NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2009

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

ANSWER BOOKLET

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

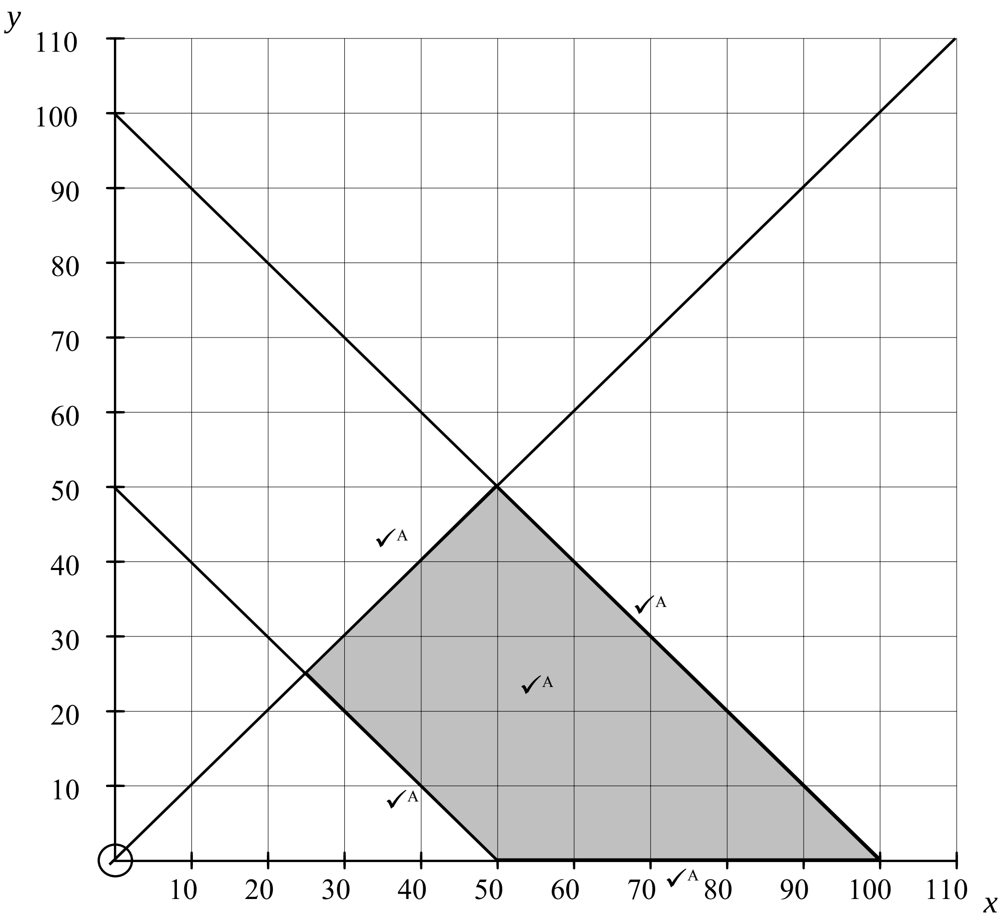
The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 6

(a) The following constraints apply to a linear programming problem.

$x \geq 0, \quad y \geq 0, \quad x + y \geq 50, \quad x + y \leq 100, \quad y \leq x$

(1) Draw these constraints on the axes below and shade the feasible region.



(5)

(2) Which constraint does not influence the feasible region?

$x \geq 0$ ✓A

(1)

(3) Give the maximum value of y that satisfies all the constraints.

Max $y = 50$ ✓A

(1)

(4) Given the objective function $C = 2x + y$ determine the minimum value of C that satisfies all the constraints.

$y = -2x + C$

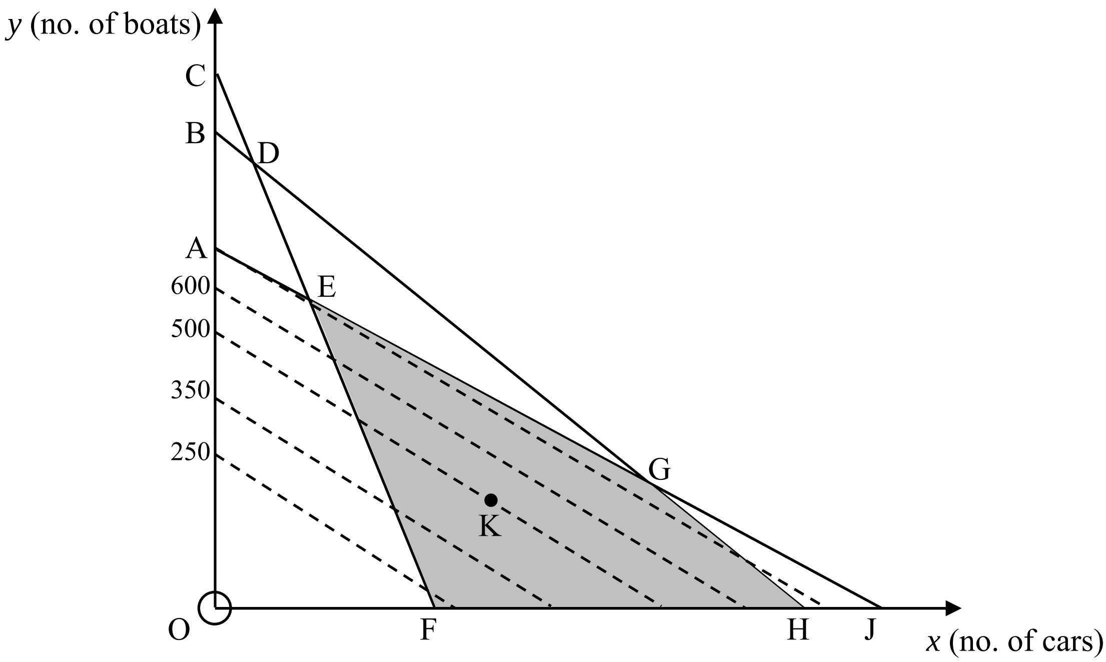
Min. at point where $x = y = 25$ ✓A

Min. $C = 2 \times 25 + 25$ ✓M

$= 75$ ✓A

(3)

(b) A linear programming technique has been applied to a situation where the variables are the number of cars (x) and the number of boats (y) produced by a toy manufacturer. The shaded area below is the feasible region:



Assume that all points A to K have integer coordinates.

Suppose that c_1 is the profit made on each car and c_2 is the profit made on each boat. Hence the values c_1 and c_2 can be assumed to be positive constants.

The profit function is $P = c_1 x + c_2 y$
and is represented on the graph above by dotted lines.

(1) Write down the profit obtained at the point K in terms of c_2.

$\frac{P}{c_2} = 500$

$\therefore P = 500c_2$ ✓A

(1)

(2) Identify the point in the feasible region that yields the maximum profit.

G ✓A

(1)

(3) As a result of an improved manufacturing process, the profit made on each car increases to become c_3, whereas the profit on each boat remains constant, such that the profit function now is $P = c_3 x + c_2 y$.

Determine which labelled point(s) in the feasible region are now likely to maximise the profit.

Gradient of first profit function: $-\dfrac{c_1}{c_2}$

Gradient of new profit function: $-\dfrac{c_3}{c_2}$ ✓A

$c_3 > c_1 \quad \therefore$ Steeper search line. ✓M

G or H could yield optimal solution. ✓A

(3)

15 marks

QUESTION 7

Alex decides to include both swimming and running in her exercise plan.
On day 1, Alex swims 100 m and runs 500 m.
Each day she will increase the distance she swims by 50 m and the distance she runs will increase by 3,5% of the distance she ran on the previous day.

(a) Determine, in terms of n, the distance that Alex

(1) swims on the n^{th} day of her exercise plan.

A.P. $a_1 = 100, \quad d = 50$ ✓A

$T_n = 100 + (n-1)\,50$ ✓M✓A

$= 100 + 50n - 50$

$= 50n + 50 \quad n \ \varepsilon\ N$ ✓A

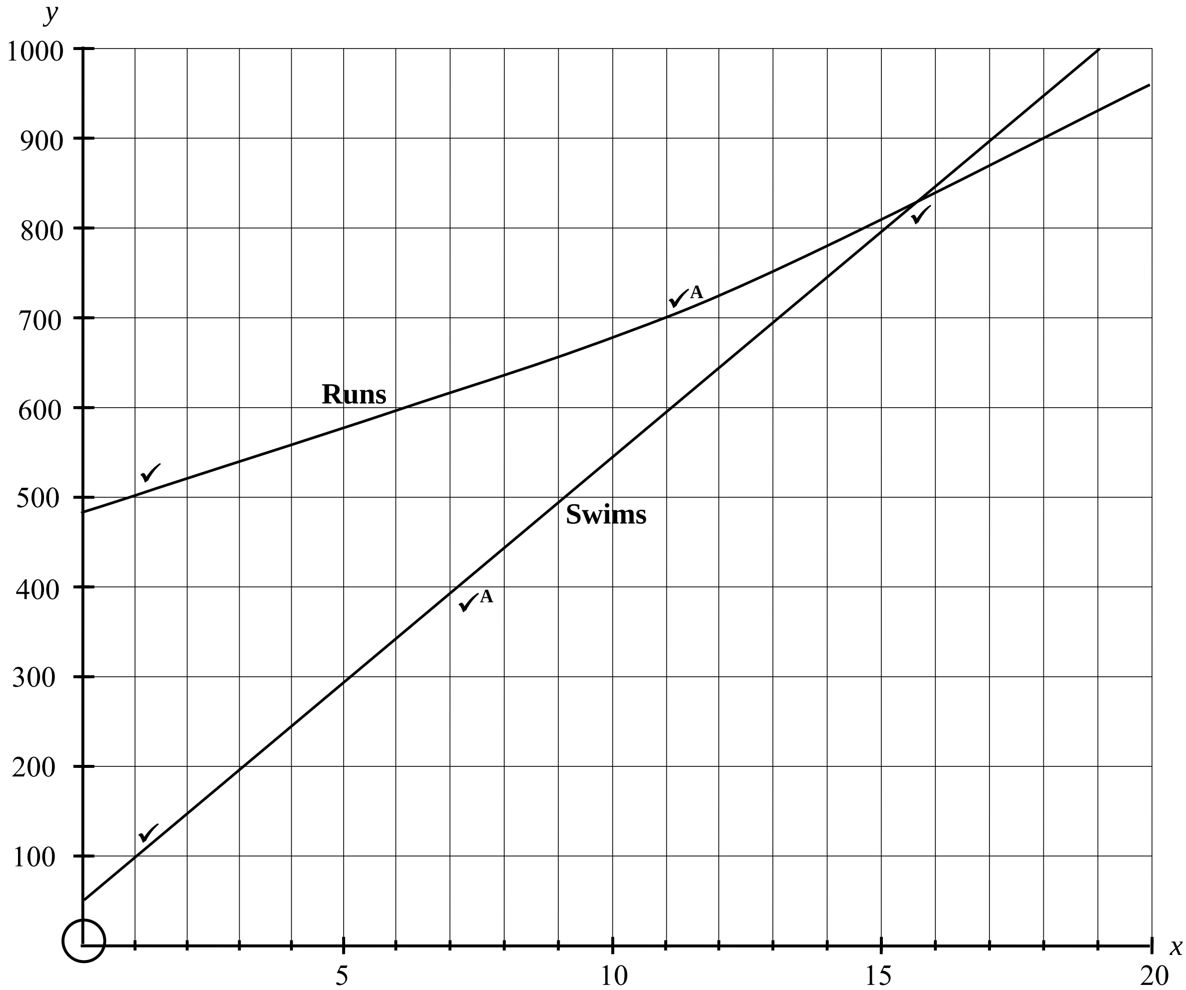
(2) runs on the n^{th} day of her exercise plan.

G.P. $a_2 = 500, \quad r = 0{,}035$ ✓A

$T_k = 500(0{,}035)^{k-1} \quad k \ \varepsilon\ N$ ✓M✓A

(6)

(b) On the set of axes provided, plot points for each of the exercise types. You may join the points to illustrate the trend.



(5)

(c) **Use your graphs** to determine the first day on which the distance Alex swims will be greater than the distance she runs.

On the 16th day. ✓CA

(1)

12 marks