

NUMBER

Types of Number

Prime Numbers

A prime number is a number which can only be divided by 1 or itself.

The smallest prime number is 2.

2 can only be divided by 2 or 1.

Here are the first 20 prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71.

Some numbers you may not be sure about, for example: 51. In this case, try dividing by 3 or 7.

e.g. $51 \div 3 = 17$

So 51 is not a prime number.

Square Numbers

A square number is formed by multiplying a number by itself.

For example, $2 \times 2 = 4$.

4 is a square number.

Here are the first ten square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

The **square root** of a square number is an integer (a whole number).

For example, the square root of 16 is 4.

The meaning of square root is, 'that figure which multiplied by itself gives the number'. So, 4 is the square root of 16 because $4 \times 4 = 16$. The symbol for square root is $\sqrt{\quad}$ for example $\sqrt{16} = 4$.

Cubic Numbers

A cubic number is formed by multiplying a number by itself three times.

For example, $2 \times 2 \times 2 = 8$.

8 is a cubic number.

The first five cubic numbers are: 1, 8, 27, 64, 125.

Factors

Factors are numbers which will divide into a number without any remainder.

Example 1: Factors of 12 are 1, 2, 3, 4, 6 and 12, because:

$$12 \div 1 = 12$$

$$12 \div 2 = 6$$

$$12 \div 3 = 4$$

$$12 \div 4 = 3$$

$$12 \div 6 = 2$$

$$12 \div 12 = 1$$

Example 2: Factors of 20 = 1, 2, 4, 5, 10 and 20.

Note: All prime numbers have only two factors i.e. 1 and themselves.

For example, factors of 7 = 1 and 7 only.

Multiples

Multiples are formed by multiplying by any number.

Multiples of 5 = 5, 10, 15, 20, etc.

Because $5 \times 1 = 5$

$$5 \times 2 = 10$$

$$5 \times 3 = 15$$

$$5 \times 4 = 20$$

Reciprocal simply means 'one over.'

For example, the reciprocal of 4 is $\frac{1}{4}$.

Four Rules of Number

Introduction

The four rules of number are Add, Subtract, Multiply and Divide.

The symbols are +, −, x and ÷.

Other words that are sometimes used for these operations include:

- **Add:** sum, total, plus
- **Subtract:** difference, minus, take away
- **Multiply:** product, times
- **Divide:** share, quotient

Adding

Adding, for example, $156 + 378$.

Always set the sum down with the figures under one another in columns, as below:

$$\begin{array}{r} 156 \\ + 378 \\ \hline 534 \end{array}$$

Subtracting

Subtracting, for example, $734 - 589$.

$$\begin{array}{r} \overset{-7^1}{-} \overset{3^1}{3} 4 \\ - 589 \\ \hline 145 \end{array}$$

We say that we cannot take 9 from 4 so we borrow a 10 from the next column and take 9 from 14, leaving 5.

Having borrowed from the 3, it must now be reduced to 2.

So 8 must be taken from 2.

Again, we borrow and take 8 from 12.

In the last column we take 5 from 6.

Therefore, $734 - 589 = 145$.

Multiplying

Short multiplication: For example, 75×4 .

Set this down as:

$$\begin{array}{r} 75 \\ \times 4 \\ \hline 300 \end{array}$$

4×5 is 20 so we put down the 0 and carry the 2.

Next, 4×7 is 28, add on the 2, giving 30.

Long multiplication: For example, 45×32 .

$$\begin{array}{r} 46 \\ \times 32 \\ \hline 92 \quad (46 \times 2) \\ 1380 \quad (46 \times 30) \\ \hline 1472 \end{array}$$

Note: Put down the 0 and multiply by 3.

Dividing

Short division: For example, $56 \div 2 = 28$

We say 2 goes into 5 and the answer is 2 remainder 1.

This 1 is put in the 6 column as a 10, giving 16.

Again, we say 2 goes into 16, 8 times, giving the answer 28.

Long division: For example, $7682 \div 14 =$

$$\begin{array}{r} 543 \\ 14 \overline{)7682} \\ \underline{70} \\ 68 \\ \underline{64} \\ 42 \\ \underline{42} \\ 0 \end{array}$$

$14 \times 5 = 70$ so 14 goes into 76, 5 times remainder 6.

We bring down the 8 to make 68.

Again 14 goes into 68, 4 times remainder 4, because $14 \times 4 = 64$.

We bring down the 2 to make 42.

Now 14 goes into 42, 3 times, because $14 \times 3 = 42$, no remainder.

BODMAS

B O D M A S tells us the order in which to do things when simplifying an expression.

B for brackets

O for order

D for division

M for multiplication

A for addition

S for subtraction

For example, work out $5 + 3 \times 2 = 5 + 6 = 11$.

We must do the multiplication before the addition.

$$(5 + 3) \times 2 = 8 \times 2 = 16.$$

In this case, we must do the brackets before the multiplication.

$$12 - 6 \div 2 = 9.$$

Here we do the division before the subtraction.

Directed Numbers

Positive and Negative Numbers

Number line: -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

This is a section of the number line from -10 to $+10$.

Note, we can write positive numbers without the $+$ sign.

With these numbers, we will have **double signs:** $+ -$, $++$, $--$ and $- +$.

It is easier to replace these with single signs: $+$ or $-$.

$- +$ can be replaced with $-$.

$++$ or $--$ can be replaced with $+$.

Example 1:

$$6+ - 4 = 6 - 4 = 2$$

$$6- + 4 = 6 - 4 = 2$$

$$6++ 4 = 6 + 4 = 10$$

$$6-- 4 = 6 + 4 = 10$$

Rule: If the double signs are different, they are $-$.

If they are the same, they are $+$.

We can now add and subtract on the number line.

Example 2:

$$4 - 7 = -3$$

We start at 4 on the number line and move 7 places to the left, as illustrated below.

-7 - 6 - 5 - 4 - 3 - 2 - 1 0 1 2 3 4 5 6 7

\wedge \wedge
 End Start
 Move 7

$$-3 + 7 = 4$$

We start at -3 and move 7 places to the right, as shown below.

-4 -3 -2 -1 0 1 2 3 4 5 6

\wedge \wedge
 Move 7

$$-1 - 5 = -6$$

We start at -1 and move 5 to the left.

Rule: Start with the first number and move right for $+$ and left for $-$.

Multiplying and Dividing

We need to learn two new rules.

Rule 1: When multiplying or dividing two numbers with the same signs, we get a positive number.

For example, $+3 \times +5 = +15$; $-3 \times -5 = +15$; $+6 \div +3 = +2$; $-6 \div -3 = +2$

Rule 2: When multiplying or dividing two numbers with different signs, we get a negative number.

For example, $+3 \times -5 = -15$; $-3 \times +5 = -15$; $6 \div -2 = -3$; $-6 \div 2 = -3$

Decimals

Place Value

The figures in a decimal have a value, depending on their place in the number.

The value is given by the following headings:

Headings	Tens	Units	,	Tenths	Hundredths
Number	6	5	,	2	5

The first figure 5 is 5 units because it is in the units column.

The second figure 5 is 5 hundredths because it is in the hundredths column.

When working with decimals, it is important to keep the figures in the correct columns.

Adding and Subtracting Decimals

Rule: Keep the points in line and add or subtract in columns.

Example: $6,23 + 17,58 + 0,452$

$$\begin{array}{r} 6,23 \\ 17,58 \\ + 0,452 \\ \hline 24,262 \end{array}$$

Otherwise, we add in the same way as integers and subtract.

$$\begin{array}{r} 45,37 \\ - 3,76 \\ \hline 41,61 \end{array}$$

Multiplying Decimals

a. Short multiplication

$$\begin{array}{r} 4,5 \times 7 = 4,5 \\ \times \quad 7 \\ \hline 31,5 \end{array}$$

Done in the same way as integers but putting in the decimal comma as we reach it.

b. Long multiplication

$$4,32 \times 5,2$$

Treat as $432 \times 52 = 22\,464$

We do this in the same way as long multiplication of integers. Then put the decimal comma in the answer, in the following way.

4,32 has two figures after the comma.

5,2 has one figure after the comma.

$2 + 1 = 3$, which gives the number of figures after the point in the answer.

$$22,464$$

$$4,32 \times 5,2 = 22,464$$

Division of Decimals

a. Dividing a decimal by a whole number (an integer)

Done in the same way as dividing integers but keep the decimal point in line in the answer.

Example: $12,45 \div 5 = 2,49$

$$\begin{array}{r} 2,49 \\ 5 \overline{) 12,45} \\ \underline{10} \\ 24 \\ \underline{20} \\ 45 \\ \underline{40} \\ 50 \\ \underline{50} \\ 0 \end{array}$$

^
commas in line

b. Dividing a decimal by a decimal

$$6,25 \div 0,5 = 12,5$$

$$\times 10 \quad \times 10$$

We make the 0,5 into a whole number by multiplying by 10. We must also do the same to the 6,25.

$$62,5 \div 5 = 12,5$$

Rule: To divide by a decimal, we must make it into a whole number by multiplying by 10, 100, etc. We must multiply the other number by the same amount.

We can deal with remainders: $17 \div 4 = 4,25$.

$$\begin{array}{r} 4,25 \\ 4 \overline{) 17,00} \\ \underline{16} \\ 10 \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

As many zeros as we need can be added after the decimal point and we continue to divide as for whole numbers.

Other Operations

Multiplying and dividing decimals by 10, 100...

$$5,76 \times 10 = 57,6$$

$$5,76 \times 100 =$$

$$576,0$$

$$165,8 \div 10 = 16,58$$

$$165,8 \div 100 = 1,658$$

Note: We can use a simple rule to describe these results.

The point moves by the same number of places as there are zeros.

It moves to the right for multiplication and to the left for division.

$5,76 \times 1\,000 = 5\,760,0$
(Point moves three places to right)

$165,8 \div 1\,000 = 0,1658$
(Point moves three places to left)

Changing a decimal to a fraction

		,	Tenths	Hundredths
0,25	=	0,	2	5

Put the decimal over its last heading:

$$0,25 = \frac{25}{100} = \frac{1}{4}$$

FRACTIONS

Equivalent Fractions

This rectangle has been divided into eight equal parts.

Each part is one eighth of the rectangle ($\frac{1}{8}$).

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Shading fractions of diagrams

We can shade three quarters of the rectangle as below.

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
---------------	---------------	---------------	---------------

Note that each quarter is made up of two eighths. So $\frac{3}{4}$ is the same as $\frac{6}{8}$.

These are called **equivalent fractions**. We say $\frac{3}{4} = \frac{6}{8}$.

If we multiply the top and the bottom of a fraction by the same number, we get an equivalent fraction.

$$\frac{1}{2} = \frac{3}{6} \text{ (x top and bottom by 3).}$$

$$\frac{2}{3} = \frac{8}{12} \text{ (x top and bottom by 4).}$$

Other Types of Fractions

1. Improper or top-heavy fractions:

Where the top number is bigger than the bottom. For example, $\frac{3}{2}$ is a top-heavy fraction.

2. Mixed numbers:

This is a mixture of whole numbers and fractions. For example, $1\frac{1}{2}$.

This can be changed to a top-heavy fraction as follows:

$$1\frac{1}{2} = \frac{2 \times 1 + 1}{2} = \frac{3}{2}$$

We multiply the whole number by the bottom number of the fraction and add the top of the fraction, i.e. $2 \times 1 + 1 = 3$, over the bottom, which gives $\frac{3}{2}$

e.g. $5\frac{2}{3} = \frac{3 \times 5 + 2}{3} = \frac{17}{3}$

Changing a top-heavy to a mixed fraction:

$\frac{17}{3} = 17 \div 3 = 5$, remainder 2

This is written as 5 and $\frac{2}{3}$ ($5\frac{2}{3}$).

Cancelling a Fraction

This involves converting a fraction to its simplest form (or lowest terms).

We do this by dividing the top and bottom by the same number.

Example: $\frac{5}{10} = \frac{1}{2}$ (\div top and bottom by 5)

$\frac{6}{8} = \frac{3}{4}$ ($\div 2$)

$\frac{12}{20} = \frac{3}{5}$ ($\div 4$)

Note: $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{3}{5}$ cannot be simplified further.

Writing as a fraction

Example: I have 20 sweets and I eat 15. What fraction have I eaten?

The fraction eaten is 15 out of 20. This is written as a fraction, i.e. $\frac{15}{20}$.

We must always simplify so $\frac{15}{20} = \frac{3}{4}$ ($\div 5$).

Finding 'a fraction of'

Example 1: What is $\frac{3}{4}$ of 8?

Rule: Divide by the bottom and multiply (times) by the top.

$$8 \div 4 \times 3 = 6$$

$$\text{So } \frac{3}{4} \text{ of } 8 = 6$$

Example 2: $\frac{2}{5}$ of 15 = $15 \div 5 \times 2 = 6$

Adding and Subtracting

Examples: $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

$$\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$

$$\frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$$

When the bottom numbers are the same, we add or subtract the top numbers.

When the bottom numbers are different, we must make them the same by multiplying.

Example 1: $\frac{1}{2} + \frac{1}{3} = ?$

$$2 \times 3 = 6$$

By making the bottom numbers 6, we must multiply the top numbers by the same amount.

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

In other words, $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$ and these can now be added to get $\frac{5}{6}$.

Example 2: $\frac{4}{5} - \frac{2}{3} = ?$

$$(5 \times 3 = 15)$$

$$\frac{4}{5} = \frac{12}{15} \text{ and } \frac{2}{3} = \frac{10}{15}$$

$$\text{So, } \frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

Mixed numbers

We can add or subtract the whole numbers and then the fractions.

Example:

$$3 \frac{1}{4} + 2 \frac{1}{3} = 5 \frac{7}{12}$$

$$5 \frac{7}{12} - 2 \frac{1}{3} = 3 \frac{3}{12} = 3 \frac{1}{4}$$

Multiplying

When multiplying fractions, we multiply the top numbers and multiply the bottom numbers. Then we simplify the answer, if possible.

$$\text{Example 1: } \frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12} = \frac{1}{2}$$

If the numbers are large, we can cancel diagonally.

$$\text{Example 2: } \frac{15}{16} \times \frac{24}{35} = ?$$

$$15 \text{ and } 35 \text{ cancel by } 5 \text{ giving } \frac{3}{16} \times \frac{24}{7}.$$

$$16 \text{ and } 24 \text{ cancel by } 8 \text{ giving } \frac{3}{2} \text{ and } \frac{3}{7}.$$

$$\text{So } \frac{15}{16} \times \frac{24}{35} = \frac{3}{2} \times \frac{3}{7} = \frac{9}{14}.$$

For mixed numbers

$$\text{Example 3: } 3 \frac{1}{4} \times 2 \frac{1}{3} = ?$$

$$\text{Change to a top-heavy fraction, so } \frac{13}{4} \times \frac{7}{3} = 9 \frac{1}{12} = 7 \frac{7}{12}$$

Dividing

Rule: Change the sign from \div to \times and turn over the second fraction. Then use the same method as multiplying.

Example: $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = 1 \frac{2}{4} = 1 \frac{1}{2}$

For mixed numbers

$$3 \frac{1}{4} \div 2 \frac{1}{3} = \frac{13}{4} \times \frac{3}{7} = \frac{39}{28} = 1 \frac{11}{28}$$

Change to a top-heavy fraction first, then use the above rule.

Changing a Fraction to a Decimal

Rule: Divide the top by the bottom.

Example 1: $\frac{3}{4} = 3 \div 4 = 0,75$

Or $\frac{3}{4} = \frac{75}{100} = 0,75$ (x top and bottom by 25)

Example 2: $\frac{3}{5} = 3 \div 5 = 0,6$

Or $\frac{3}{5} = \frac{6}{10} = 0,6$ (x top and bottom by 2)

Percentages

Understanding Percentages

Percentages are fractions with a denominator of 100.

For example, 50 percent (50 %) = $\frac{50}{100} = \frac{1}{2}$

20 percent (20 %) = $\frac{20}{100} = \frac{1}{5}$

Changing a fraction or decimal to a percentage - simply multiply by 100.

Examples: $\frac{1}{2} \times 100 = 50$ so $\frac{1}{2} = 50\%$
 $\frac{1}{4} \times 100 = 25$ so $\frac{1}{4} = 25\%$
 $\frac{1}{3} \times 100 = 33\frac{1}{3}$ so $\frac{1}{3} = 33\frac{1}{3}\%$
 $0,35 \times 100 = 35$ so $0,35 = 35\%$
 $0,625 \times 100 = 62,5$ so $0,625 = 62,5\%$

Changing a percentage to a decimal or fraction - simply divide by 100.

Examples: $28\% = \frac{28}{100} = \frac{7}{25}$
 $75\% = \frac{75}{100} = \frac{3}{4}$
 $28\% = \frac{28}{100} = 0,28$
 $75\% = \frac{75}{100} = 0,75$

Finding a Percentage

Example 1: Work out 25 % of 40.

Method: Write 25 % as a fraction and then multiply by 40.

$$\begin{aligned} 25\% \text{ of } 40 &= \frac{25}{100} \times 40 \\ &= \frac{1}{4} \times 40 \\ &= 10 \end{aligned}$$

Example 2: Calculate 50 % of 60.

$$\begin{aligned} &= \frac{50}{100} \times 60 \\ &= \frac{1}{2} \times 60 \\ &= 30 \end{aligned}$$

$10\% = \frac{10}{100} = \frac{1}{10}$ so 10 % of a quantity is simply one tenth, which can be worked out by dividing by 10.

$$10\% \text{ of } 30 = 30 \div 10 = 3$$

$$10\% \text{ of } 80 = 8$$

$$10\% \text{ of } 250 = 25$$

$$10\% \text{ of } 16 = 1,6$$

$$10\% \text{ of } 38 = 3,8$$

We can use this to work out other percentages:

$$20\% \text{ of } 30 = 3 \times 2 = 6 \text{ (simply double the 10 \%)}$$

$$30\% \text{ of } 30 = 3 \times 3 = 9 \text{ (x the 10 \% by 3)}$$

$$\begin{aligned} 15\% \text{ of } 30 &= 10\% \text{ of } 30 + 5\% \text{ of } 30 \\ &= 3 + 1,5 \\ &= 4,5 \end{aligned}$$

Using this method, we can work out any percentage which is a multiple of 10 or 5, without using a calculator.

Increasing or Decreasing by a Percentage

Example 1: Increase R40 by 8 %.

Method: Work out the 8 % and add it on.

$$\begin{aligned} 8\% \text{ of } 40 &= \frac{8}{100} \text{ of } 40 \\ &= 40 \div 100 \times 8 \\ &= 3,2 \text{ (calculator method)} \end{aligned}$$

So, 8 % of R40 = R3,20 (note: calculator will give 3,2, but this must be written as R3,20 for money)

Now add this on to R40 to give R43,20.

OR

Work out (100 + 8) % of 40 = 108 % of 40

$$\begin{aligned} &\frac{108}{100} \text{ of } 40 \\ &= 40 \div 100 \times 108 \text{ (on calculator)} \\ &= R43,20 \end{aligned}$$

Example 2: Decrease R40 by 8 %.

In this case, we subtract the 8 %.

$$R40 - 3,20 = R36,80$$

OR

$$100 - 8 = 92 \text{ so } 92 \% \text{ of } 40 = R36,80 \text{ (giving the answer directly)}$$

Example 3: In a sale, all prices are reduced by 30 %. What is the sale price of a coat costing R80 before the sale?

$$\text{Reduction} = 30 \% \text{ of } 80 = R24$$

$$\text{Sale price} = 80 - 24 = R56$$

Writing an Amount as a Percentage

This is the same thing as writing as a fraction, then changing it to a percentage.

Example 1: I score 18 out of 20 in a test. What is this as a percentage?

$$\text{Fraction} = \frac{18}{20}$$

$$\frac{18}{20} \times 100 = 100 \div 20 \times 18$$
$$= 90$$

$$\text{So, } \frac{18}{20} = 90 \%$$

Example 2: I have 40 sweets and I eat 8. What fraction have I eaten?

$$\frac{8}{40} = 20 \%$$

What percentage do I have left?

$$100 - 20 = 80 \text{ so I have } 80 \% \text{ left.}$$

Decimal Places, Significant Figures and Estimates

Approximate Values

To write an approximate value for a number, we can use several methods.

1) Rounding to the nearest whole number

$2,4 = 2$ to the nearest whole number, because it is nearer to 2 than to 3.

$76,2 = 76$ to the nearest whole number, because it is nearer to 76 than to 77.

$7,8 = 8$ to the nearest whole number, because it is nearer to 8 than to 7.

Rule: If the digit after the units is 5 or more, then round up the units. Otherwise leave the units as they are. Note the decimal part is left off.

2) Rounding to the nearest ten

$13 = 10$ (to the nearest ten)

$68 = 70$ (to the nearest ten)

$25 = 30$ (to the nearest ten)

Rule: If the digit after the tens is 5 or more then round up the tens.

In the same way we can round to the nearest 100, 1 000, and so on.

$1\ 467 = 1\ 000$ to the nearest 1 000.
 $= 1\ 500$ to the nearest 100.

Decimal Places

We use similar rules as above but we start counting after the decimal comma.

For example, $3,42 = 3,4$ to one decimal place.

One decimal place is the first figure after the point. Because it is followed by a 2, it stays the same.

$6,58 = 6,6$ to one decimal place.

Because the 5 is followed by an 8 it is rounded to a 6.

$47,3948 = 47,39$ to two decimal places.

The 9 is the second decimal place, followed by a number less than 5, so it stays the same.

Rule: Count the figures after the comma to find the decimal place. If this is followed by a number less than 5, it stays as it is. Otherwise it is rounded up.

3,98 = 4,0 to one decimal place.

Because the 9 is rounded up to a 10 which makes the 3 into a 4. Note the 0 is needed this time.

Significant Figures

Examples for whole numbers

129 = 100 to 1 s.f.

129 = 130 to 2 s.f.

3 684 = 3 680 to 3 s.f.

Note: We start counting from the first figure in the number. Figures left off must be replaced by zeros, otherwise we lose the place value.

45 793 = 50 000 to 1 s.f.

Examples for decimals

0,0047 = 0,005 to 1 s.f.

Note: We start counting from the first non-zero figure.

0,07649 = 0,076 to 2 s.f.

Figures left off are not replaced by zeros.

Estimates

When estimating the answer to a calculation we round off the numbers to 1 s.f. We then use the closest convenient number in order to make the calculation as simple as possible.

Example: Estimate the value of $19,6 \div 3,28$.

19,6 = 20

3,28 = 3 (to 1 s.f.)

$19,6 \div 3,28 = 20 \div 3$ (to 1 s.f.)

$21 \div 3 = 7$ (21 being more convenient)

The estimate is 7.

Note: Estimates should always be used when doing calculations on the calculator, to check the answer. They can also be asked as specific questions on the exam paper.

Ratio

Simplifying Ratios

This is one of the three types of questions that learners can get on ratio.

Example 1: Writing down and simplifying a ratio.

Jane and her sister Sally are 80 cm and 120 cm tall respectively. Write down the ratio of their heights.

$$\begin{array}{ccc} \text{Jane} & & \text{Sally} \\ 80 & : & 120 \end{array}$$

Write down the numbers given with : in between. They must be in the same units.

Then simplify by dividing both sides by the same number.

$$\begin{array}{ccc} 80 & : & 120 \\ & \div & 40 \\ 2 & : & 3 \end{array}$$

The ratio of Jane's height to Sally's is 2 : 3.

If the units are different, for example 90 cm : 1,5 m, then they must be changed to the same as below.

$$\begin{array}{ccc} 90 & : & 150 \\ & \div & 30 \\ 3 & : & 5 \end{array}$$

Dividing in a Given Ratio

Example 1: Share R80 between Tsepo and Pravesh in the ratio 3 : 5.

This means that Tsepo gets 3 parts and Pravesh gets 5 parts.

$3 + 5 = 8$ parts altogether.

So $R80 \div 8 = R10$ (the value of one part)

Tsepo = 3 parts so $3 \times 10 = R30$ and Pravesh = 5 parts so $5 \times 10 = R50$.

The **rules** for this operation are as follows:

- Add the numbers in the ratio.
- Divide the amount by this total.
- Multiply the numbers in the ratio by this value.

Finding the Missing Value

Example 1: A line is divided in the ratio 2 : 3. If the longer section is 15 cm long, how long is the shorter section?

The longer section = 3 parts.

3 parts = 15 cm

1 part = $15 \div 3 = 5$ cm

The shorter section = 2×5 cm = 10 cm.

Direct and Inverse Proportion

Direct Proportion

If one quality is directly proportional to another it changes in the same way.

As it increases, so does the other – as it decreases, the other decreases also.

Example: The cost of sweets is directly proportional to the number of sweets bought.

If 1 sweet costs 10 c, 5 sweets cost $5 \times 10 = 50$ c.

Or

If 10 sweets cost 60 c, 1 sweet costs $60/10 = 6$ c.

Method

For direct proportion, we find the value of 1 by division and then multiply to find the total value.

For example, a car uses 20 litres of petrol in travelling 140 km. How much would be used on a journey of 35 km?

$$1 \text{ km} = \frac{20}{140}$$

$$35 \text{ km} = \frac{20}{140} \times 35 = 5 \text{ litres}$$

Rule: Divide to find 1 and then multiply.

Inverse Proportion

If one quantity is inversely proportional to another, it changes in the opposite way – as it increases, the other decreases.

Example: If 8 men take 4 days to build a wall, how long would it take 2 men (assuming they work at the same rate)?

First, we decide whether the problem is direct or inverse proportion.

In this case, if less men are used, they will take longer, so it is inverse proportion.

Method

8 men take 4 days

1 man takes $8 \times 4 = 32$ days

$$2 \text{ men take } \frac{32}{2} = 16 \text{ days}$$

Again we find the value of 1 **by multiplying**. Then divide to find the final answer.

Note: This process is the opposite of direct proportion.

Boundaries (Upper and Lower Class)

Upper and Lower Boundaries

The degree of accuracy in a measurement is often given in a question. This means that we have a maximum value (upper boundary) and a minimum value (lower boundary).

Example: A length is measured as 15 cm to the nearest cm.

Minimum value = 14,5 cm

This is because 14,4 would be 14 cm to the nearest cm.

Maximum value = 15,49999...cm

We are allowed to write this as 15,5 cm.

Rule: If a measurement is accurate to a given amount, then the boundaries are half of that amount, above and below the measurement.

For example, 2,8 cm to the nearest mm; given amount = 0,1.

$$\text{Maximum} = 2,8 + \frac{0,1}{2} = 2,8 + 0,05 = 2,85$$

$$\text{Minimum} = 2,8 - \frac{0,1}{2} = 2,8 - 0,05 = 2,75$$

Application to Problems

Example 1: Calculate the maximum value for the area of a rectangle of length 12 cm and width 10 cm measured to the nearest cm.

$$\text{Maximum length} = 12 + 0,5 = 12,5 \text{ cm}$$

$$\text{Maximum width} = 10 + 0,5 = 10,5 \text{ cm}$$

$$\text{Maximum area} = 12,5 \times 10,5 = 131,25 \text{ cm}^2$$

Example 2: Find the maximum value of $500 \div 100$ (figures given to the nearest 100).

$$\text{Max. value of } 500 = 500 + 50 = 550$$

$$\text{Min. value of } 100 = 100 - 50 = 50$$

$$\text{Max. value of } 500 \div 100 \text{ is } 550 \div 50 = 11$$

Note: For division,

maximum is **max. \div min.**

minimum is **min. \div max.**

For subtraction,

maximum = **max. $-$ min.**

minimum = min. – max.

Standard Form

Standard Form

This is a way of simplifying how very large or very small numbers are written. A number in standard form is written as:

	$A \times 10^n$	
--	-----------------	--

Where $1 \leq A < 10$ and n is the number of places the decimal comma has moved.

Example 1: Write 253000000 in standard form.

Method:

1) Make the number between 1 and 10 by putting in a decimal comma.

$$= 2,53$$

2) Restore the place values by multiplying 10^n ($n = 8$).

2,53000000

←

87654321

3) Write as $2,53 \times 10^8$.

Example 2: Write 0,00000067 in standard form.

1) 6,7 (number between 1 and 10)

2) $6,7 \times 10^n$

$n = -7$ (because the point has moved the other way)

3) $6,7 \times 10^{-7}$

Note: Numbers less than 1 have a negative value of n .

Problems with Using Standard Form

Example 1: Calculate the value of $(16,5 \times 10^4) \times (8 \times 10^2)$.

Write answers in standard form.

Calculator method

Key sequence is as follows:

$$\begin{aligned} 16,5 \text{ exp } 7 \times 8 \text{ exp } 3 &= 132000000 \\ &= 1,32 \times 10^8 \end{aligned}$$

Note: The calculator sometimes writes standard form in a different way. Do not simply copy the display.

Non-calculator method

Multiply the numbers $16,5 \times 8 = 132,0$.

Multiply the powers $10^4 \times 10^2 = 10^6$.

$$132 \times 10^6 = 1,32 \times 10^2 \times 10^6$$

$$= 1,32 \times 10^8$$

Example 2: Calculate the value of $2,4 \times 10^{11} \div 1,2 \times 10^{-4}$.

Calculator method

Use the calculator in the same way but use \div .

Non-calculator method

$$2,4 \div 1,2 = 2$$

$$10^{11} \div 10^{-4} = 10^{15}$$

$$2,0 \times 10^{15}$$

Adding and Subtracting

For adding and subtracting in standard form, the following methods may be applied:

Calculator: The same method but use + or –.

Non-calculator: Change to ordinary numbers, do the calculations and then change the answers to standard form.

Practical Use of Numbers

Exchange Rates

This is a practical use of direct proportion.

Example 1: Given \$1 = R6,5, change \$10 into rands.

$$\$10 = 6,5 \times 10 = R65$$

Example 2: Change R28 into dollars.

$$\begin{aligned} R 1 &= \$1/6,5 \\ R28 &= 1/6,5 \times 28 = \$4,31 \text{ (to the nearest cent)} \end{aligned}$$

Rule: Work out what one unit is in the money you are changing. Then multiply by the amount you are changing.

Simple Interest (I)

This is a practical use of percentages. It is a way of working out the interest gained on an investment.

The **formula** for simple interest is: $I = \frac{PTR}{100}$

where
P = Principal (amount of money invested)
T = Time (in years)
R = Rate of interest (%)

Example 1: R600 is invested for 8 years at 15 % simple interest. How much interest will it earn? How much will the amount be after 8 years?

$$P = 600$$

$$T = 8$$

$$R = 15$$

$$I = \frac{600 \times 8 \times 15}{100}$$

$$I = R720$$

$$\text{Amount} = 600 + 720 = R1\ 320$$

Note: Questions on this topic can ask for I, P, T, or R.

Simply make the required term the subject of the formula and substitute in the values given.

Example 2: R6 000 was invested for 1 year and 6 months. If the simple interest gained was R1 000, what was the rate of interest?

$$P = 6\ 000$$

$$T = 1 \frac{1}{2} \text{ yrs}$$

$$R = ?$$

$$I = 1\ 000$$

$$R = \frac{100 I}{PT} \quad \text{(making R the subject)}$$

$$R = \frac{100 \times 1\,000}{6\,000 \times 1 \frac{1}{2}} = 11 \frac{1}{9}$$

$$R = 11 \frac{1}{9} \%$$

[Back to top](#)

Compound Interest

In this case, the interest gained is added on each year and the next year's interest worked out.

Example 1: Work out the compound interest on R900 invested for 3 years at 5 %.

$$1^{\text{st}} \text{ year } \frac{5}{100} \times 900 = R45$$

$$2^{\text{nd}} \text{ year } \frac{5}{100} \times 945 = R47,25$$

$$3^{\text{rd}} \text{ year } \frac{5}{100} \times 992,25 = R49,61 \text{ (to nearest cent)}$$

Total compound interest = 45 + 47,25 + 49,61 = R141,86

Example 2: Calculate the amount after 3 years.

$$\text{Amount} = 900 + 141,86 = R1\,041,86$$

For a larger number of years, we use a shorter method.

Using the figures in this example:

The amount at the end of the first year = 105 % of 900 = 1,05 x 900

At the end of the second year = 105 % of the first year = 1,05 x 1,05 x 900

So each year we multiply by 105 % = 1,05

Final amount = $(1,05)^3 \times 900 = R1\,041,86$

Note: This can be done on the calculator as follows:

$1,05 \text{ (power button)} 3 \times 900$ \wedge
--

For 15 years, we simply change the power to 15.

$$(1,05)^{15} \times 900 = R1\,871,04 \text{ (nearest cent)}$$

[Back to top](#)

Speed

Speed is defined as the distance travelled in one unit of time.

Example 1: A car travels 150 km in two hours. What was its average speed?

$$\text{In 1 hour the car travels } 150 \div 2 = 75 \text{ km}$$

$$\text{Speed} = 75 \text{ km/hr}$$

The **units of speed** can be km/hr or m/s.

Note: Always use the units asked for in a question.

Example 2: A particle travels 30 000 cm in 0,5 mins. What is its speed in m/s?

$$30\,000 \text{ cm} = 300 \text{ m}$$

$$0,5 \text{ mins} = 30 \text{ s}$$

$$\text{Speed} = 300/30 = 10 \text{ m/s}$$

Formula for average speed (S)
$S = \frac{D}{T}$
Where S = Speed D = Distance T = Time

To use this formula two of the values are given. By changing the subject of the formula, we can calculate the third.

Example 3: A car travels at a speed of 80 km/hr for 2 hours. What distance did it travel?

$$S = 80$$

$$T = 2$$

$$D = S \times T \text{ (making D the subject)}$$

$$D = 80 \times 2 = 160 \text{ km}$$

Example 4: A car travels 300 km at a speed of 60 km/hr. How long did it take?

$$S = 60$$

$$D = 300$$

$$T = \frac{D}{S}$$

(making T the subject)

$$T = \frac{300}{60} = 5 \text{ hours}$$

[Back to top](#)

Density

Density is the mass per unit volume.

Formula for density (D)
$D = \frac{M}{V}$
Where M = Mass (g) V = Volume (cm ³)

Example 1: The mass of a solid is 500 g and the volume is 1 000 cm³. Calculate the density.

$$M = 500$$

$$V = 1\,000$$

$$D = \frac{500}{1\,000} = 0,5 \text{ g/cm}^3$$

Example 2: V = 3 000cm³, D = 0,2 g/cm³, so M = ?

$$M = D \times V \text{ (making M the subject)}$$

$$M = 0,2 \times 3\,000 = 600\text{g}$$

[Back to top](#)

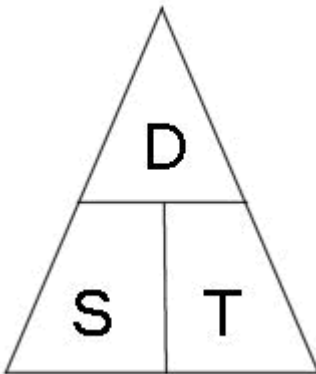
Triangle Method

Both types of problems relating to speed and density can be solved by using a triangle. This is known as the Triangle Method for Speed and Density.

Speed

The vertical line in the triangle means multiply and the horizontal line means divide.

So if two of the values of D, T or S are known, then you can work out the other using one of the following:



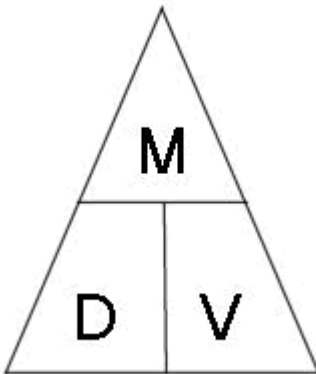
$$D/T = S$$

$$D/S = T$$

$$S \times T = D$$

Density

The triangle for density is used in the same way.



$$M/D = V$$

$$M/V = D$$

$$D \times V = M$$

Rational and Irrational Numbers

Rational Numbers

A number that can be written exactly as a fraction is called a rational number.

For example, $\frac{3}{4}$, $\frac{1}{2}$, 5,3333... , 6, are all rational numbers.

Note: 5,3333... = $5\frac{1}{3}$ (rational)

If a decimal can be written down exactly, or is a recurring decimal, it is rational.

Irrational Numbers

A number such as $\sqrt{2}$ is irrational because it cannot be written exactly as a fraction or decimal.

π (pi) is irrational because it is a neverending, non-recurring decimal.

All square roots and cube roots that are not exact are irrational.

Example 1: Find an irrational number between 27 and 35.

$(27 + \sqrt{2})$ is irrational.

Example 2: If we square an irrational number, we always get a rational number. Investigate:

$(\sqrt{2})^2 = 2$ If we square a square root we get a rational number.

But $(\pi)^2$ is still irrational.

So the statement is incorrect.

SURDS

Definition

When the square root of a number is the only exact way of writing down the number, it is called a surd.

So $\sqrt{2}$ is called a surd.

Simplifying Surds

a. Adding and subtracting

Rule: We can add or subtract the square roots of the same numbers only.

$$\sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

$$5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$$

$$\sqrt{2} + \sqrt{3} \text{ cannot be simplified.}$$

b. Multiplying and dividing

Rule: We can multiply and divide surds.

$$\sqrt{2} \times \sqrt{3} = \sqrt{6}$$

$$\sqrt{6} \div \sqrt{3} = \sqrt{2}$$

Note: $\sqrt{6} \div 2$ cannot be simplified.

But $\sqrt{6} \times 2 = 2\sqrt{6}$

We must be aware of this when multiplying or dividing surds by integers.

Changing the Form of a Surd

$$\sqrt{24} = \sqrt{4 \times 6} = \sqrt{4} \times \sqrt{6} = 2 \times \sqrt{6} = 2\sqrt{6}$$

So $\sqrt{24} = 2\sqrt{6}$

If one of the factors of the number inside the square root is a square number, then it can be square rooted and taken outside.

$$\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$$

Problems

Example 1: A rectangle has a length of $(2 + \sqrt{3})$ cm and width $(3 - \sqrt{3})$ cm. Work out the area of the rectangle.

$$\text{Area} = (2 + \sqrt{3})(3 - \sqrt{3}) = 6 - 2\sqrt{3} + 3\sqrt{3} - 3$$

$$\text{Area} = (3 + \sqrt{3}) \text{ cm}$$

Example 2: Simplify $\sqrt{8} \times \sqrt{3} =$
 $= \sqrt{8} \times \sqrt{3} = \sqrt{24} = 2\sqrt{6}$

[Back to top](#)

Rationalising the Denominator

This is a way of removing the surd from the denominator.

Example 1: $\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}}$

Rule: To rationalise the denominator, multiply above and below by the surd.

Example 2: When the denominator is an expression:

$$\begin{aligned} \frac{5}{(2 + \sqrt{3})} &= \frac{5}{(2 + \sqrt{3})} \times \frac{(2 - \sqrt{3})}{(2 - \sqrt{3})} = \frac{5(2 - \sqrt{3})}{4 - 2\sqrt{3} + 2\sqrt{3} - 3} \\ &= \frac{5(2 - \sqrt{3})}{1} = 5(2 - \sqrt{3}) \end{aligned}$$

Rule: Multiply above and below by the expression with the sign of the surd changed. This eliminates the surd from the denominator.

Changing a Recurring Decimal to a Fraction

Examples

This is a common question and requires good preparation.

Example 1: Change 1,7777... to a fraction.

Method:

$$\begin{aligned} \text{Let } x = 1,7 & \quad \text{then} & \quad 10x = 17,7 & \quad \text{and subtracting gives} \\ & & \quad 9x = 16 \\ & & \quad x = \frac{16}{9} \\ & & \quad x = 1\frac{7}{9} \end{aligned}$$

Example 2: Change 0,3232323232... to a fraction.

Let $x = 0,32323232\dots$ (Because there are two figures recurring, multiply by 100).

$$\begin{aligned} 100x &= 32,323232\dots \\ x &= 0,323232\dots & \quad \text{(subtracting)} \\ 99x &= 32 \\ x &= \frac{32}{99} \end{aligned}$$

Example 3: Change 0,084848484... to a fraction.

(Note: The 0 is not recurring).

$$\begin{aligned} 1\,000x &= 84,848484\dots & \quad \text{(x 1 000 to move the point)} \\ 10x &= 0,848484\dots & \quad \text{(x 10 to move the point)} \\ 990x &= 84 & \quad \text{(subtracting)} \\ x &= \frac{84}{990} & \quad \text{(cancel)} \\ x &= \frac{14}{165} \end{aligned}$$