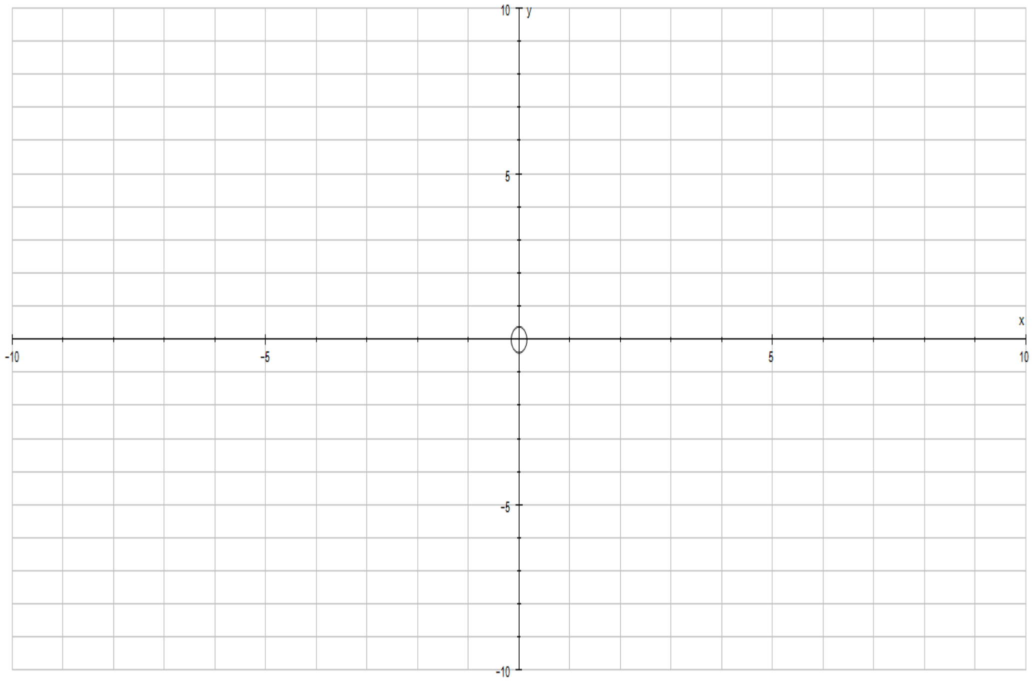
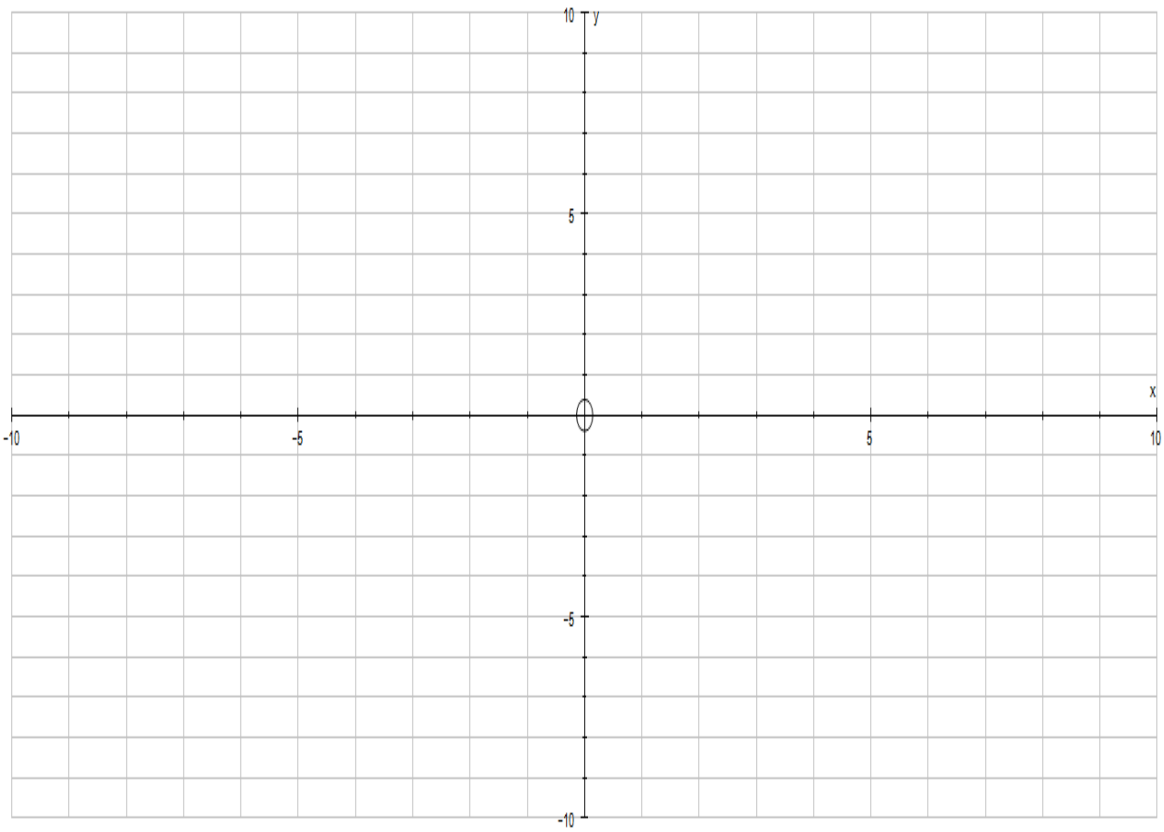
ANSWER SHEET

NAME: ___________________________________

QUESTION 4(a)



QUESTION 11(a)



Question 1

Solve for all real value(s) of x:

(a) $3x^2+4x=5$ correct to 2 decimal places (3)

(b) $(x+1)(x^2+8)(x^3+27)=0$ (2)

(c) $2\log_4 x=-1$ (2)

(d) $9\times 4^x=1$ (3)

(e) $\sqrt{x^2+x+10}=x^2+x-2$ (6)

16 marks

Question 2

(a) Given the expression: $2x^3+5x^2-23x+10$

(i) Show that $(x-2)$ is a factor (2)

(ii) Hence factorise the expression fully (2)

(b) In a scientific analysis the following formula defines the relationship between two variables x and y:

$$\log(y) = 2\log(x) + \log 4.$$

Solve for y in terms of x (3)

(c) Find a value for x which satisfies **both** of the following equations:

$$2x^2+3x=2$$

$$2x^2+2^{-\frac{1}{x}}=\frac{3}{4}$$

(4)

(d) Simplify:

(i) $$\frac{\left(5^{2x}\right)^{-2}.20^{x+1}.125^{x+1}}{2^{1+2x}}$$ (4)

(ii) $$\frac{2^{2011}+4^{1005}}{8^{670}}$$ (4)

19 marks

Question 3

It is possible to express all fractions as the sum of two ***unit fractions***. A unit fraction is a fraction with a numerator of 1. The table below illustrates a sequence of some of these fractions.

n	fraction identity
1	$\frac{2}{3}=\frac{1}{2}+\frac{1}{6}$
2	$\frac{2}{5}=\frac{1}{3}+\frac{1}{15}$
3	$\frac{2}{7}=\frac{1}{4}+\frac{1}{28}$
4	$\frac{2}{9}=\frac{1}{5}+\frac{1}{45}$
5	$\frac{2}{11}=\frac{1}{6}+\frac{1}{66}$

(a) Write down the next fraction identity in the sequence (2)

(b) Write a formula for the n^{th} fraction identity in the sequence (5)

(c) Hence, or otherwise, express $\frac{2}{999}$ as the sum of two unit fractions (2)

9 marks

Question 4

The following functions are given:

$$f(x)=\frac{1}{x+1}+2$$

$$g(x)=-3\left(x+1\right)^2+3$$

(a) Sketch f and g on the axes given on the ANSWER SHEET. Show all turning points, intercepts and asymptotes (8)

(b) Show that the equation below can be solved using the graphs above

$$\left(3x^2+6x+2\right)\left(x+1\right)=-1$$ (3)

(c) For what values of k will the equation $g(x)=k$ have no real solutions (2)

13 marks

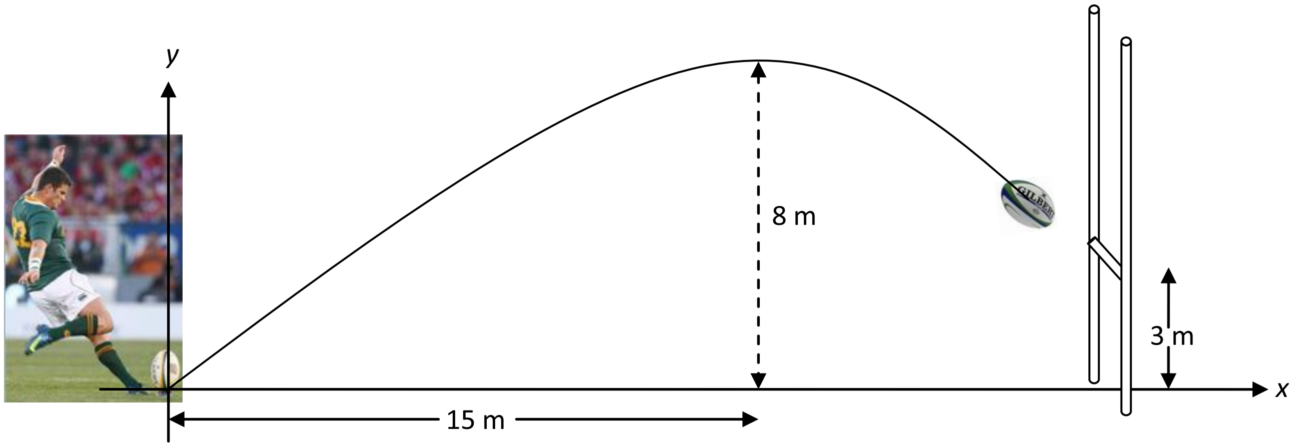
Question 5

Morné Steyn takes a penalty in a rugby match for the Springboks, as illustrated in the diagram below.

The ball follows exactly the path of a parabola with equation: $y = a(x-p)^2 + q$.

The origin is taken to be the point at which the ball is kicked. The ball reaches its maximum height of 8 metres when it is 15 metres away from where the ball was kicked. The cross-bar is at a height of 3 metres.

NOT TO SCALE



(a) Determine the values of a, p and q (4)

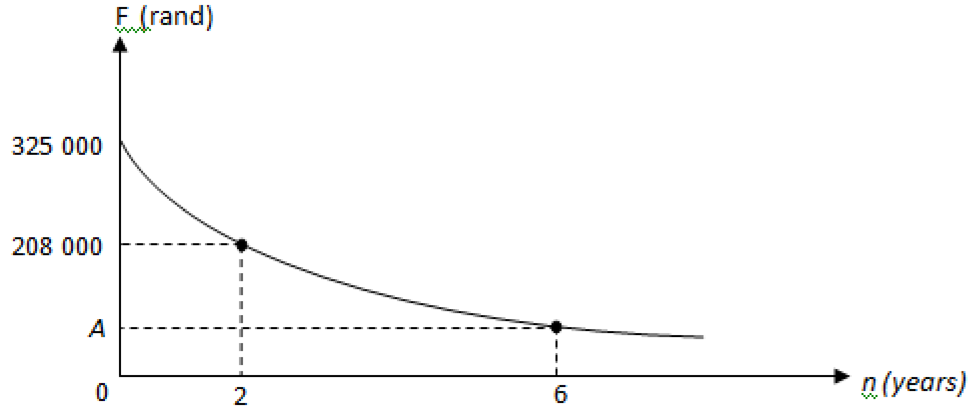
(b) If the ball is struck at a distance of 24 metres from the goal, show that the ball clears the cross-bar (2)

(c) With the same kick, what is the maximum distance from which the ball could have been struck such that the ball would have cleared the cross-bar? (3)

9 marks

Question 6

(a) The graph below shows the depreciating value of a car over a period of time:



(i) What is the cost of the car new? (1)

(ii) What type of depreciation is illustrated? (1)

(iii) Use the information on the graph to find the rate of depreciation. (3)

(iv) Calculate A, the value of the car after 6 years. (3)

(b) Case A: Siphiwe invests a lump sum of R25 000 immediately. The investment earns compound interest.

Case B: David decides to wait for 4 years and then starts a monthly annuity of R500, the payments being made at the end of each month.

In both cases the effective rate is 15% per annum and the investments mature 20 years from now.

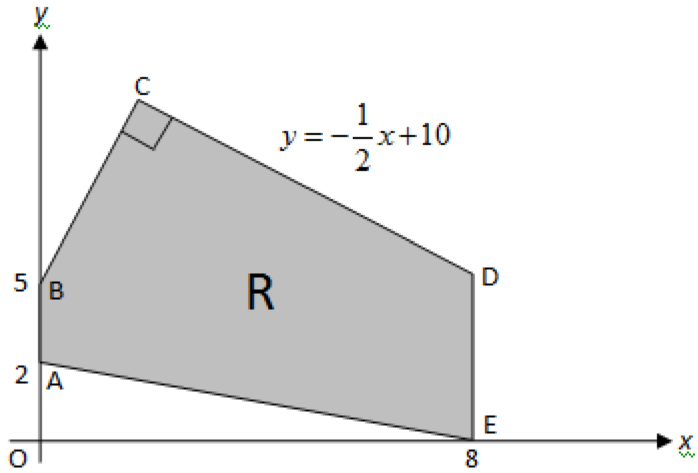
(i) Calculate the per annum nominal interest rate for David's investment. (2)

(ii) Showing all necessary working, determine who will have more cash saved after 20 years? (6)

16 marks

Question 7

In the diagram below (not to scale), the shaded area, R, represents the feasible region in a linear programming problem. The equation of the line joining C and D is $y = -\frac{1}{2}x + 10$ The coordinates A(0; 2), B(0; 5) and E(8; 0) are given. BC $\perp$ CD and DE $\perp$ OE.



(a) Write down the set of inequalities that describes **R**. (6)

(b) Find the coordinates of points D and C. (4)

(c) An objective function, P = $ax + by$, is to be maximised

(i) If $a = 2$ and $b = 3$, at which point is P maximised? (2)

(ii) If $a = 4$, write down the value of b such that P is maximised at any point on CD. (2)

14 marks

Question 8

(a) Find $f'(-2)$ if $f(x) = \frac{1}{2}x^2 - \frac{3}{x^3}$ (3)

(b) Find $\frac{dy}{dx}$ if $y = \left(\frac{\sqrt{x} - 1}{x}\right)^2$ (4)

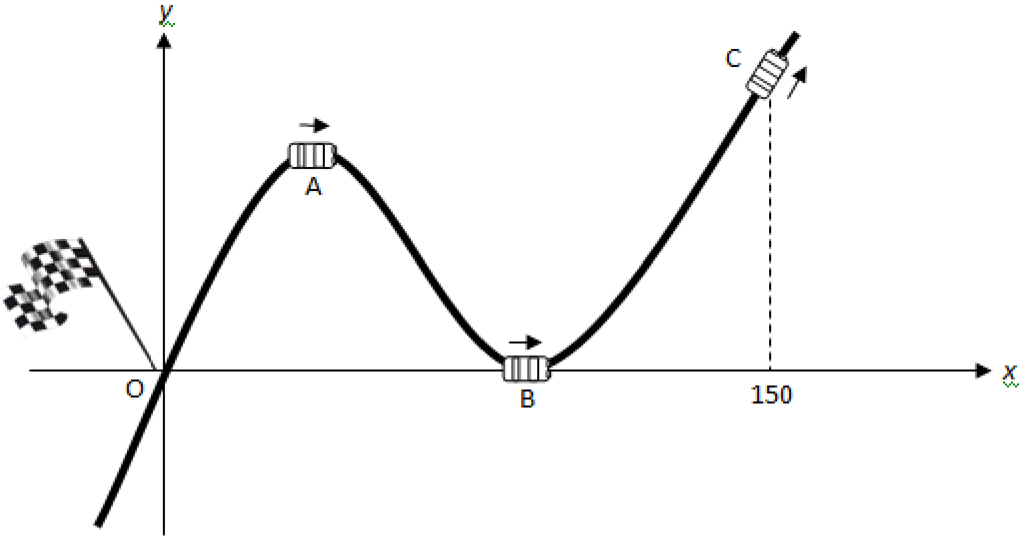
(c) If $g'(x) = \frac{1}{2}x$ write down a possible function, $g(x)$ (1)

(d) Find $f'(x)$ from first principles if: $f(x) = -\frac{3}{2}x^2$ (5)

13 marks

Question 9

Part of a rally track follows the path of a cubic curve. A plan view of this section of the track is shown in the diagram below. At a certain instant, cars A and B are at the turning points of the curve. Car C is at the point where $x = 150$. The race starts at the origin, O.



The function which describes this part of the track is: $f(x) = \frac{1}{900}x^3 - \frac{1}{5}x^2 + 9x$

(a) Determine the co-ordinates of cars A and B. (6)

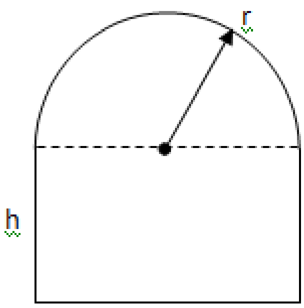
(b) Find the average gradient of the curve between car A and car C. (4)

(c) Find the equation of the tangent to the track at the starting flag. (2)

(d) At which point between A and B does a car stop turning to the right and start turning to the left? (2)

14 marks

Question 10



The given shape consists of a semi-circle of radius r cm and a rectangle with a height of h cm.

(a) Write an expression for the area of the shape. (2)

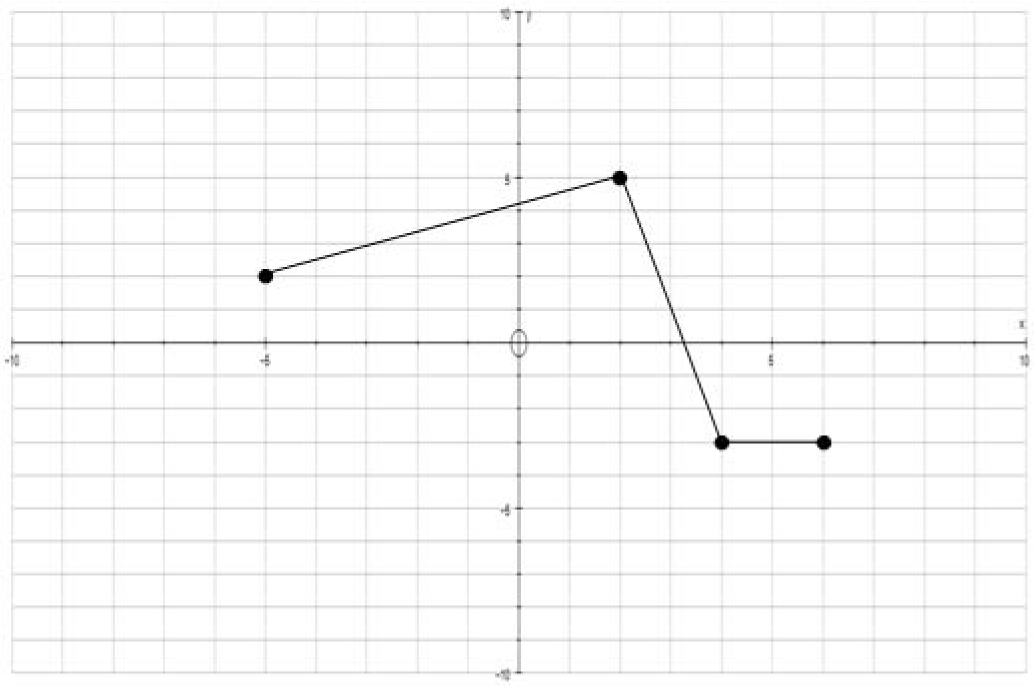
(b) If the shape is subject to the constraint that the perimeter of the rectangle ABCD must be 20 cm, determine the value of $\boldsymbol{r}$ that will yield the maximum possible area of the shape. (5)

(c) Prove that the value found is a maximum. (2)

9 marks

Question 11

(a) The graph of $f(x)$ is given below:



On the axes given in the ANSWER SHEET, sketch:

(i) $f(x-2)+1$ (2)

(ii) $2f(-x)$ (2)

(b) Given the two functions:

$$f(x)=\frac{x}{x-2}$$

$$g(x)=\frac{2x}{x-1}$$

Prove that $g(x)=f^{-1}(x)$ (5)

9 marks

Question 12

The following formula may be used in this question:

$$1+4+9+16+.......+n^2=\frac{n(n+1)(2n+1)}{6}$$

At Vodacom World of Golf, the golf balls are arranged in the form of a square based pyramid, as shown in Figure 1 below. The number of balls in each layer follows the pattern 1, 4, 9, 16, .. etc. After a number of complete layers of balls have been used the shape would then resemble what is known as a '*frustum*' of a pyramid, as shown in Figure 2.

Figure 1

Figure 2

(a) In <u>Figure 2</u>, how many golf balls have been ***used*** at this stage? (1)

(b) How many balls would there be in the 40^{th} layer of the golf ball pyramid (2)

(c) If there are m balls in a certain layer, how many would there be in the layer immediately above? (Give your answer in terms of m) (2)

(c) Determine the number of balls between and including the 41^{st} and 80^{th} layers (4)

9 marks

END OF PAPER

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} \left[a + (i-1)d \right] = \frac{n}{2}\left[2a + (n-1)d \right]$$

$$\sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n - 1)}{r-1} \; ; \quad r \neq 1 \qquad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}; \; -1 < r < 1, \; r \neq 0$$

$$T_n = an^2 + bn + c$$

$$T_n = T_1 + (n-1)f + \frac{(n-1)(n-2)}{2}s$$

where f is the first term of the first difference and s is the second difference

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$A = P(1+ni) \qquad A = P(1-ni)$$

$$A = P(1+i)^n \qquad A = P(1-i)^n$$

$$F = x\left[\frac{(1+i)^n - 1}{i}\right] \qquad P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2} \; ; \; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2} ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x\ ;\ y) = ((x_A\cos\alpha - y_A\sin\alpha)\ ;\ (y_A\cos\alpha + x_A\sin\alpha))$$

$$\bar{x} = \frac{\sum x}{n} \qquad \bar{x} = \frac{\sum fx}{n}$$

$$\text{var} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \qquad \text{var} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$s.d = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$