

JEM: Q3 & Q4

DETAILS OF ASSESSMENT:

FV PII 2010.

ANALYSIS OF PAPER AND MEMORANDUM

Mathematics Taxonomy Levels:

level 1 Knowledge
 level 2 Performing Routine Procedures
 level 3 Performing Complex Procedures
 level 4 Problem Solving

QUESTION	LO	POSSIBLE SOLUTION	MARKING NOTES	TAXONOMY
1(a)		$AC = \sqrt{(5+3)^2 + (-3-1)^2} \checkmark$ $= 4\sqrt{5} \checkmark$		
(b)		$m = \frac{1+3}{-3-5} = -\frac{1}{2} \checkmark$ $y+3 = -\frac{1}{2}(x-5) \checkmark$ $y = -\frac{1}{2}x - \frac{1}{2} \checkmark$		
(c)		$m_{AC} \cdot m_{BD} = -\frac{1}{2} \times 2 = -1 \checkmark$ $\therefore AC \perp BD \checkmark$ $\text{midpt } BD: \left(\frac{4+2}{2}, \frac{0-4}{2}\right) = (3, -2) \checkmark$ $\text{if } x=3: y = -\frac{1}{2}(3) - \frac{1}{2} \checkmark$ $= -2 \checkmark$ $\therefore (3, -2) \text{ lies on the line } AC \checkmark$ $\therefore AC \text{ is the perpendicular bisector of } DB$		
(d)		$\text{Area} = 2 \times \text{Area } \triangle ACD \checkmark$ $\text{height} = \sqrt{(4-3)^2 + (0+2)^2} \checkmark$ $= \sqrt{5} \checkmark$ $\therefore \text{Area} = 2 \times \frac{1}{2} (\sqrt{5})(4\sqrt{5}) \checkmark$ $= 20 \text{ units}^2 \checkmark$		
(e)		$\tan \theta = m_{AB} = \frac{1+4}{-3-2} = -1 \checkmark$ $\theta = 135^\circ \checkmark$		
(f)		$\tan \hat{ADX} = \frac{0-1}{4+3} = -\frac{1}{7} \checkmark$ $\hat{ADX} = 8^\circ \checkmark$ $\therefore \hat{BAD} = 37^\circ \checkmark$		

DETAILS OF ASSESSMENT:

ANALYSIS OF PAPER AND MEMORANDUM

Mathematics Taxonomy Levels:

level 1	Knowledge
level 2	Performing Routine Procedures
level 3	Performing Complex Procedures
level 4	Problem Solving

QUESTION	LO	POSSIBLE SOLUTION	MARKING NOTES	TAXONOMY
2(a)		$(x-2)^2 + (y+3)^2 = r^2 \checkmark$ $(6-2)^2 + (-1+3)^2 = r^2 \checkmark$ $20 = r^2 \checkmark$ eqn: $(x-2)^2 + (y+3)^2 = 20 \checkmark$		
(b)		midpt PQ: $(\frac{6+0}{2}; \frac{-1-7}{2}) = (3; -4) \checkmark$ $m_{PQ} = \frac{-1-7}{6-0} = -1 \checkmark$ eqn of \perp bisector: $y+4 = -1(x-3)$ $y = -x-1 \checkmark$ sub $x=2$: $y = -2-1$ $= -3 \checkmark$ $\therefore \perp$ bisector of PQ passes through center of the circle. \checkmark		
(c)		$(-1-2)^2 + (2+3)^2 = 34 \checkmark$ \rightarrow this is longer than the radius of the circle \checkmark \therefore the point lies outside the circle. \checkmark		

DETAILS OF ASSESSMENT:

ANALYSIS OF PAPER AND MEMORANDUM

Mathematics Taxonomy Levels:

- level 1 Knowledge
- level 2 Performing Routine Procedures
- level 3 Performing Complex Procedures
- level 4 Problem Solving

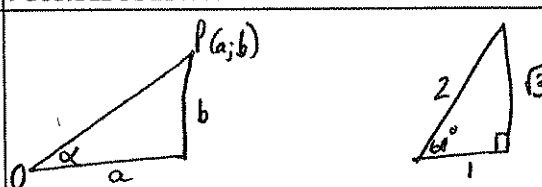
QUESTION	LO	POSSIBLE SOLUTION	MARKING NOTES	TAXONOMY
3 (a)(i)		$(5; -1)$		
(ii)		$(1; -5)$		
(b) (i)		<ul style="list-style-type: none"> • coordinates <u>reduced</u> by a factor of <u>2</u> ✓ • area <u>$\frac{1}{4}$</u> that of original ✓ 		
(ii)		<ul style="list-style-type: none"> • vertices reflected about <u>x-axis</u> ✓ • vertices shifted <u>2 units to the right</u> ✓ • area remains the same ✓ 		

DETAILS OF ASSESSMENT:

ANALYSIS OF PAPER AND MEMORANDUM

Mathematics Taxonomy Levels:

- level 1 Knowledge
- level 2 Performing Routine Procedures
- level 3 Performing Complex Procedures
- level 4 Problem Solving

QUESTION	LO	POSSIBLE SOLUTION	MARKING NOTES	TAXONOMY
4.		 $\frac{a}{OP} = \cos \alpha \qquad \frac{b}{OP} = \sin \alpha$ $\therefore a = OP \cdot \cos \alpha \qquad b = OP \cdot \sin \alpha$ $P' = [OP \cdot \cos \alpha \cdot \cos 60^\circ - OP \cdot \sin \alpha \cdot \sin 60^\circ; OP \cdot \sin \alpha \cdot \cos 60^\circ + OP \cdot \cos \alpha \cdot \sin 60^\circ]$ $= [a \cdot \cos 60^\circ - b \cdot \sin 60^\circ; b \cdot \cos 60^\circ + a \cdot \sin 60^\circ]$ $= \left[\frac{a}{2} - \frac{b \cdot \sqrt{3}}{2}; \frac{b}{2} + \frac{a \cdot \sqrt{3}}{2} \right]$		

DETAILS OF ASSESSMENT:

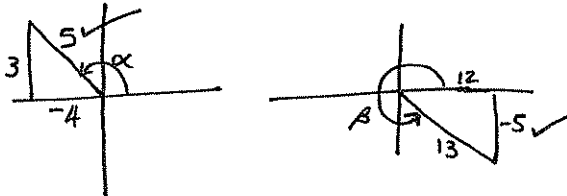
ANALYSIS OF PAPER AND MEMORANDUM

Mathematics Taxonomy Levels:

level 1	Knowledge
level 2	Performing Routine Procedures
level 3	Performing Complex Procedures
level 4	Problem Solving

QUESTION	LO	POSSIBLE SOLUTION	MARKING NOTES	TAXONOMY
----------	----	-------------------	---------------	----------

5(a)



$$\sin \alpha \cdot \sin \beta = \left(\frac{3}{5}\right) \left(-\frac{5}{13}\right)$$

$$= -\frac{3}{13} \checkmark$$

(b)(i)

$$\frac{\sin(\pi) \cdot (-\cos \pi) \cdot (\tan \pi)}{(-\sin \pi) \cdot (\cos \pi)}$$

$$= \tan \pi \checkmark \checkmark$$

(ii)

$$\frac{\sin 80^\circ \cdot (-\cos 60^\circ)}{\sin 80^\circ \cdot (\sin 30^\circ)}$$

$$= \frac{(-1/2)}{(1/2)} \checkmark$$

$$= -1 \checkmark$$

DETAILS OF ASSESSMENT:

ANALYSIS OF PAPER AND MEMORANDUM

Mathematics Taxonomy Levels:

- level 1 Knowledge
- level 2 Performing Routine Procedures
- level 3 Performing Complex Procedures
- level 4 Problem Solving

QUESTION	LO	POSSIBLE SOLUTION	MARKING NOTES	TAXONOMY
6.		$2\sin\alpha(\cos\alpha+1) + \cos\alpha(\cos\alpha+1) = 0$ $(\cos\alpha+1)(2\sin\alpha + \cos\alpha) = 0 \quad \checkmark$ <p>$\therefore \cos\alpha+1 = 0 \quad \checkmark$ OR $2\sin\alpha + \cos\alpha = 0$</p> $\cos\alpha = -1 \qquad 2\sin\alpha = -\cos\alpha$ $\alpha = 180^\circ; -180^\circ \quad \checkmark \qquad 2\tan\alpha = -1$ $\qquad \qquad \qquad \tan\alpha = -2 \quad \checkmark$ $\qquad \qquad \qquad \alpha = 297^\circ; 117^\circ \quad \checkmark$ $\therefore \alpha = 180^\circ; -180^\circ; 117^\circ$		

DIAGRAM SHEET

EXAMINATION NUMBER:

--	--	--	--	--	--	--

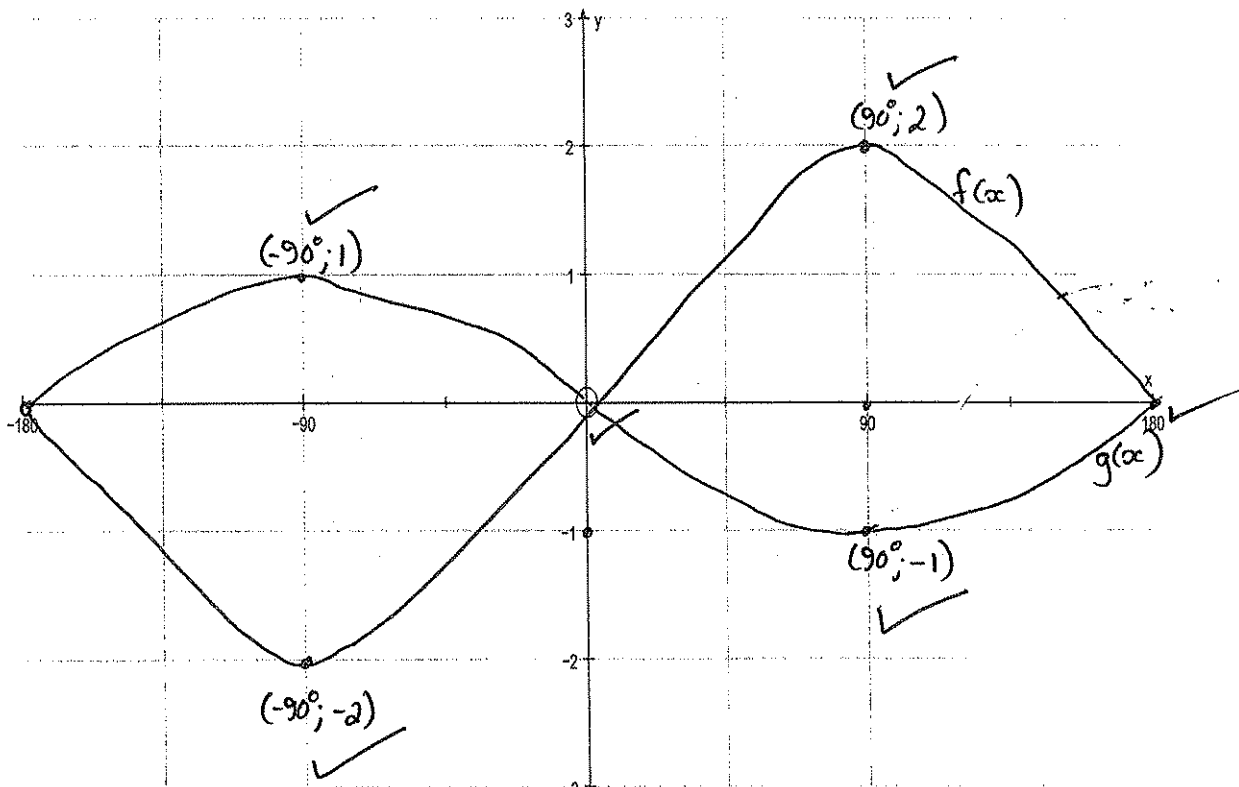
QUESTION 7

Consider the functions f and g such that:

$$f(x) = 2 \sin x \text{ and } g(x) = -\sin x$$

- (a) On the same system of axes sketch the graphs of f and g for the interval $[-180^\circ; 180^\circ]$. Indicate the intercepts with the axes and the coordinates of the turning points.

(6)



- (b) What is the range of f ?

$$-2 \leq y \leq 2$$

(2)

- (c) For what value(s) of x is: $f(x) < g(x)$?

$$-180^\circ \leq x \leq 0^\circ$$

(3)

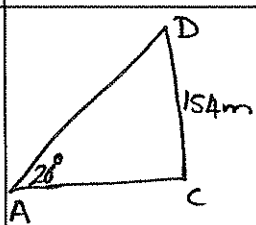
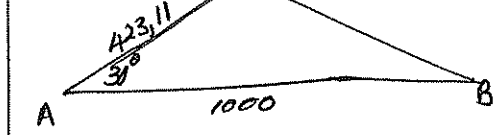
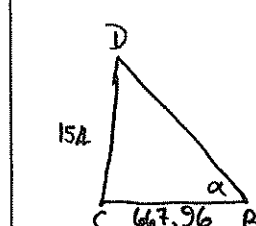
[11]

DETAILS OF ASSESSMENT:

ANALYSIS OF PAPER AND MEMORANDUM

Mathematics Taxonomy Levels:

- level 1 Knowledge
- level 2 Performing Routine Procedures
- level 3 Performing Complex Procedures
- level 4 Problem Solving

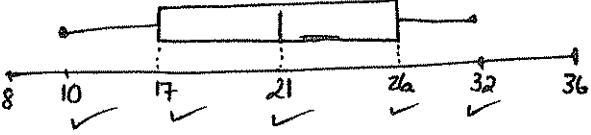
QUESTION	LO	POSSIBLE SOLUTION	MARKING NOTES	TAXONOMY
8 (a)		 $\tan 20^\circ = \frac{154}{AC} \checkmark$ $AC = \frac{154}{\tan 20^\circ}$ $= 423,11 \text{ m} \checkmark$		
(b)		 $BC^2 = (1000)^2 + (423,11)^2 - 2(1000)(423,11) \cos 30^\circ \checkmark$ $\therefore BC = 667,96 \text{ m} \checkmark$  $\tan \alpha = \frac{154}{667,96} \checkmark$ $\alpha = 12,98^\circ \checkmark \checkmark$		

DETAILS OF ASSESSMENT:

ANALYSIS OF PAPER AND MEMORANDUM

Mathematics Taxonomy Levels:

- level 1 Knowledge
- level 2 Performing Routine Procedures
- level 3 Performing Complex Procedures
- level 4 Problem Solving

QUESTION	LO	POSSIBLE SOLUTION	MARKING NOTES	TAXONOMY
9 (a)		21 ✓✓		
(b)		$Q_1 = 17$ ✓		
(c)		$Q_3 = 26$ ✓		
(d)				
(d)		any reasonable comment (1x2) ✓✓		

DETAILS OF ASSESSMENT:

ANALYSIS OF PAPER AND MEMORANDUM

Mathematics Taxonomy Levels:

- level 1 Knowledge
- level 2 Performing Routine Procedures
- level 3 Performing Complex Procedures
- level 4 Problem Solving

QUESTION	LO	POSSIBLE SOLUTION	MARKING NOTES	TAXONOMY
10 (a)		$\cos 2D = 2\cos^2 D - 1 \checkmark$ $7p = 2(2p)^2 - 1 \checkmark$ $0 = 8p^2 - 7p - 1 \checkmark$ $0 = (8p+1)(p-1) \checkmark$ $\therefore p = -1/8 \text{ or } p = 1 \checkmark$		
(b) (i)		$\text{LHS} = \overbrace{\sin A \cos B + \cos A \sin B} \checkmark - \overbrace{(\sin A \cos B - \cos A \sin B)} \checkmark$ $= 2 \cos A \sin B \checkmark$ $= \text{RHS}$		
(ii)		$\sin 5\alpha - \sin \alpha = \sin(3\alpha + 2\alpha) - \sin(3\alpha - 2\alpha) \checkmark$ $= 2 \cos 3\alpha \cdot \sin 2\alpha \checkmark$		
(iii)		$2 \cos 3\alpha \cdot \sin 2\alpha = 0$ $\therefore \cos 3\alpha = 0 \checkmark \text{ or } \sin 2\alpha = 0 \checkmark$ $3\alpha = 90^\circ + 360^\circ n \text{ or } 3\alpha = 270^\circ + 360^\circ n \text{ or } 2\alpha = 0^\circ + 360^\circ n$ $\alpha = 30^\circ + 120^\circ n \checkmark \quad \alpha = 90^\circ + 120^\circ n \checkmark \quad \alpha = 180^\circ n \checkmark$		

DETAILS OF ASSESSMENT:

ANALYSIS OF PAPER AND MEMORANDUM

Mathematics Taxonomy Levels:

level 1	Knowledge
level 2	Performing Routine Procedures
level 3	Performing Complex Procedures
level 4	Problem Solving

QUESTION	LO	POSSIBLE SOLUTION	MARKING NOTES	TAXONOMY
11. (i)		<p>Centre of circle: $\left(\frac{-1+3}{2}; \frac{0-2}{2}\right) = (1; -1)$</p> <p>radius = $\frac{1}{2}$ diameter</p> $= \frac{1}{2} \sqrt{(3+1)^2 + (-2-0)^2}$ $= \frac{1}{2} \sqrt{20}$ $= \sqrt{5} \quad \checkmark$ <p>$\therefore (x-1)^2 + (y+1)^2 = 5 \quad \checkmark \checkmark$</p>		
(ii)		<p>$(x-1)^2 + (0+1)^2 = 5 \quad \checkmark$</p> $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0 \quad \checkmark$ $x = 3 \text{ or } -1 \quad \checkmark$		
(iii)		<p>if 2 circles touch each other, then the distance between their centres is equal to the sum of their radii</p> <p>centres: $(1; -1)$ and $(3; 3)$.</p> $\text{distance between centres} = \sqrt{(1-3)^2 + (-1-3)^2}$ $= 2\sqrt{5} \quad \checkmark$ $\text{sum of radii} = \sqrt{5} + \sqrt{5} \quad \checkmark \checkmark$ $= 2\sqrt{5} \quad \checkmark \checkmark$ <p>\therefore circles touch.</p>		

DETAILS OF ASSESSMENT:

ANALYSIS OF PAPER AND MEMORANDUM

Mathematics Taxonomy Levels:

- level 1 Knowledge
- level 2 Performing Routine Procedures
- level 3 Performing Complex Procedures
- level 4 Problem Solving

QUESTION	LO	POSSIBLE SOLUTION	MARKING NOTES	TAXONOMY
12.		<p>mean: $\frac{2+x+y}{3} = 5 \checkmark$</p> <p>$x+y = 13$</p> <p>$y = 13 - x \checkmark \text{---} \textcircled{1}$</p> <p>std. deviation: $\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{6} \checkmark$</p> <p>$\frac{\sum(x-\bar{x})^2}{n} = 6 \checkmark$</p> <p>$(2-5)^2 + (x-5)^2 + (y-5)^2 = 18 \checkmark$</p> <p>$9 + x^2 - 10x + 25 + y^2 - 10y + 25 = 18$</p> <p>$x^2 - 10x + y^2 - 10y + 41 = 0 \checkmark \text{---} \textcircled{2}$</p> <p>substitute $\textcircled{1}$ into $\textcircled{2}$</p> <p>$x^2 - 10x + (13-x)^2 - 10(13-x) + 41 = 0$</p> <p>$x^2 - 10x + 169 - 26x + x^2 - 130 + 10x + 41 = 0$</p> <p>$2x^2 - 26x + 80 = 0 \checkmark$</p> <p>$x^2 - 13x + 40 = 0 \checkmark$</p> <p>$(x-8)(x-5) = 0 \checkmark$</p> <p>$x = 8 \text{ OR } x = 5 \checkmark$</p> <p>$\therefore y = 5 \text{ OR } y = 8 \checkmark$</p>		