XT - MATHS Grade 11

Subject: Linear Programming **Date:** 2010/06/29

Total Marks: 156

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1. FALSE 4

Explanation: There is a limitation of a maximum number of computer games as well as TV games that can be made in a day.
Therefore no more than 80 computer games and 50 TV games can be made in one day.
Thus:

$$x \leq 80\ ;\quad y \leq 50$$

The **total** number of games produced in a day, i.e. the sum of the two different types, cannot be more than 100.
Therefore:

$$x + y \leq 100$$

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2. TRUE 20

Explanation: **<u>Constraints:</u>**
Standing order per week:

$$x \geq 6\ ;\ y \geq 6$$

Shop's stocks per week:

$$x + y \leq 20$$

<u>Last week's profit:</u>

$$P_1 = 40x + 60y$$

$$\therefore\ y = -\frac{40}{60}x + \frac{P_1}{60}$$

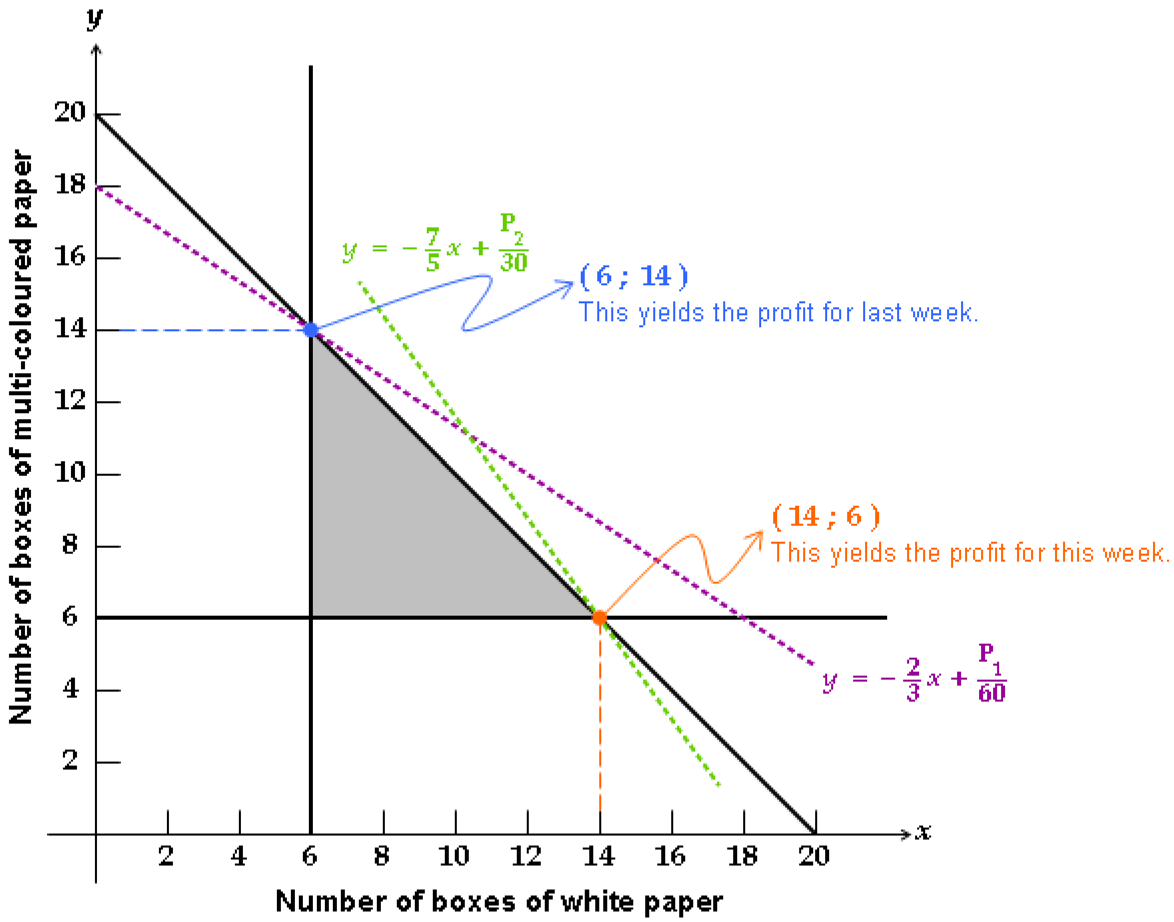
$$\therefore\ y = -\frac{2}{3}x + \frac{P_1}{60}$$

<u>This week's profit:</u>

$$P_2 = 42x + 30y$$

$$\therefore\ y = -\frac{42}{30}x + \frac{P_2}{30}$$

$$\therefore\ y = -\frac{7}{5}x + \frac{P_2}{30}$$



Now:

$$\begin{aligned} \text{Max. } P_1 &= 40x + 60y \\ &= 40(6) + 60(14) \\ &= 240 + 840 \\ &= 1080 \end{aligned}$$

and

$$\begin{aligned} \text{Max. } P_2 &= 42x + 30y \\ &= 42(14) + 30(6) \\ &= 588 + 180 \\ &= 768 \end{aligned}$$

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3. TRUE 4

Explanation: $2y - 3x + 5 \geq 0$

$$\therefore 2y \geq 3x - 5$$

$$\therefore y \geq \frac{3}{2}x - \frac{5}{2}$$

Boundary: $y = \frac{3}{2}x - \frac{5}{2}$

y-intercept: $c = -\frac{5}{2}$

$= -2\frac{1}{2}$

x-intercept: $0 = \frac{3}{2}x - \frac{5}{2}$

$\therefore \frac{3}{2}x = \frac{5}{2}$

$\therefore 3x = 5$

$\therefore x = \frac{5}{3}$

$\therefore x = 1\frac{2}{3}$

The one line must be included and thus solid, while the other must be excluded and thus broken.

$y \geq \frac{3}{2}x - \frac{5}{2}$ shows that all values above and on its boundary should be included.

$x < 2$ shows that all values to the left of $x = 2$ must be included.

Therefore, graph A does represent the given information.

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4. A 6

Explanation: $x + y \geq -3$

$\therefore y \geq -x - 3$

Boundary: $y = -x - 3$

y-intercept: $c = -3$

x-intercept: $0 = -x - 3$

$\therefore x = -3$

These intercepts correspond to the solid lines on each of the given graphs. The ≥-sign indicates that the line must be solid and the values above it must be included. All four graphs show exactly this!

$2y - 1 > 3x$

$\therefore 2y > 3x + 1$

$\therefore y > \frac{3}{2}x + \frac{1}{2}$

Boundary: $y = \frac{3}{2}x + \frac{1}{2}$

y-intercept: $c = \frac{1}{2}$

x-intercept: $0 = \frac{3}{2}x + \frac{1}{2}$

$\therefore \frac{3}{2}x = -\frac{1}{2}$

$\therefore 3x = -1$

$\therefore x = -\frac{1}{3}$

These intercepts eliminate graph B. The >-sign indicates that the line must be broken and the values above it must be included. The other three graphs show exactly this!

$4y - 4 - x < 0$

$\therefore 4y < x + 4$

$\therefore y < \frac{1}{4}x + 1$

Boundary: $y = \frac{1}{4}x + 1$

y-intercept: $c = 1$

x-intercept: $0 = \frac{1}{4}x + 1$

$\therefore \frac{1}{4}x = -1$

$\therefore x = -4$

These intercepts eliminate graph C. The <-sign indicates that the line must be broken and the values below it must be included. This eliminates graph D.

Therefore, graph A is the correct one.

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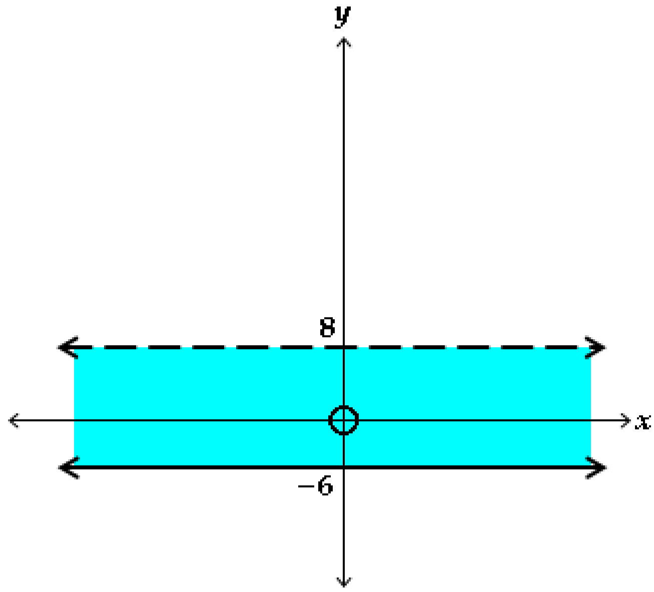
5. D 6

Explanation: $8 > y \geq -6$:



$2 - \frac{4}{11}y > x$: $2 - \frac{4}{11}y > x$

$\therefore -\frac{4}{11}y > x - 2$

$\therefore 4y < -11x + 22$

$\therefore y < -\frac{11}{4}x + \frac{11}{2}$

Boundary: $y = -\frac{11}{4}x + \frac{11}{2}$

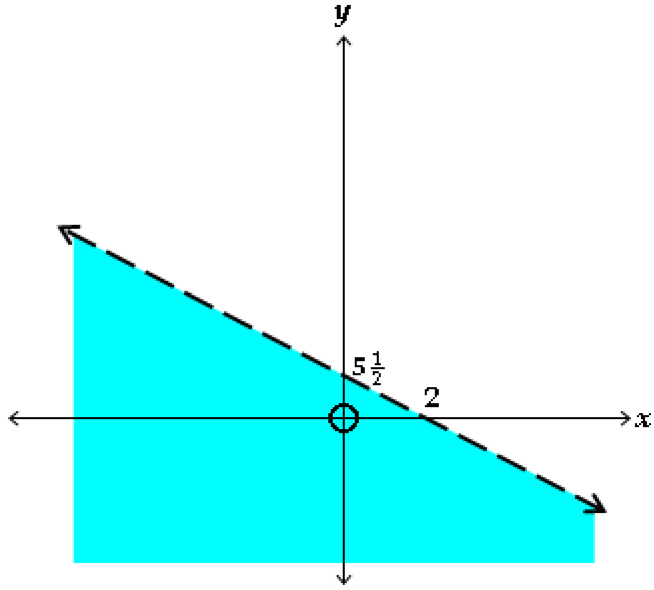
y-intercept: $c = \frac{11}{2}$

x-intercept: $0 = -\frac{11}{4}x + \frac{11}{2}$

$\therefore \frac{11}{4}x = \frac{11}{2}$

$\therefore 11x = 22$

$\therefore x = 2$



$\underline{11x - y + 33 \geq 0}$: $11x - y + 33 \geq 0$

$\therefore -y \geq -11x - 33$

$\therefore y \leq 11x + 33$

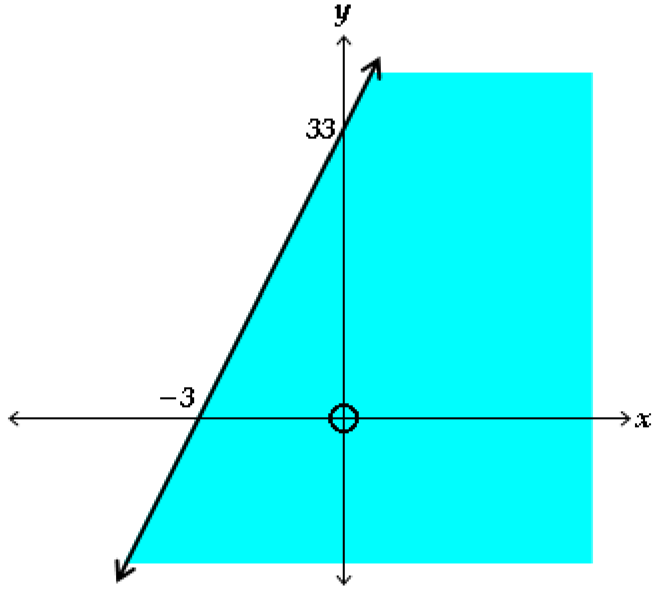
Boundary: $y = 11x + 33$

y-intercept: $c = 33$

x-intercept: $0 = 11x + 33$

$\therefore 11x = -33$

$\therefore x = -3$



When all three inequalities are drawn on the same set of axes, the overlapping shaded areas will correspond to that of graph D.

6. E 18

Explanation: Let x be the number of analogue clocks.
Let y be the number of digital clocks.

IMPLICIT CONSTRAINTS:

$x \geq 0 \; ; \; x \in \mathbb{N}_0$ [Only whole analogue clocks can be sold.]

$y \geq 0 \; ; \; y \in \mathbb{N}_0$ [Only whole digital clocks can be sold.]

CONSTRAINTS:

Total Income: $80x + 100y \geq 2\,200$

[≥ as the income must be R 2 200 or more]

Products / week: Analogue: $x \leq 20$

Digital: $y \leq 40$

Packaging dept.: $x + y \leq 50$

[≤ as they cannot pack more than 50]

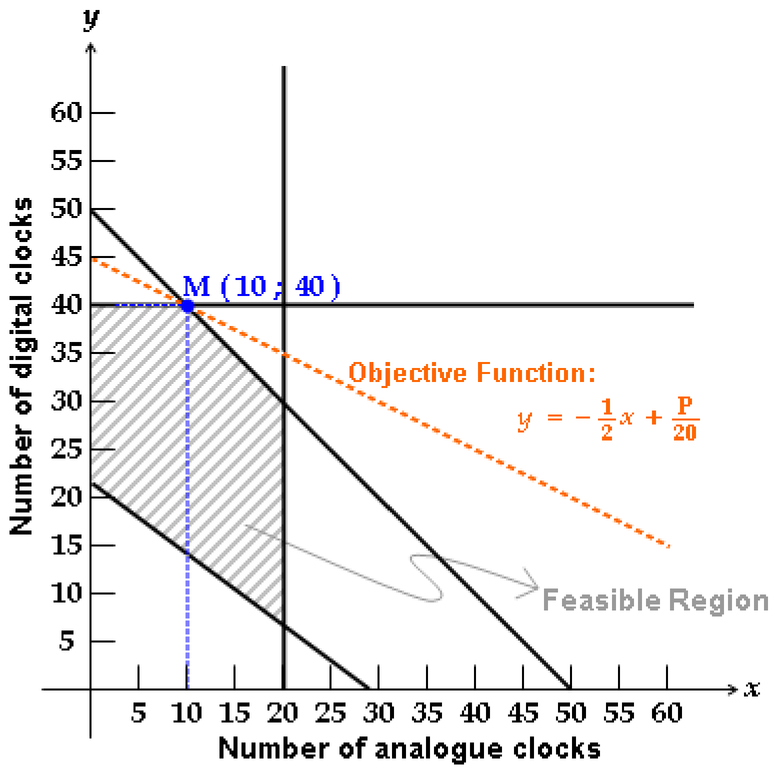
OBJECTIVE FUNCTION:

$$P = 10x + 20y$$

$$\therefore 20y = -10x + P$$

$$\therefore y = -\frac{1}{2}x + \frac{P}{20}$$

Sketch the above constraints and objective function:

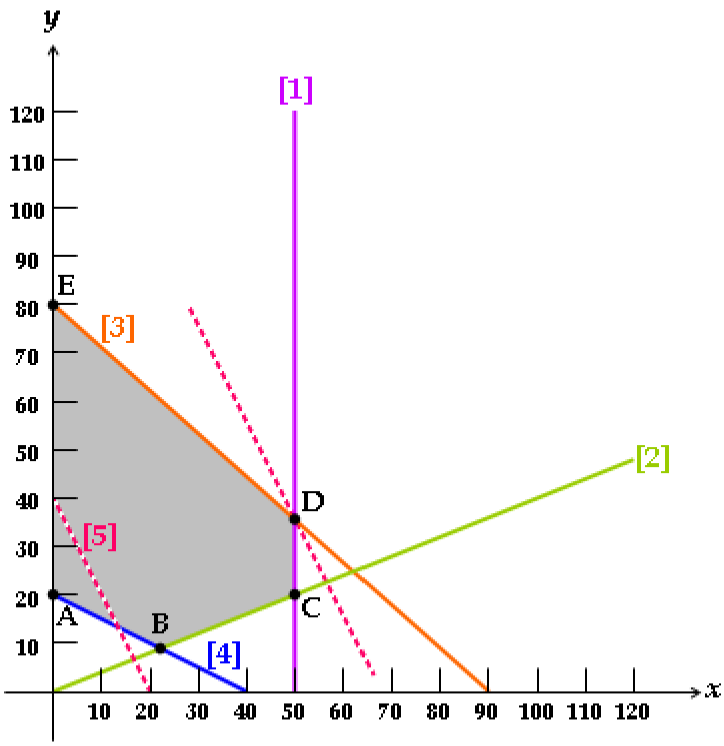


Reading from the sketch, point M will give the maximum profit.
The coordinates of M: (10 ; 40)
This means that 10 analogue and 40 digital clocks must be produces per week to ensure a maximum profit under the given constraints.
Now:

$$\text{Max. } P = 10(10) + 20(40)$$
$$= 100 + 800$$
$$= R\,900$$

7. D 2

Explanation: By increasing the y-intercept of the objective function, the profit increases. Look at all the lines parallel to the shown objective function. The one with the highest y-intercept that still touches or passes through the feasible region will pass through the point giving the maximum profit.



Point $\mathbf{D}$ is the highest single point (vertex) of the feasible region through which lines parallel to the objective function can pass, therefore at $\mathbf{D}$ the profit will be a maximum.

8. 600 12
600 l
600 litres

Explanation: The quantity of red wine produced is dependent on the quantity of white wine produced. The maximum of the total produced per day is $1\,800\ \ell$, therefore the maximum red wine produced is $600\ \ell$ since there would be $1\,200\ \ell$ white wine, i.e. twice as much as red wine, produced.

You can also solve this problem graphically.
Let x be the amount of white wine pressed and y be the amount of red wine pressed.

<u>CONSTRAINTS:</u>

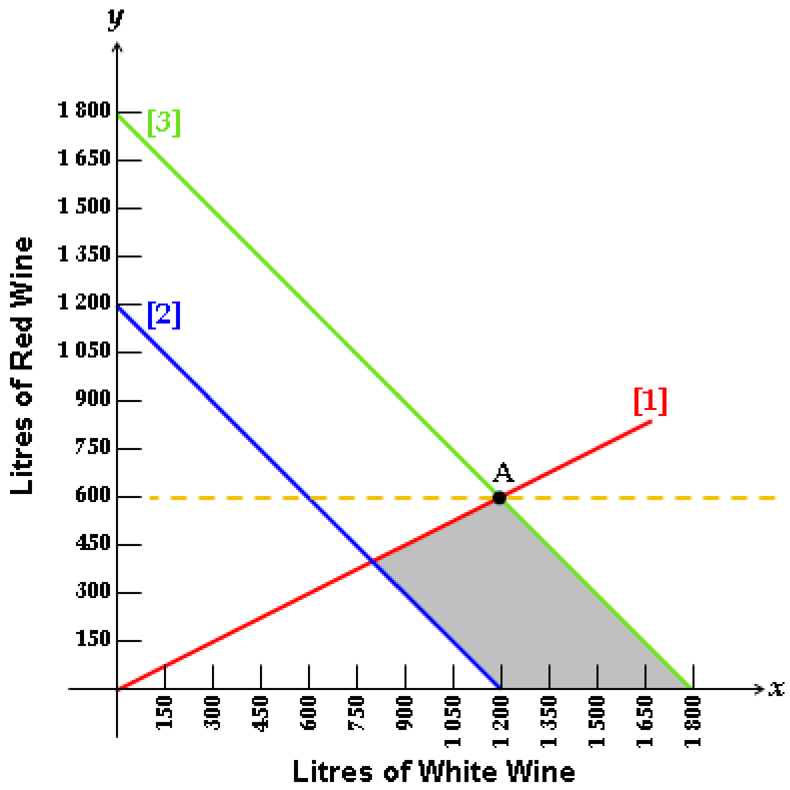
Red and white balance: $\frac{x}{y} \geq \frac{2}{1}$ [the ratio $x : y$ is at least twice as much white than red]

$\therefore x \leq 2y$

$\therefore y \leq \frac{1}{2}x$ .... **[1]**

Productivity: $x + y \geq 1\,200$ [must be more than $1\,200\ \ell$] .... **[2]**

Availability: $x + y \leq 1\,800$ [cannot be more than $1\,800\ \ell$] .... **[3]**



By sliding the yellow dotted line upwards to the highest point of the feasible region, it will give the y-value for the maximum litres of red wine which can be produced per day. As the corresponding y-value is 600, the maximum amount of red wine that can be produced per day is $600\ \ell$.

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9. (7 ; - 4) 4

Explanation: The only lines that are used are ***AB: y = - 4*** and ***AD: y = - x + 3***

$\mathbf{A}$ is the point of intersection of $y = -4$ and $y = -x + 3$.

Therefore:

$$-4 = -x + 3$$

$$\therefore\ x = 7$$

The coordinates of $\mathbf{A}$ are $(\,7\,;\,-4\,)$.

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10. square 36

Explanation: $\underline{-2 \le 2y - 3x \le 24}$:

- $2y - 3x \le 24$

$$\therefore\ 2y \le 3x + 24$$

$$\therefore\ y \le \frac{3}{2}x + 12$$

<u>Boundary:</u> $y = \frac{3}{2}x + 12$

y-intercept: $c = 12$

x-intercept: $0 = \frac{3}{2}x + 12$

$$\therefore\ \frac{3}{2}x = -12$$

$$\therefore\ 3x = -24$$

$$\therefore\ x = -8$$

- $2y - 3x \geq -2$

  $\therefore 2y \geq 3x - 2$

  $\therefore y \geq \frac{3}{2}x - 1$

  Boundary: $y = \frac{3}{2}x - 1$

  y-intercept: $c = -1$

  x-intercept: $0 = \frac{3}{2}x - 1$

  $\therefore \frac{3}{2}x = 1$

  $\therefore 3x = 2$

  $\therefore x = \frac{2}{3}$

$-8 \leq \frac{3}{2}y + x \leq 5$:

- $\frac{3}{2}y + x \leq 5$

  $\therefore \frac{3}{2}y \leq -x + 5$

  $\therefore 3y \leq -2x + 10$

  $\therefore y \leq -\frac{2}{3}x + \frac{10}{3}$

  $\therefore y \leq -\frac{2}{3}x + 3\frac{1}{3}$

  Boundary: $y = -\frac{2}{3}x + 3\frac{1}{3}$

  y-intercept: $c = 3\frac{1}{3}$

  x-intercept: $0 = -\frac{2}{3}x + \frac{10}{3}$

  $\therefore 2x = 10$

  $\therefore x = 5$

- $\frac{3}{2}y + x \geq -8$

  $\therefore \frac{3}{2}y \geq -x - 8$

  $\therefore 3y \geq -2x - 16$

  $\therefore y \geq -\frac{2}{3}x - \frac{16}{3}$

  $\therefore y \geq -\frac{2}{3}x - 5\frac{1}{3}$

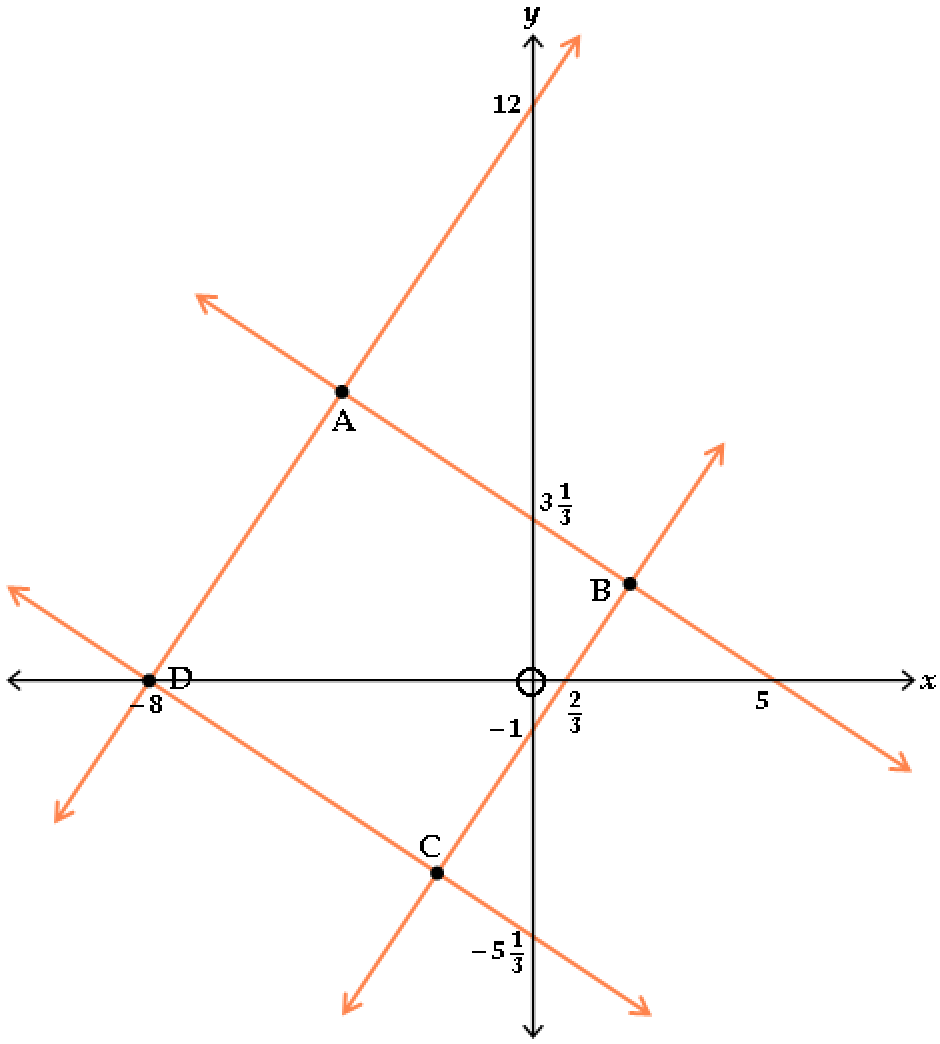
  Boundary: $y = -\frac{2}{3}x - 5\frac{1}{3}$

  y-intercept: $c = -5\frac{1}{3}$

  x-intercept: $0 = -\frac{2}{3}x - \frac{16}{3}$

  $\therefore 2x = -16$

  $\therefore x = -8$



<u>AB</u>: $y = -\frac{2}{3}x + 3\frac{1}{3}$

<u>BC</u>: $y = \frac{3}{2}x - 1$

<u>CD</u>: $y = -\frac{2}{3}x - 5\frac{1}{3}$

<u>DA</u>: $y = \frac{3}{2}x + 12$

According to the gradients:

$$AB \parallel DC\ ;\ \ BC \parallel AD$$
$$AB \perp BC\ ;\ \ AB \perp AD$$
$$CD \perp BC\ ;\ \ CD \perp AD$$

Therefore, this quadrilateral is either a square or a rectangle.

The lengths of the sides of this quadrilateral must be calculated to determine whether it is a square or a rectangle.
Calculate the coordinates of A, B, C and D by using the distance formula from Analytical Geometry.

<u>Coordinates of A:</u>

$$AB = AD$$
$$\therefore -\frac{2}{3}x + \frac{10}{3} = \frac{3}{2}x + 12$$
$$\therefore -4x + 20 = 9x + 72$$
$$\therefore 13x = -52$$
$$\therefore x = -4$$

Substitute this value into $\mathbf{AD}$:

$$\begin{aligned} y &= \frac{3}{2}x + 12 \\ &= \frac{3}{2}(-4) + 12 \\ &= -6 + 12 \\ &= 6 \end{aligned}$$

Therefore:

$$\mathrm{A} = (-4; 6)$$

<u>Coordinates of $\mathbf{B}$:</u>

$$\begin{aligned} \mathrm{AB} &= \mathrm{BC} \\ \therefore -\frac{2}{3}x + \frac{10}{3} &= \frac{3}{2}x - 1 \\ \therefore -4x + 20 &= 9x - 6 \\ \therefore 13x &= 26 \\ \therefore x &= 2 \end{aligned}$$

Substitute this value into $\mathbf{BC}$:

$$\begin{aligned} y &= \frac{3}{2}x - 1 \\ &= \frac{3}{2}(2) - 1 \\ &= 3 - 1 \\ &= 2 \end{aligned}$$

Therefore:

$$\mathrm{B} = (2; 2)$$

<u>Coordinates of $\mathbf{C}$:</u>

$$\begin{aligned} \mathrm{BC} &= \mathrm{CD} \\ \therefore \frac{3}{2}x - 1 &= -\frac{2}{3}x - \frac{16}{3} \\ \therefore 9x - 6 &= -4x - 32 \\ \therefore 13x &= -26 \\ \therefore x &= -2 \end{aligned}$$

Substitute this value into $\mathbf{BC}$:

$$\begin{aligned} y &= \frac{3}{2}x - 1 \\ &= \frac{3}{2}(-2) - 1 \\ &= -3 - 1 \\ &= -4 \end{aligned}$$

Therefore:

$$\mathrm{C} = (-2; -4)$$

<u>Coordinates of $\mathbf{D}$:</u>

$\mathbf{D}$ is the x-intercept for both $\mathbf{CD}$ and $\mathbf{AD}$.
Therefore:

$$\mathrm{D} = (-8; 0)$$

Now:

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(-4 - 2)^2 + (6 - 2)^2} = \sqrt{52}$$

$$BC = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(-2 - 2)^2 + (-4 - 2)^2} = \sqrt{52}$$

$$CD = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(-8 - (-2))^2 + (-4 - 0)^2} = \sqrt{52}$$

$$DA = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(-8 - (-4))^2 + (0 - 6)^2} = \sqrt{52}$$

All the sides of this quadrilateral are equal in length and the adjacent sides are perpendicular, therefore it is a square.

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11. (1) ***x, y*** are natural numbers less than or equal to ***6***. 3

(2) ***x*** and ***y*** are positive numbers

(3) ***x*** and ***y*** are counting numbers

Explanation: (1) The only values that can be obtained when a die is rolled are ***1, 2, 3, 4, 5, 6***.
These are natural numbers, all less than or equal to six.
Thus $x, y \leq 6$ and $x, y \in \mathbb{N}$

(2) When liquids are mixed together, they can be fractional amounts,
eg ***2,5 kl*** of ***x*** and ***2,25kl*** of ***y***.
Practically, x and y can be any positive numbers.
$x, y \in \mathbb{R}$

(3) When dealing with items that can be counted (eg animals, clockes, cars)
the values must be positive whole numbers.
$x, y \in \mathbb{N}_0$

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12. (1) $2x + 3y \geq 125$ 15

(2) $3x + 2y \geq 150$

(3) 40

(4) 15

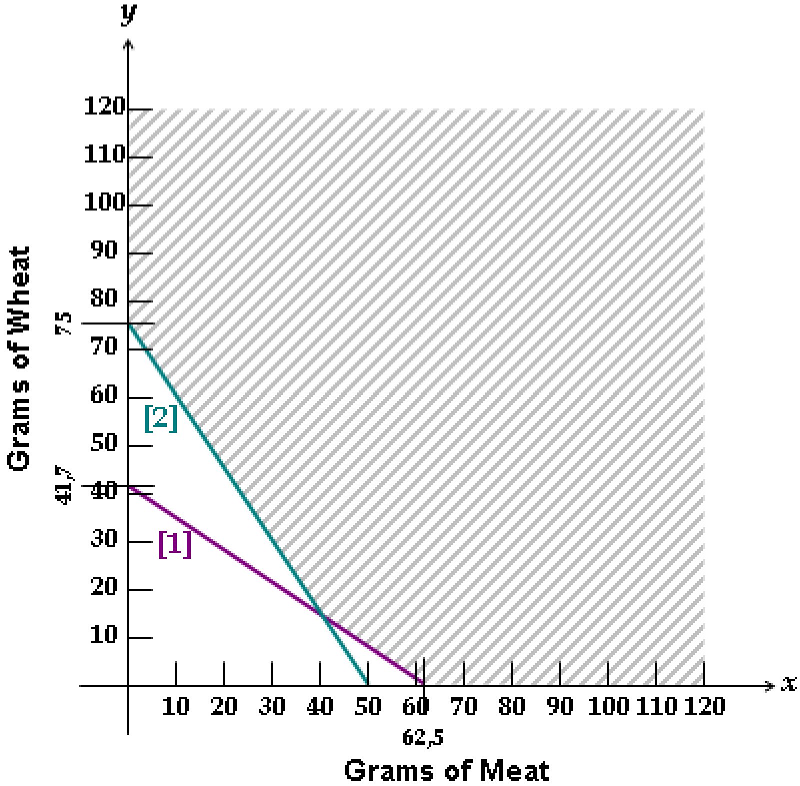
Explanation: (1) Vitamins: **2 units** per gram of meat (x): $2x$
3 units per gram of wheat (y): $3y$
The meat and wheat combined must contain **125 units** of vitamins or more.
Therefore:

$$2x + 3y \geq 125 \quad \ldots\ldots\ldots \quad [1]$$

(2) Fatty Acids: **3 units** per gram of meat (x): $3x$
2 units per gram of wheat (y): $2y$
The meat and wheat combined must contain **150 units** of fatty acids or more.
Therefore:

$$3x + 2y \geq 150 \quad \ldots\ldots\ldots \quad [2]$$

The feasible region using the above constraints:



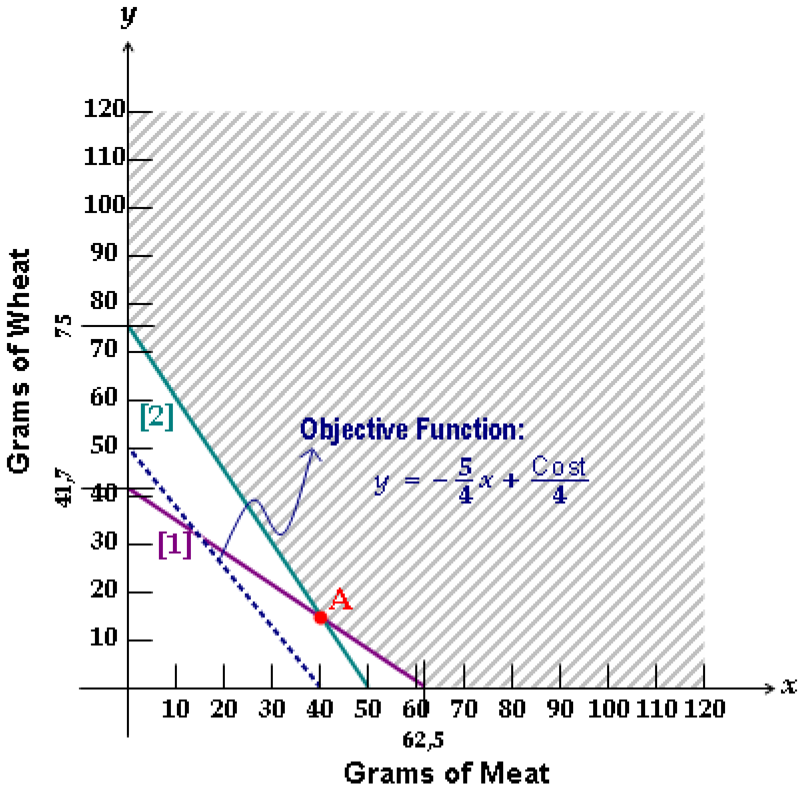
(3) & (4) Cost of meat : Cost of wheat = 5 : 4
Therefore:

$$\text{Cost} = 5x + 4y$$

$$\therefore 4y = -5x + \text{Cost}$$

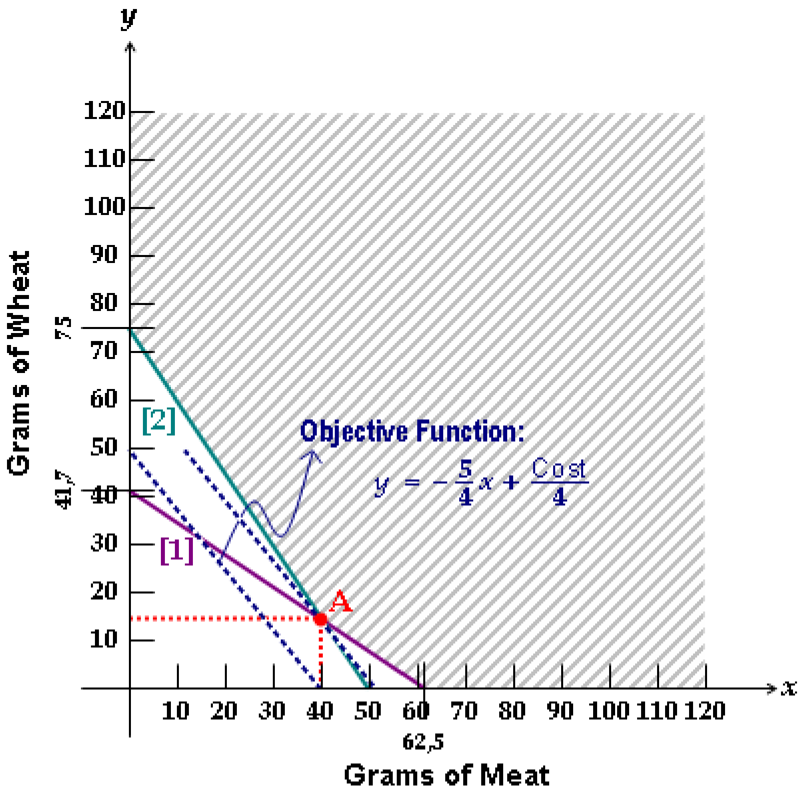
$$\therefore y = -\frac{5}{4}x + \frac{\text{Cost}}{4}$$

Now:



When sliding the objective function upwards, the first point of the feasible region cut by the objective function will give the values which will achieve a minimum cost. This will happen at point $\mathbf{A}$. The coordinates of $\mathbf{A}$ will give the grams of meat and the grams wheat respectively which will ensure a minimum cost. The coordinates of $\mathbf{A}$ can be read off the graph if your sketch is accurately drawn or you can calculate those coordinates algebraically.

<u>Graphic solution:</u>



The coordinates of $\mathbf{A}$ are $(40\,;\ 15)$.
This means that $40\ \text{g}$ of meat and $15\ \text{g}$ of wheat will ensure a minimum cost to produce.

13. (1) $5y \geq -12x - 15$ 12

(2) $\left(2\frac{1}{12}; 4 \right)$

Explanation: (1) <u>CD</u>: y-int.: $(0; -3)$

$\therefore y = mx - 3$

x-int.: $\left(-1\frac{1}{4}; 0 \right)$

$\therefore 0 = m\left(-\frac{5}{4}\right) - 3$

$\therefore 0 = -5m - 12$

$\therefore 5m = -12$

$\therefore m = -\frac{12}{5}$

Boundary line: $y = -\frac{12}{5}x - 3$

The boundary line is solid and the values above it are included, therefore the inequality sign $\geq$ must be used.

The inequality:

$$y \geq -\frac{12}{5}x - 3$$

$$\therefore 5y \geq -12x - 15$$

(2) $\mathbf{A}$ is the point of intersection of $y = 4$ and $\mathbf{AB}$.
The line passing through $\mathbf{A}$ and $\mathbf{B}$ is parallel to $\mathbf{DC}$ and will have the same gradient.

<u>AB</u>: gradient: $m = -\frac{12}{5}$

$\therefore y = -\frac{12}{5}x + c$

x-int.: $\left(3\frac{3}{4}; 0 \right)$

$\therefore 0 = -\frac{12}{5}\left(\frac{15}{4}\right) + c$

$\therefore 0 = -9 + c$

$\therefore c = 9$

Boundary line: $y = -\frac{12}{5}x + 9$

<u>Coordinates of A</u>:

$$-\frac{12}{5}x + 9 = 4$$

$$\therefore -\frac{12}{5}x = -5$$

$$\therefore 12x = 25$$

$$\therefore x = \frac{25}{12}$$

$$\therefore x = 2\frac{1}{12}$$

Therefore:

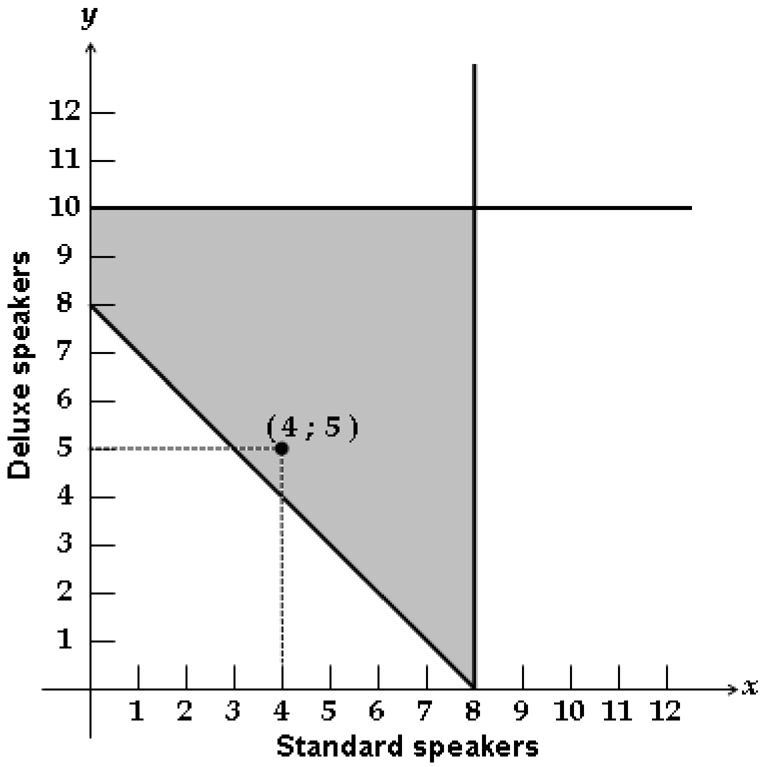
$$\mathbf{A} = \left(2\frac{1}{12}; 4 \right)$$

14. TRUE 10

Explanation: The factory can produce 4 standard and 5 deluxe speakers since ...

- together they exceed the minimum of 8 for profitability:
  Total: $x + y \geq 8$, and $4 + 5$ is greater than 8.
- separately they satisfy the daily maximum requirements:
  Standard: $x \leq 8$, and 4 is smaller than 8.
  Deluxe: $y \leq 10$, and 5 is smaller than 10.

If the constraints are sketched, it will show that the point $(4\,;\,5)$ will lie in the feasible region:



15. TRUE 4

Explanation: $y > x - 3$

Boundary: $y = x - 3$

y-intercept: $c = -3$

x-intercept: $0 = x - 3$

$\therefore x = 3$

These intercepts correspond with those of the broken line, which in turn correspond with the inequality. The values above this boundary should be included and the values in the coloured area lie above it.

$y \geq -2x + 5$

Boundary: $y = -2x + 5$

y-intercept: $c = 5$

x-intercept: $0 = -2x + 5$

$\therefore 2x = 5$

$\therefore x = \frac{5}{2}$

These intercepts correspond with those of the solid line, which in turn correspond with the inequality. The values above this boundary should be included and the values in the coloured area lie above it.

15 Questions, 16 Pages