + \hat{b} + \hat{c} + \hat{d} = 360^\circ$$

Again this property can be used to calculate missing angles.

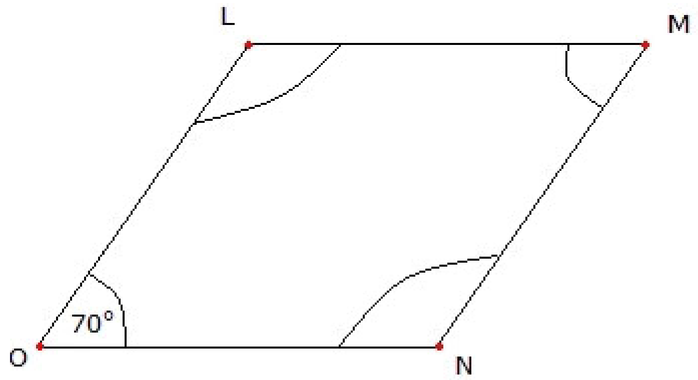
Example 1: Given the quadrilateral PQRS, calculate angle $P\hat{Q}R$.



Calculate $P\hat{Q}R$

$$\begin{aligned} P\hat{Q}R &= 360^\circ - (120^\circ + 60^\circ + 70^\circ) \quad \text{(angles in a quadrilateral)} \\ &= 360^\circ - 250^\circ \\ &= 110^\circ \end{aligned}$$

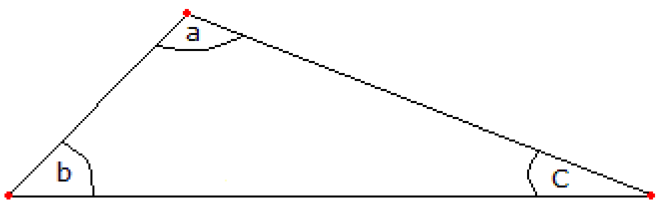
Example 2: Given the parallelogram LMNO, with angle LÔN = 70°. Calculate angles $O\hat{L}M$, $L\hat{M}N$ and $M\hat{N}O$.



$$\hat{OLM} = 180 - 70 \qquad \text{(interior angles)}$$
$$= 110^\circ$$
$$\hat{LMN} = 70^\circ \qquad \text{(interior angles)}$$
$$\hat{MNO} = 360^\circ - (110^\circ + 70^\circ + 70^\circ) \qquad \text{(angles in a quadrilateral)}$$
$$= 360^\circ - 250^\circ$$
$$= 110^\circ$$

Angles in a Triangle

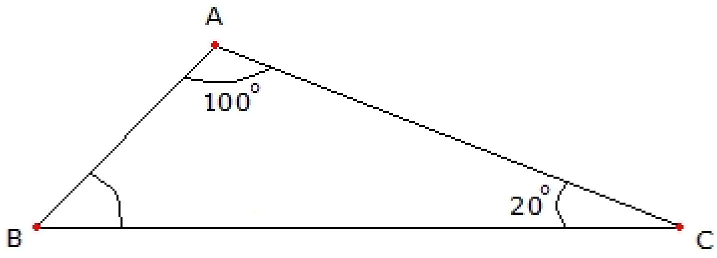
The sum of the angles in a triangle is 180°.



$$\hat{a} + \hat{b} + \hat{c} = 180^\circ$$

This property can be used to solve problems on angles.

Example 1: Given triangle ABC with angle $B\hat{A}C = 100^\circ$ and angle $B\hat{C}A = 20^\circ$, Calculate angle $A\hat{B}C$.

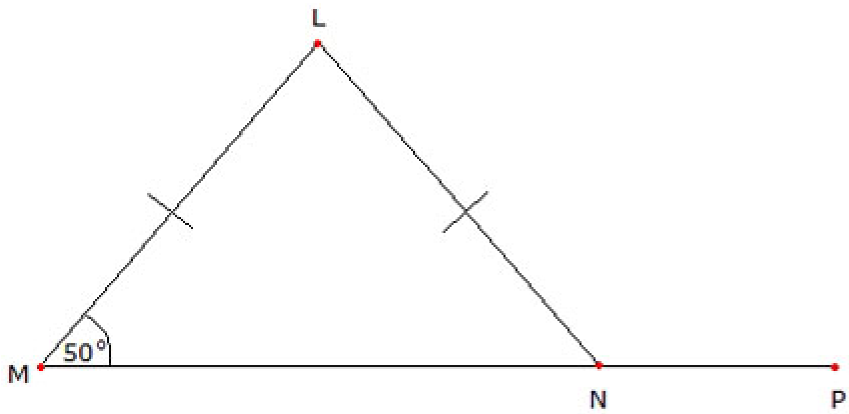


$$\hat{ABC} = 180^\circ - (100^\circ + 20^\circ) \quad \text{(angles in a triangle)}$$
$$= 180^\circ - 120^\circ$$
$$= 60^\circ$$

The method is to add the two angles given and subtract this total from 180° .

Note: Reasons are often asked for and should be given briefly, in brackets.

Example 2: Given triangle LMN with angle $L\hat{M}N$ =50° and LM= LN, calculate angle $L\hat{N}M$, angle $M\hat{L}N$ and angle $L\hat{N}P$.

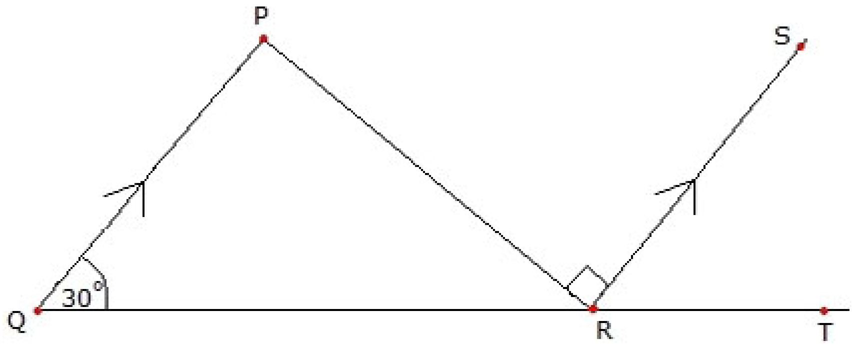


$$L\hat{N}M = L\hat{M}N = 50^\circ \quad \text{(isosceles triangle)}$$

$$M\hat{L}N = 180 - 100 \quad \text{(angles in a triangle)}$$
$$= 80^\circ$$

$$L\hat{N}P = 180 - 80 \quad \text{(angles on a straight line)}$$
$$= 100^\circ$$

Example 3: Given triangle PQR with $P\hat{Q}R$ =30°, the line RS is parallel to PQ and $P\hat{R}S$=90°. Calculate angles $Q\hat{P}R$, $Q\hat{R}P$ and $S\hat{R}T$.



$$\hat{QPR} = 90^\circ \qquad \text{(alternate angles)}$$

$$\hat{QRP} = 180^\circ - 120^\circ = 60^\circ \qquad \text{(angles in a triangle)}$$

$$\hat{SRT} = 180^\circ - (90^\circ + 60^\circ) = 30^\circ \qquad \text{(angles on a straight line)}$$

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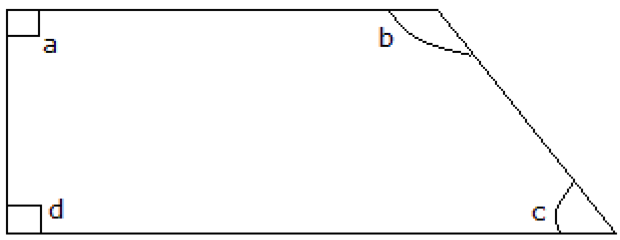
Angles in a Quadrilateral

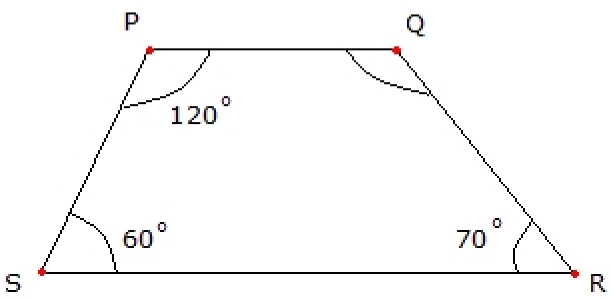
The sum of the angles in a quadrilateral is 360°.



$$\hat{a} + \hat{b} + \hat{c} + \hat{d} = 360^\circ$$

Again this property can be used to calculate missing angles.

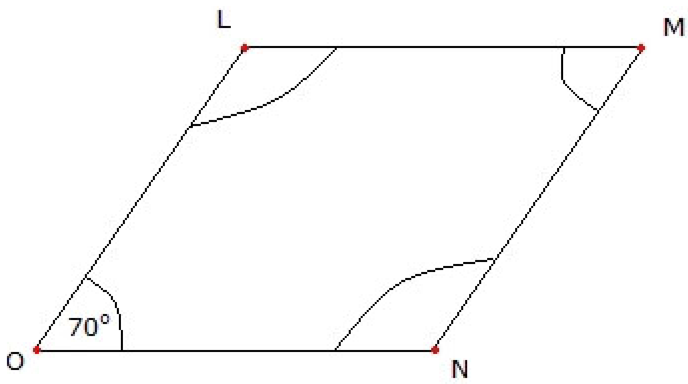
Example 1: Given the quadrilateral PQRS, calculate angle $\hat{PQR}$.



Calculate $P\hat{Q}R$

$$\begin{aligned} P\hat{Q}R &= 360^\circ - (120^\circ + 60^\circ + 70^\circ) \quad \text{(angles in a quadrilateral)} \\ &= 360^\circ - 250^\circ \\ &= 110^\circ \end{aligned}$$

Example 2: Given the parallelogram LMNO, with angle LÔN = 70°. Calculate angles $O\hat{L}M$, $L\hat{M}N$ and $M\hat{N}O$.



$$\begin{aligned} O\hat{L}M &= 180 - 70 \quad \text{(interior angles)} \\ &= 110^\circ \end{aligned}$$

$$L\hat{M}N = 70^\circ \quad \text{(interior angles)}$$

$$\begin{aligned} M\hat{N}O &= 360^\circ - (110^\circ + 70^\circ + 70^\circ) \quad \text{(angles in a quadrilateral)} \\ &= 360^\circ - 250^\circ \\ &= 110^\circ \end{aligned}$$

Length and Area

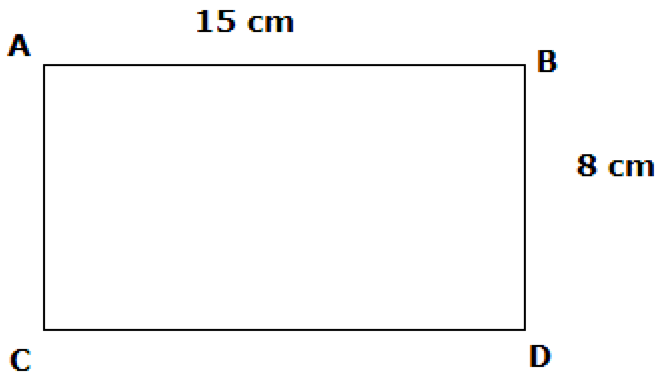
Units of Length

On the metric system, we use the following units for length: millimetres (mm), centimetres (cm), metres (m) and kilometres (km).

10 mm = 1 cm
100 cm = 1 m
1 000 m = 1 km

Perimeter

This is the distance around the outside of a shape. To calculate the perimeter, we add together the lengths of the sides of the shape.
Example 1: Calculate the perimeter of the rectangle ABCD.

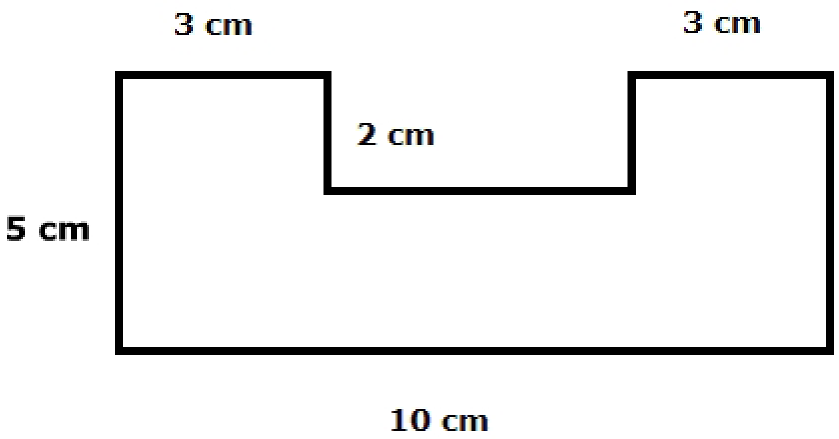


Perimeter = 15 + 15 + 8 + 8 = 46 cm

Note: If we use L for length and W for width, the perimeter (P) can be written as a formula:

$\boxed{P = 2L + 2W}$

Example 2: Calculate the perimeter (P) of the shape below.



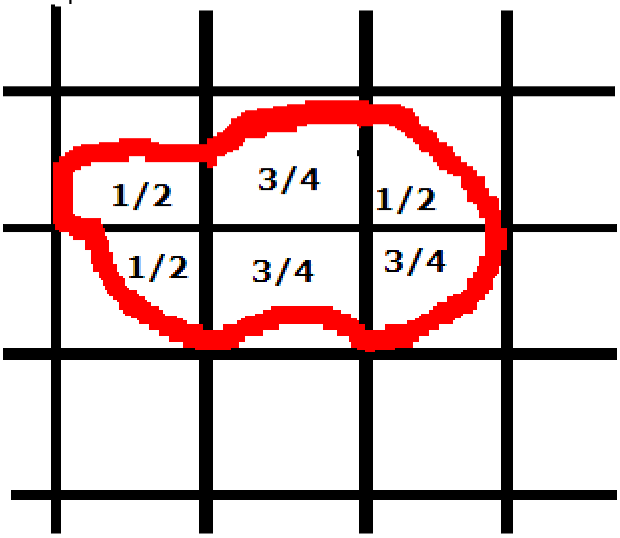
P = 5 + 5 + 3 + 3 + 2 + 2 + 10 + 4 = 34 cm
Note: In this example we need to work out the length of 4 cm (10 – 6 = 4). The other missing lengths 2 cm and 5 cm can be found from the shape.

Area

Area is the space inside a shape. It is measured by dividing the shape into squares and counting them. If the squares' sides are 1 cm in length, then we can use the units cm^2.

Irregular shapes can be drawn on a grid and the area estimated by counting the squares. Parts of a square need to be added to make a whole square.

For example: Estimate the area of the shape below:



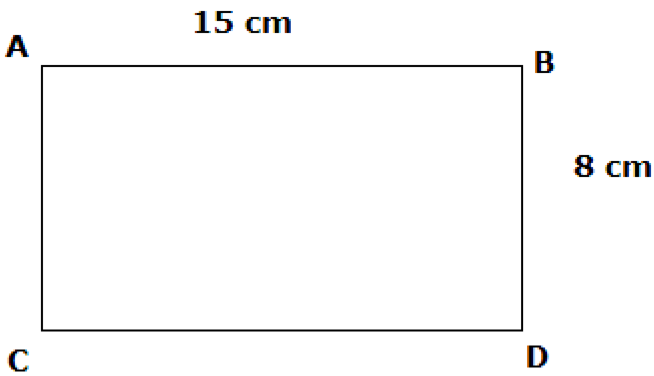
Area = 3½ squares

Regular shapes for example, triangles, rectangles and kites, have a formula for calculating the area.

Area of a rectangle

Area = Length x Width

For example: Calculate the volume of the cuboid shown below. Calculate the area of the rectangle ABCD.

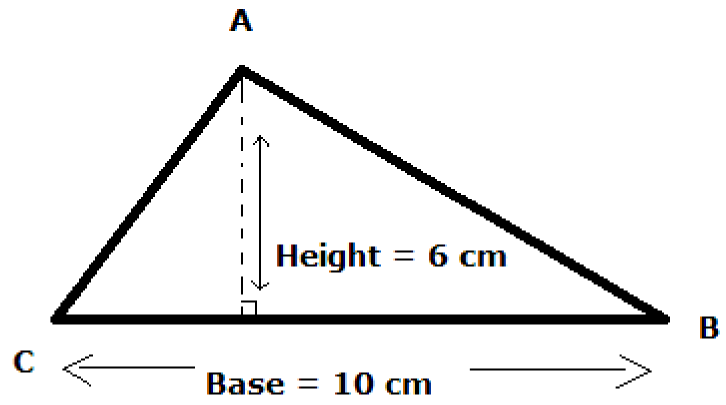


Area = 15 x 8= 120 cm^2 (Note the units of area: cm^2)

Area of a triangle

Area = ½ x Base x Height

For example: Calculate the area of triangle ABC.



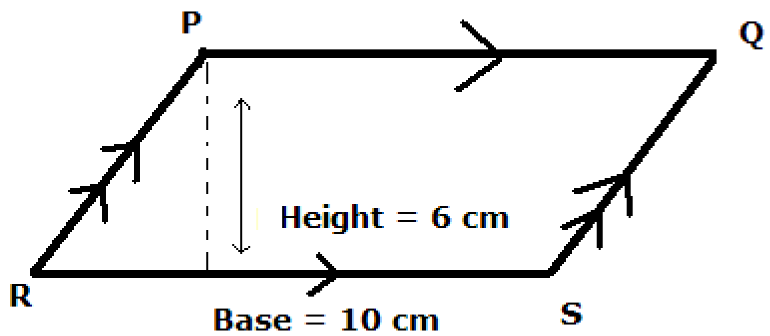
Area = ½ x 10 x 6 = ½ x 60 = 30 cm²

(**Note:** 10 x 60 would give the area of the rectangle standing on BC, the area of the triangle is half this area).

Area of a parallelogram

Area = Base x Height

For example: Calculate the area of the parallelogram PQRS.

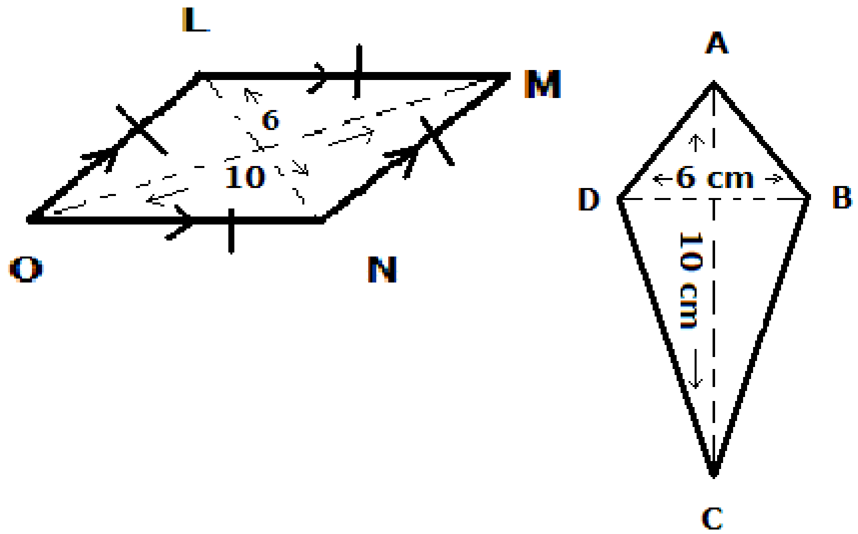


Area = 10 x 6 = 60 cm²

Area of a kite and rhombus

Area = ½ (the product of the diagonals)

For example: Calculate the areas of the kite ABCD and the rhombus LMNO.

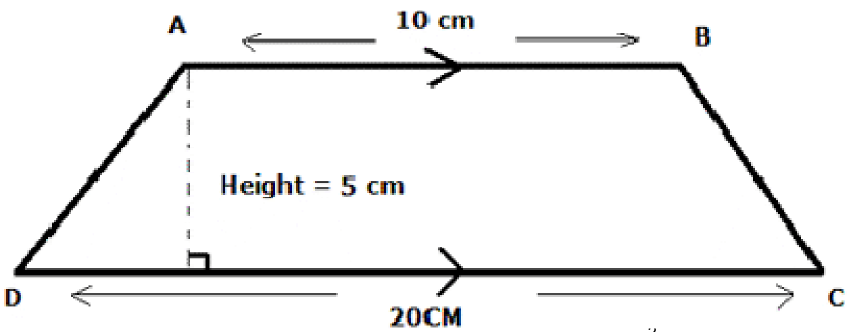


Area of ABCD and LMNO = ½ x 10 x 6 =30 cm²

Area of a trapezium

Area = ½ (the sum of the parallel sides) x the height

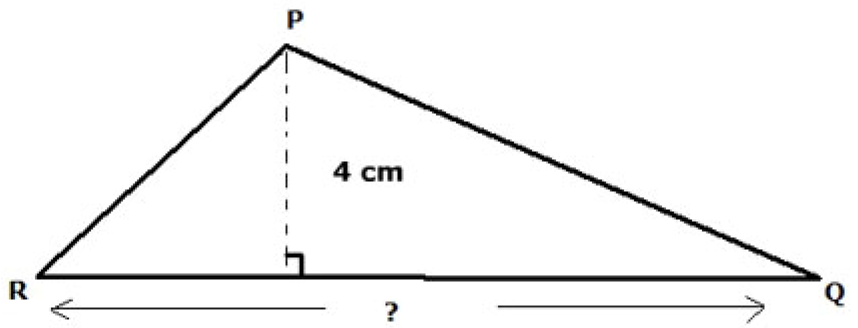
For example: Calculate the area of the trapezium ABCD.



Area = ½ (10+20) x 5 = ½ x 30 x 5 = 75 cm^2

Note: Sometimes the area is given in a problem and we are asked to calculate the length of one of the sides.

For example: Calculate the length of QR in the triangle, given that the area is 20 cm^2.

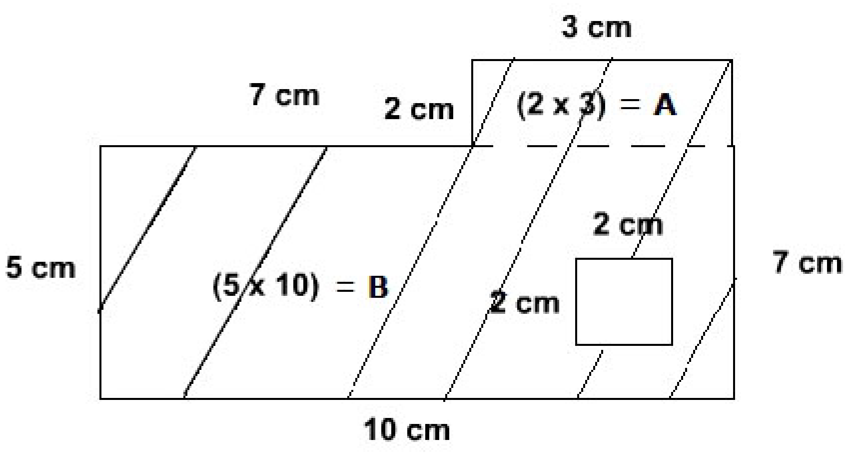


20 = ½ x 4 x QR
20 = 2 x QR
QR = 10 cm

Compound shapes

In some problems it is necessary to divide the shape into regular shapes. We can add or subtract areas.

For example: Calculate a) the total area and b) the shaded area in the diagram below.



a) Total area = area A + area B
= (2 x 3) + (5 x 10)
= 6 + 50
= 56 cm^2

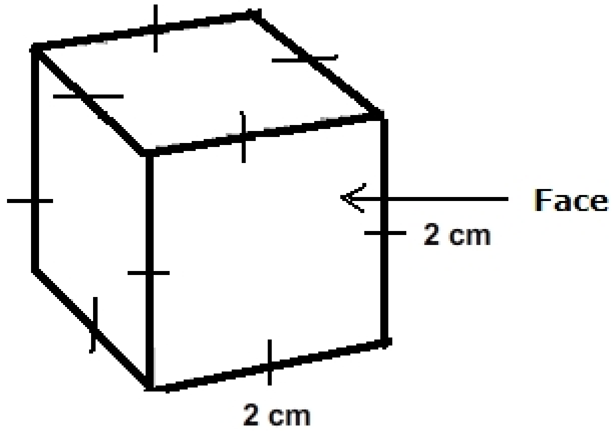
b) Shaded area = 56 – (2 x 2)
= 52 cm^2

Volume and 3D Objects

Volume is the space inside a 3D object. It is measured by the number of cubes which will fit inside the object.
If the cubes are 1 cm in length then the unit of volume is cm^3 .

Cube

A **Cube** has six equal square faces.
This is a 2 cm cube:



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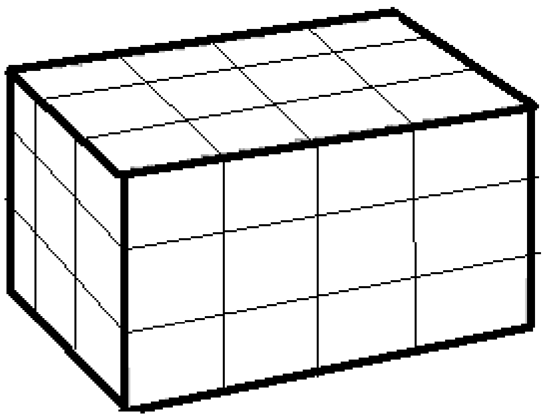
Cuboid

A **cuboid** has some rectangular faces. We can be expected to find the volume of a cuboid by counting the cubes.

For example: Calculate the volume of the cuboid, drawn below.



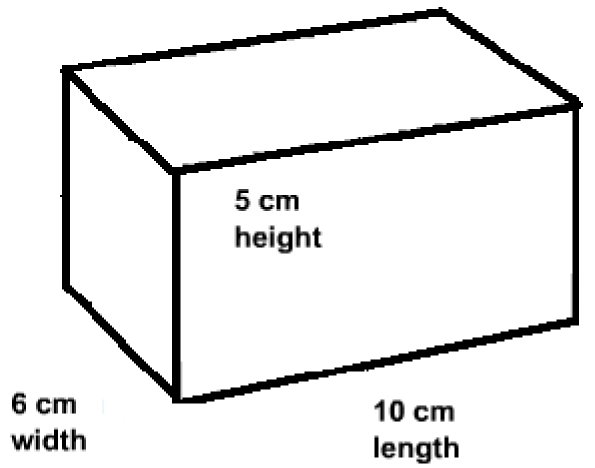
It is best to work in layers as not all of the cubes can be seen. We can see 12 cubes in the top layer. There are 3 layers, so the total number of cubes is 3 x 12 = 36 cubes.
Volume of the cuboid = 36 cubes.

Formula for the volume of a cuboid

This can be used when the dimensions of the cuboid are given.

Volume of a cuboid = Length x Width x Height

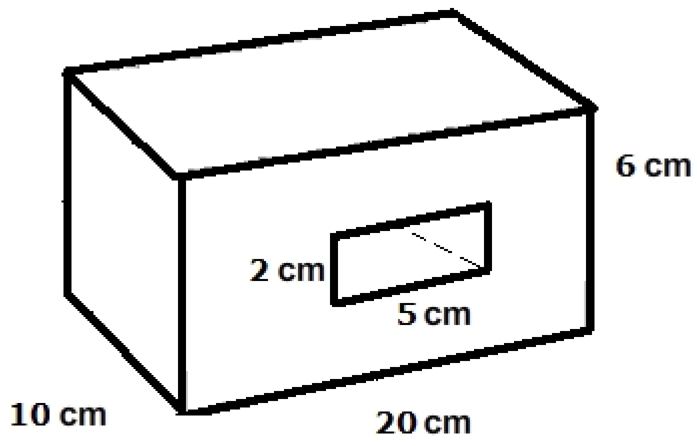
For example: Calculate the volume of the cuboid shown below.



Volume = 10 x 6 x 5 = 300 cm^3 (Note the cubic units for volume)

Note: As with area, volumes can be added or subtracted.

For example: A cuboid has had a rectangular section removed from the centre, as shown in the diagram below. Calculate the volume of the remainder.



Total volume = 20 x 10 x 6 = 1 200 cm^3
Volume of section cut out = 5 x 10 x 2 = 100 cm^3
Volume of remainder = 1 200 – 100 = 1 100 cm^3

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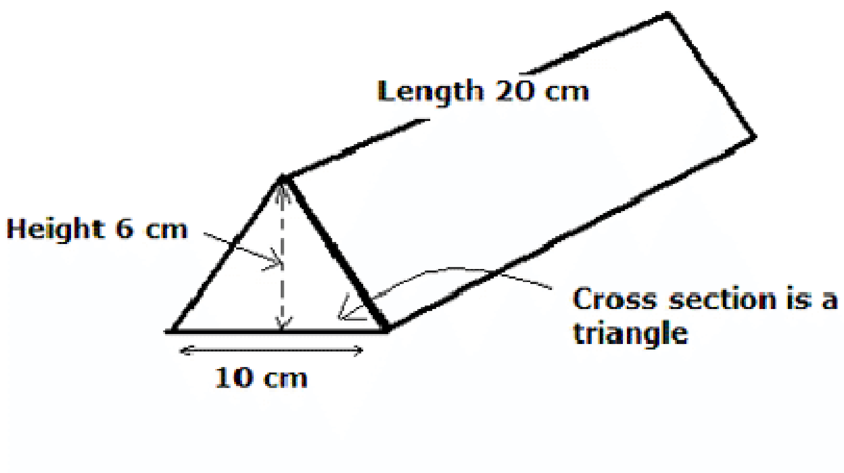
Prism

Volume of a prism

A **prism** is a 3D object whose cross section is constant, along its length. An example is drawn below.



Formula for the volume of a prism

Volume of a prism = Area of cross section x Length

For example:

Volume of the triangular prism (above) = Area of triangle x length

$= (\frac{1}{2} \times 10 \times 6) \times 20$

$= 30 \times 20$

Volume = 600 cm^3

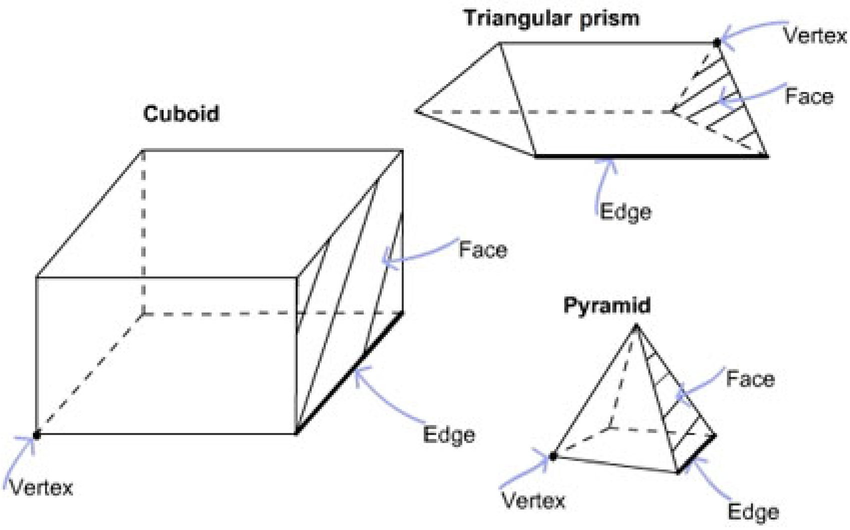
3D Objects, Nets and the Circle

3D Objects

The names of the different parts of an object are:

Faces – These are the flat surfaces.
Vertices – These are the corners (vertex for one).
Edges – These are the sides of the faces.

These are shown in the diagram below, for different objects.



Note: For all these there is a formula connecting number of faces (F), edges (E), and vertices (V). The numbers can be seen in the table below.

Object	**Vertices**	**Faces**	**Edges**
Cuboid	8	6	12
Triangular Prism	6	5	9
Pyramid	5	5	8

Check that this formula works:

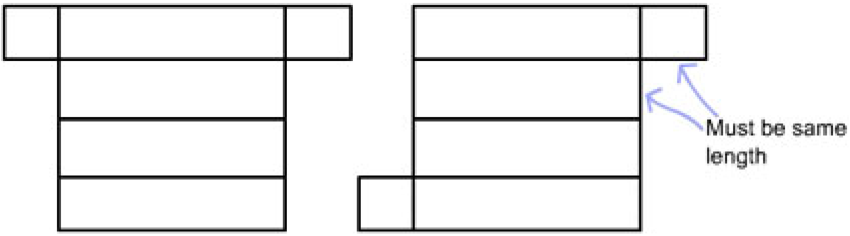
$$\boxed{V + F = E + 2}$$

Drawing 3D objects on plain or isometric paper should be practiced.

Nets

The net of a 3D object is a drawing of the object, ‘flattened out’, showing all the faces.

For example: The net of a cuboid:

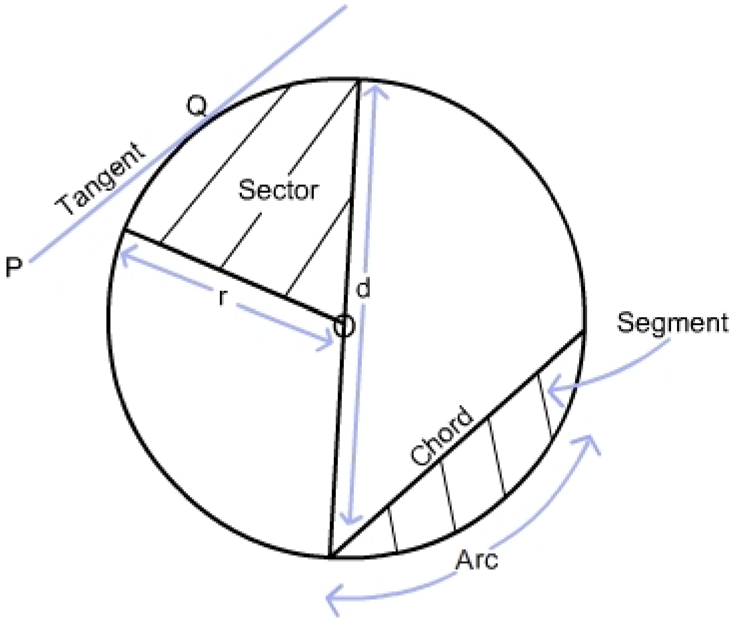


This can be drawn in a number of ways. To check, it must fold up without any overlaps.

Two nets of the cuboid are shown above. Can you think of more?

Circle

The following diagram shows all the parts of the circle, labelled. These parts of the circle need to be learned and are tested in the exam.



Key: Radius (r) Centre (O) Circumference (C) Diameter (d)

Sector – Area between 2 radii. Arc - Part of the circumference.

Chord – Line from one point on the circumference to another.

Segment – Area cut off by a chord.

Tangent – Straight line from a point (P) outside the circle touching it at one point only (Q).

Note: The diameter is twice the radius.

$$\boxed{d = 2r}$$

For example:

1) If $r = 6m$, calculate the diameter.

$$d = 2 \times 6 = 12 \text{ m}$$

2) If $d = 18cm$, calculate the radius.

$$18 = 2 \times r$$
$$r = 9 \text{ cm}$$