XT - MATHS Grade 12

Subject: Series and Sequences 1: Arithmetic **Date:** 2010/06/29

Total Marks: 84

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1. FALSE 10

Explanation: This series is arithmetic as …

$$d = 13 - 11 \quad \text{and} \quad d = 15 - 13$$
$$= 2 \qquad\qquad\qquad = 2$$

The sum of an arithmetic series is given by …

$$S_n = \frac{n}{2}[\ 2a + (n-1)d\ ]$$

a represents the first term of this series: $a = 11$
n represents the number of terms in this series: $n = 10?$
d represents the constant difference between the terms of this series: $d = 2$

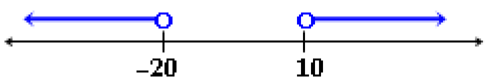
Then …

$$S_n = \frac{n}{2}[\ 2(11) + (n-1)(2)\ ]$$
$$S_n = \frac{n}{2}[\ 22 + 2n - 2\ ]$$
$$S_n = \frac{n}{2}[\ 20 + 2n\ ]$$

The sum of this series must be larger than 200.
Therefore …

$$\frac{n}{2}[\ 20 + 2n\ ] > 200$$
$$n\ [\ 10 + n\ ] > 200$$
$$10n + n^2 > 200$$
$$n^2 + 10n - 200 > 0$$
$$(n-10)(n+20) > 0$$

		-20		10	
$(n-10)$:	$-$	$-$	$-$	0	$+$
$(n+20)$:	$-$	0	$+$	$+$	$+$
	$+$	0	$-$	0	$+$



As the number of terms cannot be negative or a fraction, $n > 10$, i.e. only eleven or more terms will give a sum of larger than 200.

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2. FALSE 4

Explanation: The first and second terms of an arithmetic progression are 13 and 19.
Therefore …

$$d = T_2 - T_1$$
$$= 19 - 13$$
$$= 6$$

The n^{th} term of an arithmetic progression:

$$T_n = a + (n-1)d$$

a is the first term: $a = 13$
d is the constant difference: $d = 6$
n is the term number: the 35^{th} term must be found,
therefore $n = 35$

Therefore …

$$T_{35} = 13 + (35 - 1)(6)$$
$$= 13 + (34 \times 6)$$
$$= 13 + 204$$
$$= 217$$

The thirty-fifth term is therefore **217** and not **219**.

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3. B 6

Explanation: First determine if this is an arithmetic or a geometric series:

$$d = T_2 - T_1 = 9 - 4 = 5 \qquad d = T_3 - T_2 = 14 - 9 = 5$$

$$r = T_2 \div T_1 = 9 \div 4 = \frac{9}{4} \qquad r = T_3 \div T_2 = 14 \div 9 = \frac{14}{9}$$

There is a constant difference between successive terms and not a constant ratio, therefore this series is arithmetic.

The sum formula for an arithmetic progression:

$$S_n = \frac{n}{2}[\ 2a + (n - 1)d\ ] \quad \text{OR} \quad S_n = \frac{n}{2}[\ a + \ell\ ]$$

a represents the first term of this series: $a = 4$
ℓ represents the last term of this series: $\ell = 54$
n represents the number of terms in this series: $n = ?$
d represents the constant difference between the terms of this series: $d = 5$
T_n represents the last term of this series: $T_n = 54$
S_n is the sum of n terms of this series: $S_n = ?$

The number of terms must be determined before the sum can be calculated.

The n^{th} term of anarithmetic progression:

$$T_n = a + (n - 1)d$$
$$54 = 4 + (n - 1)(5)$$
$$54 = 4 + 5n - 5$$
$$5n = 55$$
$$n = 11$$

Now …

$$S_{11} = \frac{11}{2}[\ 2(4) + (11 - 1)(5)\ ]$$
$$= \frac{11}{2}[\ 8 + 50\ ] \quad \text{OR} \quad S_{11} = \frac{11}{2}[\ 4 + 54\ ]$$
$$= \frac{11}{2} \times 58 \qquad = \frac{11}{2} \times 58$$
$$= 11 \times 29 \qquad = 11 \times 29$$
$$= 319 \qquad = 319$$

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4. C 8

Explanation: The multiples are: $24\ ;\ 27\ ;\ 30\ ;\ \dots\ ;\ 117\ ;\ 120$
This is an arithmetic progression as **3** is added to each term to get the next term.
To find the number of terms to be added, you need to use the T_n-formula of an AP:

$$T_n = a + (n-1)d$$
$$120 = 24 + (n-1)(3)$$
$$120 = 24 + 3n - 3$$
$$3n = 99$$
$$n = 33$$

Now …

$$S_n = \frac{n}{2}[\ 2a + (n-1)d\ ]$$
$$S_{33} = \frac{33}{2}[\ 2(24) + (33-1)(3)\ ]$$
$$S_{33} = \frac{33}{2}[\ 48 + 96\ ]$$
$$S_{33} = \frac{33}{2} \times 144$$
$$S_{33} = 33 \times 72$$
$$S_{33} = 2\ 376$$

OR

$$S_n = \frac{n}{2}[\ a + \ell\ ]$$
$$S_{33} = \frac{33}{2}[\ 24 + 120\ ]$$
$$S_{33} = \frac{33}{2} \times 144$$
$$S_{33} = 33 \times 72$$
$$S_{33} = 2\ 376$$

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5. D 4

Explanation: To determine whether a sequence is arithmetic, you need to find the difference (d) between consecutive terms.

A:

$d = T_2 - T_1$ or $d = T_3 - T_2$ or $d = T_3 - T_2$

$= 7 - 9$ $= 5 - 7$ $= 3 - 5$

$= -2$ $= -2$ $= -2$

The answers are the same, therefore sequence A is arithmetic.

B:

$d = T_2 - T_1$ or $d = T_3 - T_2$

$= 9 - 3$ $= 27 - 9$

$= 6$ $= 18$

The answers are not the same, therefore sequence B is not arithmetic.

C:

$d = T_2 - T_1$ or $d = T_3 - T_2$

$= (3x + 3) - (x + 4)$ $= (5x + 2) - (3x + 3)$

$= 3x + 3 - x - 4$ $= 5x + 2 - 3x - 3$

$= 2x - 1$ $= 2x - 1$

The answers are the same, therefore sequence C is arithmetic.

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6. 12 8

Explanation:

$$\sum_{k=1}^{n}(2k-1) = 144$$
$$[2(1) - 1] + [2(2) - 1] + [2(3) - 1] + ... = 144$$
$$1 + 3 + 5 + ... = 144$$

The terms of this series differ with 2, this series is therefore arithmetic.

The sum formula for an arithmetic progression:

$$S_n = \frac{n}{2}[\ 2a + (n-1)d\ ] \quad \text{OR} \quad S_n = \frac{n}{2}[\ a + \ell\ ]$$

a represents the first term of this series: $a = 1$

ℓ represents the last term of this series: $\ell = 2n - 1$

n represents the number of terms in this series: $n = ?$

d represents the constant difference between the terms of this series: $d = 2$

T_n represents the last term of this series: $T_n = 2n - 1$

S_n is the sum of n terms of this series: $S_n = 144$

Now …

$$S_n = \frac{n}{2}[\ 2a + (n-1)d\ ] \qquad \text{OR} \qquad S_n = \frac{n}{2}[\ a + \ell\ ]$$

$$144 = \frac{n}{2}[\ 2(1) + (n-1)(2)\ ] \qquad 144 = \frac{n}{2}[\ 1 + (2n-1)\ ]$$

$$144 = \frac{n}{2}[\ 2 + 2n - 2\ ] \qquad 144 = \frac{n}{2}[\ 2n\ ]$$

$$144 = n^2 \qquad 144 = n^2$$

$$n = \pm 12 \qquad n = \pm 12$$

As n represents the number of terms, -12 is an invalid solution.

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7. - 646 4

Explanation: The S_n-formula for an arithmetic progression:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

a is the first term: $a = 11$

d is the constant difference: $d = -5$

n is the number of terms to be added: $n = 19$

Therefore …

$$\begin{aligned} S_{19} &= \frac{19}{2}\left[2(11) + (19-1)(-5)\right] \\ &= \frac{19}{2}\left[22 + (18)(-5)\right] \\ &= \frac{19}{2}\left[22 - 90\right] \\ &= \frac{19}{2} \times (-68) \\ &= 19 \times (-34) \\ &= -646 \end{aligned}$$

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8. 20,5 2

Explanation:

$$\begin{aligned} \text{Arithmetic Mean} &= \frac{T_1 + T_2}{2} \\ &= \frac{16 + 25}{2} \\ &= \frac{41}{2} \\ &= 20{,}5 \end{aligned}$$

9. (1) ***1, 3, 4, 7, 11, 18*** 6

(2) is not

(3) Lucas

Explanation: 1. This is a second order difference equation, where the first two terms are given.

$T_{n+2} = T_{n+1} + T_n,\ \ T_1 = T_2 = 1$
$T_1 = 1$
$T_2 = 3$
$T_3 = T_2 + T_1 = 3 + 1 = 4$
$T_4 = T_3 + T_2 = 4 + 3 = 7$
$T_5 = T_4 + T_3 = 7 + 4 = 11$
$T_6 = T_5 + T_4 = 11 + 7 = 18$

2. The difference between the terms is not constant.
Therefor this is not an arithmetic pattern.

3. Eduoardo Lucas discovered this number pattern, when he compared different starting points to the better known Fibonacci series.
How nice to have a number pattern named after you!

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10. (1) ***74, 102*** 6

(2) quadratic

(3) $T_n = T_{n-1} + 4n,\ \ T_1 = -6$

Explanation: 1. Study the pattern: ***-6, 2, 14, 30, 50 ...***
Now determine the differences: ***8 12 16 20***
Determine the differences of the differences: ***4 4 4***

To determine the next two terms, this pattern of differences must be maintained.
Hence the sixth and seventh terms must be ***24*** and ***28*** larger than its predecessors respectively.
$T_6 = 50 + 24 = 74$ and
$T_7 = 74 + 28 = 102$

2. A linear pattern has its first difference constant.
A quadratic pattern has its second difference constant.
A cubic pattern has its third difference constant.

3. The formula $T_n = 2n^2 + 2n - 10$ generates the pattern, but it is an explicit formula, ie not a recursive formula.

The formula $T_{n+1} = T_n + 4n,\ \ T_1 = -6$ generates
$T_1 = -6$
$T_2 = T_1 + 4(2) = -6 + 8 = 2$
$T_3 = T_2 + 4(3) = 2 + 12 = 14$
$T_4 = T_3 + 4(4) = 14 + 16 = 30$
$T_5 = T_4 + 4(5) = 30 + 20 = 50$
This is the pattern required.

3. The formula $T_{n+1} = 2T_n + 14,\ \ T_1 = -6$ generates:
$T_1 = -6$
$T_2 = 2T_1 + 14 = 2(-6) + 14 = -12 + 14 = 2$
$T_3 = 2T_2 + 14 = 2(2) + 14 = 4 + 14 = 18$
This is not the pattern required.

11. (1) $a + 12d = 14$ 6

(2) 5

(3) -46

Explanation: Substitute the two sets of given information into the T_n-formula for an arithmetic progression:

$$T_n = a + (n-1)d$$

6th term is − 2 : $T_6 = a + (6-1)d$

$$-21 = a + 5d \quad .... \quad [1]$$

13th term is 14 : $T_{13} = a + (13-1)d$

$$14 = a + 12d \quad .... \quad [2]$$

Subtract [1] from [2]:

$$-21 = a + 5d \quad .... \quad [2]$$

$$14 = a + 12d \quad .... \quad [1]$$

$$-35 = -7d$$

$$\therefore \quad 5 = d$$

a represents the first term of the sequence.
To find the value of a, substitute $d = 5$ into $-21 = a + 5d$:

$$-21 = a + 5(5)$$

$$-21 = a + 25$$

$$a = -21 - 25$$

$$a = -46$$

12. 3n - 10 3
- 10 + 3n

Explanation: The n^{th} term of an arithmetic sequence:

$$T_n = a + (n-1)d$$

a is the first term of the sequence, therefore $a = -7$.
d is the difference between the terms of the sequence:

$$d = T_2 - T_1 \qquad \text{or} \qquad d = T_3 - T_2$$

$$= (-4) - (-7) \qquad = (-1) - (-4)$$

$$= -4 + 7 \qquad = -1 + 4$$

$$= 3 \qquad = 3$$

Therefore:

$$T_n = -7 + (n-1)(3) \qquad [\ n \text{ remains } n \text{ as it is unknown }]$$

$$= -7 + 3n - 3$$

$$= 3n - 10$$

13. 23 10

Explanation:

$$\sum_{t=1}^{k}(2t-3) = [2(1)-3] + [2(2)-3] + [2(3)-3] + ... + [2(k)-3]$$

$$= -1 + 1 + 3 + ... + (2k-3)$$

This is an arithmetic series with $d = 2$.

The sum formula for an AP:

$$S_n = \frac{n}{2}[\ 2a + (n-1)d\ ]$$

a represents the first term of this series: $a = -1$
n represents the number of terms in this series: $n = k$
d represents the constant difference between the terms of this series: $d = 2$
S_n is the sum of k terms of this series: $S_k < 528$

Now …

$$S_k = \frac{k}{2}[\ 2(-1) + (k-1)(2)\ ]$$

$$= \frac{k}{2}[\ -\ 2 + 2k - 2\ ]$$

$$= \frac{k}{2}[\ 2k - 4\ ]$$

$$= k^2 - 2k$$

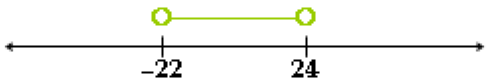
Therefore …

$$k^2 - 2k < 528$$

$$k^2 - 2k - 528 < 0$$

$$(k-24)(k+22) < 0$$

		-22		24	
$(k-24)$:	$-$	$-$	$-$	0	$+$
$(k+22)$:	$-$	0	$+$	$+$	$+$
	$+$	0	$-$	0	$+$



As k represents the term number of the last term, only $1;\ 2;\ 3;\ \ldots;\ 23$ can be valid values for k ($-22 < k < 24$).
Therefore, the greatest value that k could have is 23.

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14. A 2

Explanation:

$$T_n = 3n - 2$$

Therefore :

$$T_8 = 3(8) - 2$$

$$= 24 - 2$$

$$= 22$$

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15. B 5

Explanation: In this arithmetic sequence: $a = 3$, $d = ?$, $n = 4$ and $T_n = 63$
Then ...

$$T_n = a + (n-1)d$$

$$63 = 3 + (4-1)d$$

$$60 = 3d$$

$$20 = d$$

Therefore:

$$x = T_2 \quad \text{and} \quad y = T_3$$

$$= T_1 + d \qquad = T_2 + d$$

$$= 3 + 20 \qquad = 23 + 20$$

$$= 23 \qquad = 43$$