NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2009

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines were used as the basis for the official IEB marking session. They were prepared for use by examiners and sub-examiners, all of whom were required to attend a rigorous standardisation meeting to ensure that the guidelines were consistently and fairly interpreted and applied in the marking of candidates' scripts.

At standardisation meetings, decisions are taken regarding the allocation of marks in the interests of fairness to all candidates in the context of an entirely summative assessment.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines, and different interpretations of the application thereof. Hence, the specific mark allocations have been omitted.

Please note that learners who provided alternate correct responses to those given in the marking guidelines will have been given full credit.

SECTION A

QUESTION 1

(a) (1) $8x^2 + 1 = 7x$

$8x^2 - 7x + 1 = 0$

$x = \dfrac{7 \pm \sqrt{49 - 32}}{16}$ Quadratic formula

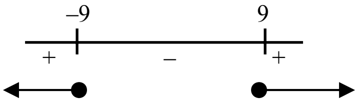
$= \dfrac{7 \pm \sqrt{17}}{16}$

$= 0{,}7 \quad \text{or} \quad 0{,}2$ (4)

(2) $x^2 \geq 81$

$x^2 - 81 \geq 0$

$(x-9)(x+9) \geq 0$ Factorising (4)



$x \geq 9 \quad \text{or} \quad x \leq -9$

(3) $\log 10^{x-5} = 7$

$10^{x-5} = 10^7$ Converting to Exps

$x - 5 = 7$

$x = 12$ (2)

ALTERNATIVELY:

$(x-5)\log 10 = 7$

$x - 5 = 7$

$x = 12$

(b) $4^2 - k.4 - 12 = 0$ Substitution of 4

$4 - 4k = 0$

$-4k = -4$

$k = 1$ (3)

(c) $\dfrac{x}{6} = \dfrac{54}{x}$ Ratios

$x^2 = 324$

$x = \pm 18$ (3)

[16]

QUESTION 2

(a) $f(x) = 2x$

$$f'(x) = \lim_{h \to o} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to o} \frac{2(x+h) - 2x}{h}$$ Substitution

$$= \lim_{h \to o} \frac{2x + 2h - 2x}{h}$$

$$= \lim_{h \to o} \frac{2h}{h}$$

$$= \lim_{h \to o} 2$$

$$= 2$$ (4)

(b) (1) $y = \frac{x^3}{6} - 6x^{-2}$

$$\frac{dy}{dx} = \frac{1}{6}.3x^2 + 12x^{-3}$$ Differentiation

$$= \frac{x^2}{2} + \frac{12}{x^3}$$ (3)

(2) $y = \frac{x^2 - 8x + 12}{3x - 6}$

$$= \frac{(x-6)(x-2)}{3(x-2)}$$ Factorising

$$= \frac{x}{3} - 2$$

$$\frac{dy}{dx} = \frac{1}{3}$$ (3)

[10]

QUESTION 3

(a) Parabola : $y = a(x-4)^2 + 8$

$(0\ ;\ 0)$: $0 = a.16 + 8$ Substitution

$a = -\frac{1}{2}$

$f(x) = -\frac{1}{2}(x-4)^2 + 8$

Straight lines :

g : $g(x) = 2x$

h : $m = -2$

$y - 0 = -2(x-8)$ Eqn. of Str. line (7)

$h(x) = -2x + 16 \quad x\ \varepsilon\ [4;8]$

(b) $y = (x-2)^2 - 3$ (2)

(c) (1) $f(1) + g(2)$

$= \frac{8\times 1 + 32}{20} + \frac{5\times 2}{2} - 4$ Substitution

$= 2 + 1$

$= 3$ (2)

(2) f^{-1}: $x = \frac{8y+32}{20}$

$20x = 8y + 32$ Simplification

$8y = 20x - 32$

$y = \frac{20x - 32}{8}$

$= \frac{5x}{2} - 4$

$\therefore\ f^{-1}(x) = g(x)$ (3)

(d) A - R

B - S

C - Q

D - V (4 × 1 = 4)

[18]

QUESTION 4

(a) (1) Loan $= \dfrac{x\left[1-(1+i)^{-n}\right]}{i}$ **240**

$\dfrac{0,155}{12}$

$$450\ 000 = \frac{x\left[1-\left(1+\dfrac{0,155}{12}\right)^{-240}\right]}{\dfrac{0,155}{12}}$$

$$= x\{7,86175222\}$$

$$x = 6092,46$$

i.e. Monthly payments : R6 092,46 (5)

(2) Total payments = $240 \times 6092,46$

$= 1462190,40$ Calculation

Interest = R1 012 190,40 (2)

(b) (1) $500\,000 = 250\,000\left(1+\dfrac{0,058}{12}\right)^{12n}$ Sub. into formula

$(1,0048333)^{12n} = 2$

$12n = \log_{1,004...} 2$ Introducing Logs

$= 143,756\ldots$

$n = 11,9796\ldots$

≈ 12 years (4)

(2) $1+i = \left(1+\dfrac{0,058}{12}\right)^{12}$

$i = 1,0048...^{12} - 1$ Manipulation

$= 0,0595669\ldots$

$\approx 6\%$ (2)

(3) No. of years $= \dfrac{72}{6}$

$= 12$

The same as the answer in (1) (2)

(c) Cost price : x

Marked price : $1,25x$

Discount $= 1,25x - 1,05x$ Difference

$= 0,2\,x$

Perc. discount $= \dfrac{0,2x}{1,25x} \times 100$ Calculation

$= 16\%$ (5)

[20]

QUESTION 5

(a) At A & B : $f'(x) = 0$

$$f'(x) = 12x^2 + 54x - 30 = 0$$

$f'(x) = 0$

$$2x^2 + 9x - 5 = 0$$

$$(2x-1)(x+5) = 0$$

$$x = \frac{1}{2} \quad \text{or} \quad x = -5$$

$$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 + 27\left(\frac{1}{2}\right)^2 - 30\left(\frac{1}{2}\right) - 1$$

$$= -\frac{35}{4} \qquad (-8,75)$$

$$f(-5) = 4(-5)^3 + 27(-5)^2 - 30(-5) - 1$$

$$= 324$$

$$\therefore \quad \text{A}\left(-5\,;324\right), \quad \text{B}\left(\frac{1}{2};-\frac{35}{4}\right)$$

(6)

(b) Ave. Grad. $= \dfrac{324-\left(-\frac{35}{4}\right)}{-5-\frac{1}{2}}$

Sub. of y & x values

$$= -\frac{121}{2} \qquad (-60,5)$$

(2)

(c) C(0 ; – 1)

$f'(0) = -30$

Eqn. of tangent : $y = -30x - 1$

(3)

(d) $4x^3 + 27x^2 - 30x - 1 = -30x - 1$

Cubic = Tangent

$$4x^3 + 27x^2 = 0$$

$$x^2(4x+27) = 0$$

$$x = 0 \quad \text{or} \quad x = -\frac{27}{4}$$

$$\therefore x = \frac{-27}{4}$$

(3)

[14]

SECTION B

QUESTION 6 ANSWER BOOKLET

QUESTION 7 ANSWER BOOKLET

QUESTION 8

(a) $\sum_{k=1}^{\infty} \frac{2}{3^k}$

$= \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots$ Expanding

$S_\infty = \frac{a}{1-r}$

$= \frac{2}{3} \div \left(1 - \frac{1}{3}\right)$ Sub. in S∞

$= 1$ (3)

(b) (1) A.P. $a = 30, \quad d = -1$

$T_n = 30 + (n-1)(-1)$ Sub. in Tn for A.P.

$= 30 - n + 1$

$= 31 - n$ (2)

(2) Max. No. of layers = 30

$S_{30} = \frac{30}{2}\left[2a + 29d\right]$ Sub. in Sn

$= 15\left[2 \times 30 + 29(-1)\right]$

$= 15\left[60 - 29\right]$

$= 465$

i.e. Max. of 465 cans

ALTERNATIVELY:

$S_{30} = \frac{30}{2}\,[a + \ell]$ ✓**A**

$= 15\,[30 + 1]$

$= 465$ (4)

[9]

QUESTION 9

(a) $f(g(9))$

$= f(9^2)$

$= \sqrt{4 \times 9^2}$

$= 2 \times 9$

$= 18$ (3)

(b) (1) $h(x) = \dfrac{x-1}{x-2}$

$= \dfrac{x-2+1}{x-2}$

$= 1 + \dfrac{1}{x-2}$ (3)

ALTERNATIVELY:

Hyperbola : $y = \dfrac{k}{x-2} + 1$

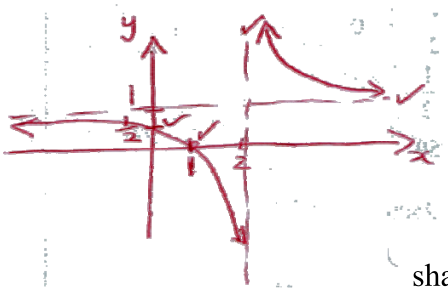
$= \dfrac{k+x-2}{x-2}$

$h(x) = \dfrac{x-1}{x-2} = \dfrac{k+x-2}{x-2}$

$\therefore\ k = 1$

$\therefore\ h(x) = \dfrac{1}{x-2} + 1$

(2)



shape (5)

(3) $1 \leq x < 2$ (1)

[12]

QUESTION 10

(a) $P(x) = 50\sqrt{x} - 0,5x - 500$

$P'(x) = 50 \times \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2} = 0$ — $P'(x) = 0$

$\frac{25}{\sqrt{x}} = \frac{1}{2}$ — Simplifying

$\sqrt{x} = 50$

$x = 2500$ (5)

(b) (1) $2x + y = 10$

$y = -2x + 10$

$\text{OP}^2 = x^2 + (-2x+10)^2$ — Sub. in Dist.

$= x^2 + 4x^2 - 40x + 100$

$= 5x^2 - 40x + 100$ (3)

(2) Min. when $\frac{d\text{OP}^2}{dx} = 0$

$10x - 40 = 0$ — Der. = 0

$x = 4$

$y = -2 \times 4 + 10$

$= 2$

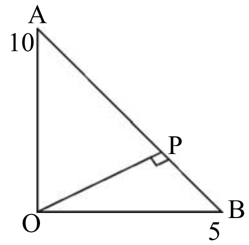
$\therefore$ P(4 ; 2) (4)

(3) Min. $\text{OP}^2 = 5 \times 4^2 - 40 \times 4 + 100$ — Substitute

$= 20$

$\therefore$ Min. OP $= 2\sqrt{5}$ (2)

ALTERNATIVELY:



$\text{AB}^2 = 125$

Area Δ AOB $= \frac{1}{2}.\text{OP}\sqrt{125} = \frac{1}{2} \times 10 \times 5$

$\therefore \text{OP} = \frac{50}{\sqrt{125}}$

$= \frac{50}{5\sqrt{5}}$

$= \frac{10}{\sqrt{5}}$

$= 2\sqrt{5}$

[14]

QUESTION 11

(a) 41 43 47 53

1^{st}. diff. 2 4 6 — Differences

2^{nd} diff 2 2

$$T_n = T_1 + (n-1)f + \frac{(n-1)(n-2)}{2}.s$$
$$= 41 + (n-1)2 + \frac{(n-1)(n-2)}{2}.2$$
$$= 41 + 2n - 2 + n^2 - 3n + 2$$

Simplifying

$$= n^2 - n + 41$$

(4)

(b) $T_{41} = 41^2 - 41 + 41$ — Substutution

$= 41^2$

which is not prime. (3)

(c) Sequence repeats cycle with units digits
1 ; 3 ; 7 ; 3 ; 1

$\frac{49\,999\,998}{5}$ gives Quotient = 9 999 999 and Rem = 3

$\therefore$ Units digit is 7. (3)

ALTERNATIVELY:

$T_{49999998} = 4999998 \times 49999997 + 41$

$8 \times 7 + 41$

$= 56 + 41$

$= 97$ ends in 7

$\therefore$ $T_{49999998}$ ends in 7

[10]