XT - MATHS Grade 12

Subject: Inverses and Logarithms **Date:** 2010/06/29

Total Marks: 66

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1. FALSE 8

Explanation:

$$\log_3(5-x) = 2 - \log_3(1+x)$$
$$\log_3(5-x) + \log_3(1+x) = 2$$
$$\log_3(5-x)(1+x) = 2$$
$$\log_3(5+4x-x^2) = 2$$
$$5+4x-x^2 = 3^2$$
$$5+4x-x^2 = 9$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)^2 = 0$$
$$x = 2$$

You could also solve for x as follows:

$$\log_3(5-x) = 2 - \log_3(1+x)$$
$$\log_3(5-x) = 2\log_3 3 - \log_3(1+x)$$
$$\log_3(5-x) = \log_3 9 - \log_3(1+x)$$
$$\log_3(5-x) = \log_3\left(\frac{9}{1+x}\right)$$
$$(5-x) = \frac{9}{1+x}$$
$$(5-x)(1+x) = 9$$
$$5+4x-x^2 = 9$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)^2 = 0$$
$$x = 2$$

If x = **-2** is substituted into the given equation:

$$\log_3(5-(-2)) = 2 - \log_3(1+(-2))$$
$$\log_3(5+2) = 2 - \log_3(1-2)$$
$$\log_3(7) = 2 - \log_3(-1)$$

But $\log_3(-1)$ is not allowed!

Therefore, x = **2** is the only valid solution.

2. FALSE 2

Explanation: According to log laws:

$$\frac{\log x}{\log y} = (\log x) \div (\log y)$$

$$\log \frac{x}{y} = (\log x) - (\log y)$$

NOTE : $\frac{\log x}{\log y}$ can also be written as $\log_y x$.

3. C 6

Explanation:

$$\frac{\log_2 9 - \log_2 3 + \log_2 27}{\log_2 81 - \log_2 27}$$

$$= \frac{\log_2 3^2 - \log_2 3 + \log_2 3^3}{\log_2 3^4 - \log_2 3^3}$$

$$= \frac{2\log_2 3 - \log_2 3 + 3\log_2 3}{4\log_2 3 - 3\log_2 3}$$ [LOG LAW 3: $\log_m b^a = a \log_m b$]

$$= \frac{4\log_2 3}{\log_2 3}$$ [Numerator and denominator simplified]

$= 4$ [Equal factors cancelled]

OR

$$\frac{\log_2 9 - \log_2 3 + \log_2 27}{\log_2 81 - \log_2 27}$$

$$= \frac{\log_2 (9 \div 3 \times 27)}{\log_2 (81 \div 27)}$$ [LOG LAW 1: $\log_m a + \log_m b = \log_m ab$
LOG LAW 2: $\log_m a - \log_m b = \log_m \frac{a}{b}$]

$$= \frac{\log_2 81}{\log_2 3}$$

$$= \frac{\log_2 3^4}{\log_2 3}$$

$$= \frac{4\log_2 3}{\log_2 3}$$ [LOG LAW 3: $\log_m b^a = a \log_m b$]

$= 4$ [Equal factors cancelled]

4. C 4

Explanation: $\frac{\log_3 27}{\log_3 81} = \frac{\log_3 3^3}{\log_3 3^4}$

$$= \frac{3\log_3 3}{4\log_3 3}$$

$$= \frac{3}{4}$$

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5. 3 4

Explanation: Substitute the given point into $y = \log_a x$:

$$-3 = \log_a \frac{1}{27}$$

Now :

$$a^{-3} = \frac{1}{27}$$

$$a^{-3} = 3^{-3}$$

$$a = 3$$

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6. 0 2

Explanation: $\log_8(\log 10) = \log_8(\log_{10} 10)$

$$= \log_8 1$$

$$= 0$$

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7. 9 6

Explanation: $\log_x 54 + \log_x 5 - \log_x 10 = 1\frac{1}{2}$

$$\therefore \log_x\left(\frac{54 \times 5}{10}\right) = 1\frac{1}{2}$$ [LOG LAW 1: $\log a + \log b = \log ab$; LOG LAW 2: $\log a - \log b = \log \frac{a}{b}$]

$$\therefore \log_x 27 = 1\frac{1}{2}$$

$$\therefore \log_x 27 = \frac{3}{2}$$

$$\therefore x^{\frac{3}{2}} = 27$$ [Exponent form]

$$\therefore x^{\frac{3}{2}} = 3^3$$

$$\therefore \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = \left(3^3\right)^{\frac{2}{3}}$$

$$\therefore x = 3^2$$

$$\therefore x = 9$$

TEST:

According to definition, the value of x must be larger than 0 but not equal to 1.
The solution $x = 9$ is therefore allowed.

8. (1) 100 4

(2) ± 10

(3) ± 10

Explanation: The complete calculation is as follows:

$$\log x^2 = 2$$

$$\log_{10} x^2 = 2$$

$$\therefore x^2 = 10^2 \quad \text{[exponential form]}$$

$$\therefore x^2 = 100$$

$$\therefore x = \pm 10$$

According to definition, x^2 in $\log x^2 = 2$ must be positive.

But the square of any number/term is always positive, therefore x may be equal to both 10 and -10.

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9. (1) $x \leq 0$ 6

(2) $x = 8$

Explanation: (1) According to definition: $x > 0$
Therefore, $\log x$ will be undefined for $x = 0$ and $x < 0$.

(2) $$\frac{\log x}{\log 4} = 1\frac{1}{2}$$

$$\therefore \frac{\log x}{\log 4} = \frac{3}{2}$$

$$\therefore 2\log x = 3\log 4 \quad \text{[Cross multiplied]}$$

$$\therefore \log x^2 = \log 4^3 \quad [\underline{\text{LOG LAW 3}}: a \log_m b = \log_m b^a]$$

$$\therefore x^2 = 4^3 \quad \text{[Logs removed]}$$

$$\therefore x^2 = 64$$

$$\therefore x = \pm 8$$

<u>TEST :</u>

According to definition, x must be larger than 0.
$8 > 0$, but -8 is not. Therefore, 8 is the only valid solution.

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10. (1) larger than 0 6

(2) 2^6

(3) 4

Explanation: (1) According to definition: $y^3 > 0$

$$\therefore y > 0$$

(2) $\log_2 y^3 = 6$

$\therefore y^3 = 2^6$ [Exponent form] .... (Ans. 2)

$\therefore y^3 = 64$

$\therefore y^3 = 4^3$

$\therefore y = 4$

OR

$\log_2 y^3 = 6$

$\therefore \log_2 y^3 = 6\log_2 2$ [$\log_m m = 1$]

$\therefore \log_2 y^3 = \log_2 2^6$ [<u>LOG LAW 3</u>: $a \log_m b = \log_m b^a$]

$\therefore y^3 = 2^6$ [Logs removed] .... (Ans. 2)

$\therefore y^3 = 64$

$\therefore y^3 = 4^3$

$\therefore y = 4$

(3) According to definition, y^3 must be larger than 0.
Therefore: $y^3 > 0$
$\therefore y > 0$
$4 > 0$, therefore $y = 4$ is a valid solution.

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11. 0,66 6

Explanation:

$$7^x \times 5^{x+2} = 263$$

$$7^x \times 5^x \times 5^2 = 263$$

$$35^x \times 25 = 263$$

$$35^x = 10,52 \quad \text{[divide by 25]}$$

$$\log 35^x = \log 10,52$$

$$x.\log 35 = \log 10,52$$

$$\therefore x = \frac{\log 10,52}{\log 35}$$

$$\therefore x = 0,66$$

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12. A 4

Explanation:

$$\log_3 \frac{a^3 b^2}{x^2}$$

$= \log_3 a^3 b^2 - \log_3 x^2$ [<u>LOG LAW 2</u>: $\log_m \frac{n}{p} = \log_m n - \log_m p$]

$= \log_3 a^3 + \log_3 b^2 - \log_3 x^2$ [<u>LOG LAW 1</u>: $\log_m np = \log_m n + \log_m p$]

$= 3\log_3 a + 2\log_3 b - 2\log_3 x$ [<u>LOG LAW 3</u>: $\log_m n^p = p\log_m n$]

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13. TRUE 2

Explanation: $\log_2 x = -3$

$\therefore x = 2^{-3}$ [Exponent form]

$\therefore x = \frac{1}{2^3}$

$\therefore x = \frac{1}{8}$

<u>TEST:</u>
According to definition, the value of x must be larger than 0.
$\frac{1}{8} > 0$, therefore this solution is valid.

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14. TRUE 4

Explanation:

$\log m^2 + 3\log m - \log 5m$

$= 2\log m + 3\log m - \log 5m$ [<u>LOG LAW 3</u>: $\log_x b^a = a \log_x b$]

$= 5\log m - \log 5m$

$= \log m^5 - \log 5m$ [<u>LOG LAW 3</u>: $a \log_x b = \log_x b^a$]

$= \log \frac{m^5}{5m}$ [<u>LOG LAW 2</u>: $\log_x a - \log_x b = \log_x \frac{a}{b}$]

OR

$\log m^2 + 3\log m - \log 5m$

$= \log m^2 + \log m^3 - \log 5m$ [<u>LOG LAW 3</u>: $a \log_x b = \log_x b^a$]

$= \log\left(m^2 \times m^3 \div 5m\right)$ [<u>LOG LAW 1</u>: $\log_x a + \log_x b = \log_x ab$
<u>LOG LAW 2</u>: $\log_x a - \log_x b = \log_x \frac{a}{b}$]

$= \log \frac{m^5}{5m}$

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15. 3 2

Explanation: $\log_3 27 = x$

$\therefore 3^x = 27$ [Exponent form]

$\therefore 3^x = (3)^3$

$\therefore x = 3$