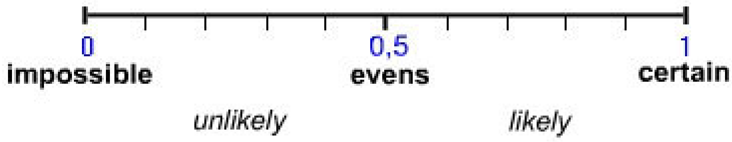
PROBIBILITY

Understanding Probability

Probability Scale and Formula

Probability is a measure of how likely an event is to happen.
A scale is used from zero to one, as shown below.



Examples

The probability of throwing a heads with a two-headed coin is 1 (certain).
The probability of throwing a tails is 0 (impossible).
The probability of throwing a heads with a fair coin is 0,5 (evens).

Calculating probability

If all the outcomes are equally likely, then the probability an event A happening is given by this formula:

$$\boxed{P(A) = \frac{\text{number of ways the event A can occur}}{\text{number of possible outcomes}}}$$

Equally Likely Outcomes

When a dice is thrown and the number shown is recorded, there are six possible results:

1,2,3,4,5,6.

If the dice is a perfect cube, each of these numbers is equally likely to come up. We say that there are six equally likely outcomes.

Example 1: The probability of getting an even number with a normal dice is as follows:

P (even)= 3/6
= 1/2 (3 even numbers 2, 4, 6)
(6 equally likely outcomes)

Example 2: The probability of getting a number greater than 4.

P (a number greater than 4) = 2/6
= 1/3 (5 and 6 are greater than 4)

Example 3: The probability of an event not happening e.g. not getting the number 6.
P (6) = 1/6 (dice)

P (not a 6) = 1 – 1/6
= 5/6

This is because it is a certainty that the event will or will not happen.

Example 4: If the probability of winning a tennis match is 0,3, what is the probability of losing?

P (winning at tennis) = 0,3

P (losing) = 1 – 0,3

= 0,7

Experimental Probability

Probability can be examined by experiment. For example, a coin is tossed 100 times and the results are recorded.

Number of heads = 47
Number of tails = 53

The **relative frequency** for heads = 47/100 and for tails = 53/100.

These are approaching the probabilities but only become accurate for a large number of trials.

$$\text{relative frequency} = \frac{\text{the number of times the event happens}}{\text{the number of attempts}}$$

Example: A drawing pin was thrown 50 times and the following results were obtained:

Landing with the sharp end down = 10

Relative frequency = 10/50
= 1/5

Landing with the flat end down = 40

Relative frequency = 40/50
= 4/5

Listing Outcomes

Listing Outcomes

Sometimes a systematic way is needed to list all the outcomes.

Example: Two bags each contain one red, one yellow and one green disc.

A disc is taken from each bag. List all the possible outcomes.

RY	RG	**RR**
YR	YG	YY
GR	GY	**GG**

Note that we start with the red disc from the first bag and list the outcomes, then the yellow disc from the first bag and finally the green disc from the first bag.

This provides us with nine possible outcomes.

Using this list, we can work out the probability of an event.

For example, P (two discs of the same colour) = 3/9 (highlighted)

= 1/3

Possibility Space

For more complicated situations, we can use a table to list all the outcomes.

Example: Throwing a coin and a dice, list all the outcomes.

This can be done by using a possibility space, as shown below.

Heads	1H	**2H**	3H	**4H**	5H	**6H**
Tails	1T	2T	3T	4T	5T	6T
Number	1	2	3	4	5	6

There are 12 possible outcomes. It can be used to calculate probabilities as follows:

Calculate the probability of getting an even number and a head.

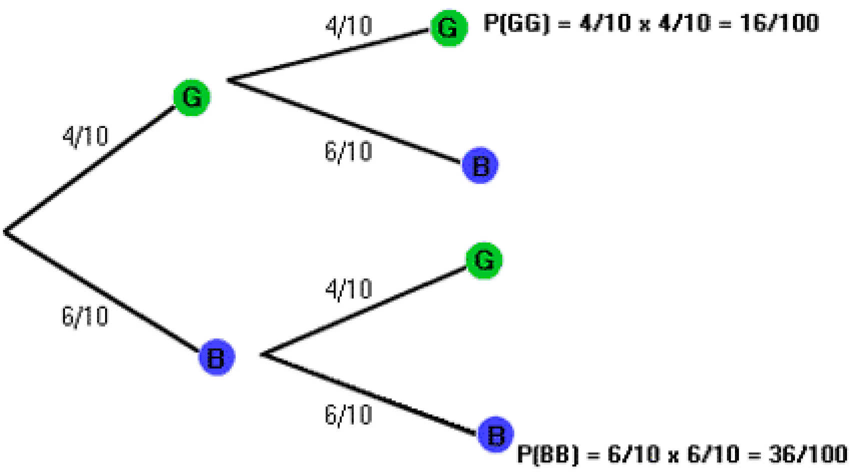
P (even number and a head) = 3/12 (highlighted above)
= 1/4

Tree Diagrams

This is a way of setting down the outcomes with their probabilities for two or more events (combined).

Example: A bag contains four green and six blue discs. A disc is selected at random and replaced, then another one is selected. What is the probability that both discs are the same colour?

The outcomes are put on the end of branches as shown. The probabilities are put on the branches.



For both discs to be the same colour, we need two greens or two blues.

When moving along the branches, we multiply the probabilities as follows:

P (GG) = 4/10 x 4/10
= 16/100

When moving across the branches, we add the probabilities.

So P (GG or BB) = 16/100 + 36/100
= 52/100
= 13/25

Expected Probability

In experimental probability:

expected number of successes = P (event) x the number of trials (large number of trials)

Example: For one thousand throws of a coin, the expected number of heads = 1/2 x 1 000 = 500.

And/Or Rules

Rules and Examples

The P (GG) can be written as P (G) and P (G). Use of the word 'and' indicates that we multiply the probabilities.

When the word 'or' can be used, it indicates (+), so we add the probabilities.

Example 1: In a bag of green and blue discs, what is the probability of selecting two discs of different colours, if the first disc is returned before selecting the second.

P(different colours) = P(GB) or P(BG)

= (4/10 x 6/10) + (6/10 x 4/10)

= 48/100

= 0,48

Example 2: Twins Lerato and Kagiso are taking a driving test. If the probability of Lerato passing is 0,6 and the probability of Kagiso passing is 0,2, what is the probability of only one twin passing?

P (Lerato fails) = 1 – 0,6 = 0,4

P (Kagiso fails) = 1 – 0,2 = 0,8

If only one twin passes then P (Lerato passes and Kagiso fails) = 0,6 x 0,8 = 0,48

OR

P (Lerato fails and Kagiso passes)= 0,4 x 0,2 = 0,08

So P (only one twin passes) = 0,48 + 0,08

= 0,56

Mutually Exclusive Events

P (A or B) = P (A) + P (B) can only be used when A and B are mutually exclusive events, i.e. they cannot happen together.

Example 1: Three cards are presented (4 of Clubs, 3 of Diamonds and 5 of Spades). If one is selected, what is the probability of getting a 4 or a Club?

We cannot say P (4 or Club) = P (4) + P (Club) because they can happen together. The 4 can also be a Club, so the events are not mutually exclusive.

Example 2: The probability that the weather will be good on Monday is 0,3.

If it is good, the probability that I can get on my bus is 0,8. If it is not good, the probability that I can get on my bus is 0,2.

Find the probability that I can get on my bus on Monday.

P (weather fine and getting on bus) = 0,3 x 0,8

= 0,24

OR

P (weather not fine and getting on bus) = 0,7 x 0,2

= 0,14

P (getting on bus) = 0,24 + 0,14

= 0,38

Example 3: When Jane goes shopping, the probability she travels by bus is 0,25, by taxi is 0,1 and on foot is 0,6. What is the probability that she returns (a) by bus or taxi, (b) by bus or on foot?

a) By bus or taxi

P = 0,25 + 0,1
= 0,35

b) By bus or on foot
P = 0,25 + 0,6
= 0,85

Conditional Probability

Example

This is when the probability of an event is influenced by a previous event.

Example: Two discs are selected from a bag of six blue and four green discs, without replacing the first disc.

a) What is the probability of getting two discs of the same colour?

P (B) = 6/10 for the first disc.

P (B) = 5/9 for the second disc because there are only 5 blue discs left and 9 discs in the bag.

So P (BB) = 6/10 x 5/9 = 30 /90

P (GG) = 4/10 x 3/9 =12/90

P (same colour) = 30/90 +12/90

= 42/90

= 7/15

b) What is the probability of getting two different coloured discs?

P (BG) = 6/10 x 4⁄9 = 24⁄90
P (GB) = 4/10 x 6⁄9 = 24⁄90
P (different colour) = 24⁄90 + 24⁄90

= 48⁄90

= 16⁄30

= 8/15

Harder Problems

'At least' problems

Example 1: In the conditional probabilities problem above, what is the probability of selecting at least one green disc?

In this case, the only outcome we are not interested in is two blue discs. So we can write:

P (at least 1 G) = 1 – P (BB)

= 1 – 1⁄3

= 2⁄3

This can be written as a general rule:

$\boxed{\text{P (at least one) = 1 – P (none)}}$

Other problems

Example 2: Boris and Aziz play three sets of tennis. The probability of Boris winning the first set is 0,4.

If Boris wins a set, the probability of his winning the next set is 0,7.

If Aziz wins a set, the probability of his winning the next set is 0,8.

a) Calculate the probability that Aziz wins all three sets.

P (Aziz wins 3 sets) = 0,4 x 0,7 x 0,7
= 0,196

b) Calculate the probability that Boris wins at least one set.

P (Boris wins at least one set) = 1 – P (wins none)
= 1 – 0,196
= 0,804

Example 3: When I go to work, I have to pass two sets of traffic lights, A and B.

The probability that I have to stop at A is 0,4.

If I have to stop at A, the probability that I have to stop at B is 0,8.

If I do not stop at A, the probability that I have to stop at B is 0,3.

What is the probability that I will have to stop at both A and B?

Calculate the probability that I have to stop at least once.

P (stop and stop) = 0,4 x 0,8
= 0,32

P (not stop at A) = 1 – 0,4

= 0,6

P (not stop at B) = 1 – 0,3

= 0,7

P (stop at least once) = 1 – P (none)
= 1 – (0,6 x 0,7)
= 0,58