XT - MATHS Grade 11

Subject: Functions 1: Parabolas and Lines **Date:** 2010/06/29

Total Marks: 66

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1. FALSE 6

Explanation: A point is an ordered pair of numbers, consisting of an ***x***- and a ***y***-coordinate.
Only the ***x***-coordinates were given here.
The points are:

$$3x^2 + x = 3x + 1$$
$$3x^2 - 2x - 1 = 0$$
$$(3x + 1)(x - 1) = 0$$
$$3x + 1 = 0 \text{ or } x - 1 = 0$$
$$x = -⅓ \text{ or } x = 1$$
$$y = 3(⅓) + 1 = 2 \text{ or}$$
$$y = 3(1) + 1 = 4$$

So the points are ***(⅓; 2)*** and ***(1; 4).***

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2. TRUE 4

Explanation: There are two ways of determining the turning point of a parabola at this stage:

- Complete the square, or
- Use the turning point formula

Metode 1: Complete the square

$$\begin{aligned} g(x) &= -2x^2 + 4x - 3 \\ &= -2\left(x^2 - 2x + \frac{3}{2}\right) \\ &= -2\left[\left(x^2 - 2x + 1\right) - 1 + \frac{3}{2}\right] \\ &= -2\left[(x - 1)^2 + \frac{1}{2}\right] \\ &= -2(x - 1)^2 - 1 \end{aligned}$$

Parabolas of the form $y = a(x - p)^2 + q$ have a turning point at $(p; q)$.

Therefore, the turning point of $g(x)$ is at $(1; -1)$.

Metode 2: Use the turning point formula

For the general parabola with equation $y = ax^2 + bx + c$:

At the turning point: $x = -\frac{b}{a}$

$$= -\frac{4}{-2}$$
$$= 1$$

Now:

$$\begin{aligned} y &= -2(1)^2 + 4(1) - 3 \\ &= -2 + 4 - 3 \\ &= -1 \end{aligned}$$

Therefore, the turning point of $g(x)$ is at $(1; -1)$.

Because the arms of the parabola will be pointing down (as $a = -2$, which is smaller than 0), the turning point will be the maximum point of the parabola.

3. FALSE 2

Explanation: y-intercept: $x = 0$

$$\therefore y = 3x^2 - 4x - 4$$
$$= 3(0)^2 - 4(0) - 4$$
$$= -4$$

Therefore, the coordinates of the y-intercept are $(0\,;\,-4)$.

4. B 4

Explanation: Substitute the turning point into $y = a(x-p)^2 + q$:

$$y = a(x-1)^2 + 4 \quad \text{..........} \ (1)$$

Now substitute the point $(2;\,6)$ into the equation:

$$6 = a(2-1)^2 + 4$$
$$\therefore 6 = a + 4$$
$$\therefore a = 2$$

Substitute $a = 2$ into (1): $y = 2(2-1)^2 + 4$

$$= 2\left(x^2 - 2x + 1\right) + 4$$
$$= 2x^2 - 4x + 2 + 4$$
$$= 2x^2 - 4x + 6$$

5. A 4

Explanation: The roots of this equation are 3 and -2.

Substitute these roots into $y = a(x-m)(x-n)$:

$$y = a(x-3)(x-(-2))$$
$$\therefore y = a(x-3)(x+2) \quad \text{..........} \ (1)$$

Substitute the point $(1;\,-12)$ into this equation:

$$-12 = a(1-3)(1+2)$$
$$\therefore -12 = a(-2)(3)$$
$$\therefore -12 = -6a$$
$$\therefore a = 2$$

Substitute $a = 2$ into (1): $y = 2(x-3)(x+2)$

$$\therefore y = 2\left(x^2 - x - 6\right)$$
$$\therefore y = 2x^2 - 2x - 12$$

6. C 3

Explanation:

For $y = ax^2 + bx + c$: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If $b^2 < 4ac$, then $b^2 - 4ac < 0$.

This would mean that the value under the square root sign is negative.

Thus the roots of the function are non-real, and hence the graph of the function will not cut the x-axis.

7. A 4

Explanation: Since the coordinates of the x-intercepts are known and since the coordinates of the turning point are not, you need to use the formula $y = a(x - m)(x - n)$, where m and n are the x-intercepts of the graph.

Substitute the x-intercepts into $y = a(x - m)(x - n)$:

$$y = a\,(x - (-5))\,(x - 2)$$

$$\therefore \quad y = a\,(x + 5)\,(x - 2) \quad \text{..........}\ (1)$$

Substitute the y-intercept into this equation:

$$-10 = a\,(0 + 5)\,(0 - 2)$$

$$\therefore \quad -10 = a\,(5)\,(-2)$$

$$\therefore \quad -10 = -10\,a$$

$$\therefore \quad a = 1$$

[**Warning:** Don't just assume $a = 1$! Work it out!]

Substitute $a = 1$ into (1): $y = 1\,(x + 5)\,(x - 2)$

$$\therefore \; y = (x + 5)\,(x - 2)$$

$$\therefore \; y = x^2 + 3x - 10$$

8. secant 6

Explanation: To determine whether the line is a tangent or a secant, first determine how many points of intersection there will be between the line and the parabola.

$x^2 - 5x + 2 = x - 3$
$x^2 - 6x + 5 = 0$
$(x - 5)(x - 1) = 0$
$x = 5$ or ***$x = 1$***
$y = (1) - 3 = -2$ or
$y = (5) - 3 = 2$

There are two points of intersection ***(1; -2)*** and ***(5; 2)*** between the line and the curve.
So the line is a **secant** to the curve.

9. tangent 6

Explanation: To determine whether the line is a tangent or a secant, first determine how many points of intersection there will be between the line and the parabola.

$x^2 + 3x - 2 = x - 3$
$x^2 + 2x + 1 = 0$
$(x + 1)^2 = 0$
$x = -1$

$y = (-1) - 3 = -4$

There is only one point of intersection ***(-1; -4)*** between the line and the curve.
So the line is a **tangent** to the curve.

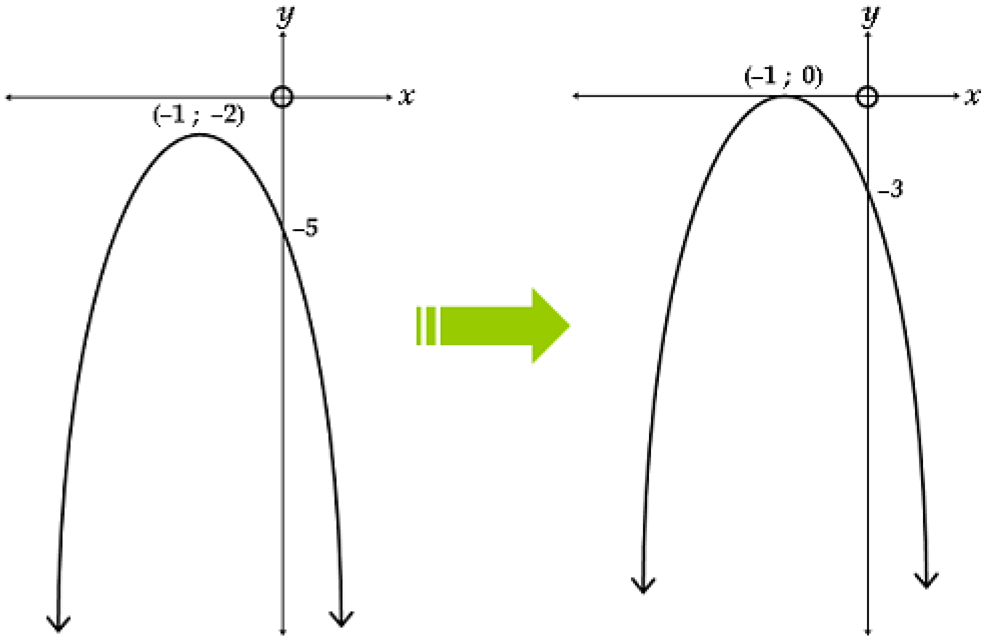
10. (0; -3) 2

Explanation: In general, a parabola of the form $y = a(x - p)^2 + q$ has $(p; q)$ as its turning point.

Hence the turning point of the function $f(x) = -3(x + 1)^2 - 2$ is $(-1; -2)$.
Because $a = -3$ ($a < 0$), the arms of the graph are pointing down.
The y-intercept of the graph:

$$f(0) = -3(0 + 1)^2 - 2$$
$$= -5$$

If the graph is now vertically translated so that its turning point rests on the x-axis, the new turning point will be $(-1; 0)$. Thus the graph (i.e. every point on the graph) must be shifted 2 units up. If the old y-intercept is shifted 2 units up, the new y-intercept will be -3.



11. maximum 1

Explanation: The coefficient of x^2 is negative, thus the arms will be pointing down.
Therefore this function will have a maximum value.

12. (1) $\left(\frac{1}{2};\ 4\frac{1}{2}\right)$ 6

(2) $y = 2x^2 - 2x + 5$

(3) non-real

Explanation:

(1) In the form ***y = a(x - p)² + q***, the turning point is given by ***(p; q)***.
The coordinates of the turning point : $\left(\frac{1}{2};\ 4\frac{1}{2}\right)$

(2) $y = 2\left(x - \frac{1}{2}\right)^2 + 4\frac{1}{2}$

$y = 2\left(x - \frac{1}{2}\right)\left(x - \frac{1}{2}\right) + 4\frac{1}{2}$

$y = 2\left(x^2 - x + \frac{1}{4}\right) + 4\frac{1}{2}$

$y = 2x^2 - 2x + \frac{1}{2} + 4\frac{1}{2}$

$y = 2x^2 - 2x + 5$

(3) To determine the roots we must use the quadratic formula,
as the equation ***$y = 2x^2 - 2x + 5$*** cannot factorise.

$$y = 2x^2 - 2x + 5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{2 \pm \sqrt{-36}}{4}$$

The fact that a negative value appears under the square root implies there are no real roots.
Thus the roots are non-real.

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13. (1) $(2;\ 3)$ 6

(2) $(2;\ 6)$

(3) $f(x) = 3x^2 - 12x + 18$

Explanation: (1) In general, a parabola of the form ***$y = a(x - p)^2 + q$*** has ***(p; q)*** as its turning point.
Thus the turning point of this function is ***(2; 3)***.

(2) When the graph is moved vertically, the intercepts on the axes and the y-coordinate of the turning point will change.
The axis of symmetry will stay the same.
The y-coordinate of the turning point is 3.
If it is moved three units up: $3 + 3 = 6$.
The turning point of the new function will then be $(2;\ 6)$.

(3) Replace the turning point of the original function by the new turning point:

$f(x) = 3(x - 2)^2 + 6$

$= 3(x^2 - 4x + 4) + 6$

$= 3x^2 - 12x + 12 + 6$

$= 3x^2 - 12x + 18$

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14. (1) $(2;\ 0)$ 8

(2) $y = -x^2 - x + 6$

(3) $y = x^2 + x - 6$

Explanation: (1) The points $(-3;\ 0)$ and $\mathbf{A}$ are the same distance from the axis of symmetry.

The point $(-3;\ 0)$ is $2\frac{1}{2}$ units from the axis of symmetry.

Therefore, the x-coordinate of point $\mathbf{A}$ is : $-\frac{1}{2} + 2\frac{1}{2} = 2$

Therefore, the coordinates of $\mathbf{A}$ are $(2;\ 0)$.

(2) You could use either one of two methods to find the equation of this graph.
- You could substitute the x-intercepts and the turning point into this formula:

$$y = a(x - m)(x - n)$$

- The turning point can be substituted into the equation:

$$y = a(x - p)^2 + q$$

$$y = a\left(x - (-\tfrac{1}{2})\right)^2 + 6\ \tfrac{1}{4}$$

$$y = a\left(x + \tfrac{1}{2}\right)^2 + \tfrac{25}{4}$$

Now, substitute the coordinates of any one of the x-intercepts into this equation:

$$y = a\left(x + \tfrac{1}{2}\right)^2 + \tfrac{25}{4}$$

$$0 = a\left(2 + \tfrac{1}{2}\right)^2 + \tfrac{25}{4}$$

$$0 = a\left(\tfrac{5}{2}\right)^2 + \tfrac{25}{4}$$

$$0 = \tfrac{25}{4}\ a + \tfrac{25}{4}$$

$$-\tfrac{25}{4}\ a = \tfrac{25}{4}$$

$$a = -1$$

Therefore …

$$y = -1\left(x + \tfrac{1}{2})\right)^2 + \tfrac{25}{4}$$

$$y = -1\left(x^2 + x + \tfrac{1}{4}\right) + \tfrac{25}{4}$$

$$y = -x^2 - x - \tfrac{1}{4} + \tfrac{25}{4}$$

$$y = -x^2 - x + \tfrac{24}{4}$$

$$y = -x^2 - x + 6$$

(3) For a reflection about the x-axis, ***(x; y)*** becomes ***(x; -y)***.
$y = -x^2 - x + 6$ becomes - ***$y = -x^2 - x + 6$***
$y = x^2 + x - 6$

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15. D 4

Explanation: The coordinates of the turning point is known.

Substitute the turning point into $y = a(x - p)^2 + q$:

$$y = a(x - (-4))^2 + 0$$

$$\therefore \quad y = a(x + 4)^2 + 0 \quad \text{..........} (1)$$

Substitute the point $(-2;\ 8)$ into this equation:

$$8 = a(-2 + 4)^2 + 0$$

$$\therefore \ 8 = a(2)^2$$

$$\therefore \ 8 = 4a$$

$$\therefore \ a = 2$$

Substitute $a = 2$ into (1): $y = 2(x + 4)^2 + 0$

$$\therefore \quad y = 2\left(x^2 + 8x + 16\right)$$

$$\therefore \quad y = 2x^2 + 16x + 32$$

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15 Questions, 7 Pages