# St. John's College Upper V

**Prelim Paper 1 2010** **Memorandum**

**TOTAL: 150 marks**

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1. (a) $3x^2+4x-5=0$

$x=\dfrac{-4\pm\sqrt{16+60}}{6}$ [3]

$x=0,79$ or $x=-2,12$

(b) $x=-1$

$x=-3$ [2]

(c) $\log_4 x=-\dfrac{1}{2}$

$x=4^{-\frac{1}{2}}$ [2]

$x=\dfrac{1}{2}$

(d) $4^x=\dfrac{1}{9}$

$x=\log_4\dfrac{1}{9}$

$=-1,58$ [3]

$\approx -1,6$

(e) $k=x^2+x$

$k+10=k^2-4k+4$

$0=k^2-5k-6$

$0=(k-6)(k+1)$

$x^2+x-6=0$ $x^2+x+1\neq 0$

$x=-3,\ x=2$ no soln. [6]

**[16]**

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2. (a) (i) $f(2) = 2(2)^3 + 5(2)^2 - 23(2) + 10$

$= 16 + 20 - 46 + 10$ [2]

$= 0$

(ii) $(x-2)(2x^2 + 9x - 5) = 2x^3 + 5x^2 - 23x + 10$ [2]

$\therefore 2x^3 + 5x^2 - 23x + 10 = (x-2)(2x-1)(x+5)$

(b) $\log y = \log x^2 + \log 4$

$\log y = \log 4x^2$

$\therefore y = 4x^2$ [3]

(c) $2x^2 + 3x - 2 = 0$

$(2x-1)(x+2) = 0$

$x = \frac{1}{2},\ x = -2$

$2(-2)^2 + 2^{1/2} \neq \frac{3}{4}$

$2\left(\frac{1}{2}\right)^2 + 2^{-2}$

$= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ [4]

(d) (i) $\frac{5^{-4x}.2^{2x+2}.5^{x+1}.5^{3x+3}}{2^{1+2x}}$

$= 2^{2x+2-1-2x}.5^{-4x+x+1+3x+3}$

$= 2 \times 5^4$ [4]

$= 1250$

(ii) $\frac{2^{2011} + 2^{2(1005)}}{2^{3(670)}}$

$= \frac{2^{2011} + 2^{2010}}{2^{2010}}$

$= \frac{2^{2010}(2^1 + 1)}{2^{2010}}$ [4]

$= 3$

**[19]**

(3) (a) $\frac{2}{13}=\frac{1}{7}+\frac{1}{91}$ [2]

(b) $\frac{2}{2n+1}=\frac{1}{n+1}+\frac{1}{(2n+1)(n+1)}$ [5]

(c) $\frac{2}{999}=\frac{1}{500}+\frac{1}{499500}$ [2]

**[9]**

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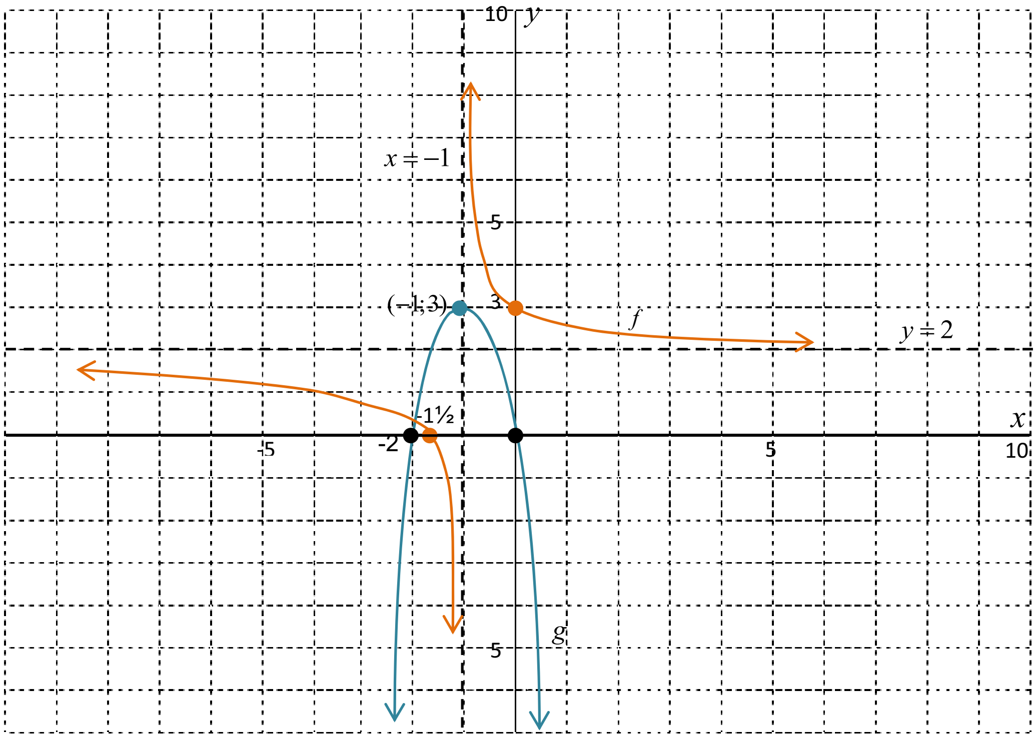
(4) (a) [8]




(b) $3(x^2+2x)=-\frac{1}{x+1}-2$

$\therefore\ 3[(x+1)^2-1]=-\frac{1}{x+1}-2$

$\therefore\ -3(x+1)^2+3=\frac{1}{x+1}+2$ [3]

(c) $k>3$ [2]

**[13]**

(5) (a) $y = a(x-15)^2 + 8$

$(0;0): \quad 0 = a(225) + 8$

$a = -\frac{8}{225} \;;\; p = 15 \;;\; q = 8$ [4]

(b) $x = 24 \therefore y = -\frac{8}{225}(24-15)^2 + 8$

$= 5{,}12 \text{ m}$

$\approx 5{,}1 \text{ m}$ [2]

which is greater than 3 m $(5{,}1 > 3)$

(c) let $y = 3$

$3 = -\frac{8}{225}(x-15)^2 + 8$

$(x-15)^2 = 140{,}625$

$x = 15 + \sqrt{140{,}625}$

$= 26{,}86 \text{ m}$

$\approx 26{,}9 \text{ m}$ [3]

**[19]**

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(6) (a) (i) R325 000 [1]

(ii) compound/reducing balance [1]

(iii) $208 = 325(1-i)^2$

$i = 20\%$ [3]

(iv) $F = 32500(0{,}8)^6$

$= \text{R}85196{,}80$ [3]

(b) (i) $0{,}15 = \left(1 + \frac{r}{12}\right)^{12} - 1$

$r = 0{,}1406$ [2]

(ii) {A} $F = 25000(1 + 0{,}15)^{20}$

$= \text{R}409163{,}43$

{B} $F = 500\left[\left(1 + \frac{0{,}1406}{12}\right)^{192} - 1 \Big/ \frac{0{,}1406}{12}\right]$

$= \text{R}356787{,}85$

$\therefore$ Siphiwe has more cash [6]

**[16]**

7. (a) $0 \le x \le 8$

$y \ge -\frac{1}{4}x + 2$

$y \le -\frac{1}{2}x + 10$ [6]

$y \le 2x + 5$

(b) $D: y = -\frac{1}{2}(8) + 10 = 6 \quad \therefore D(8;6)$

$C: 2x + 5 = -\frac{1}{2}x + 10$

$\frac{5}{2}x = 5$

$x = 2 \qquad \therefore C(2;9)$ [4]

(c) (i) $p = 2x + 3y$

$y = \boxed{-\frac{2}{3}}x + \frac{p}{3}$

$\therefore$ point $D$ [2]

(ii) $p = 4x + by$

$y = -\frac{4x}{b} + \frac{p}{b}$

$\therefore \quad -\frac{4}{b} = -\frac{1}{2}$

$\therefore \quad b = 8$ [2]

**[14]**

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8. (a) $f'(x) = x + \frac{9}{x^4}$

$\therefore f'(-2) = -2 + \frac{9}{(-2)^4}$

$= \frac{-23}{16} = -1,4$ [3]

(b) $y = \frac{x}{x^2} - \frac{2x^{\frac{1}{2}}}{x^2} + \frac{1}{x^2}$

$= x^{-1} - 2x^{\frac{-3}{2}} + x^{-2}$

$\therefore \quad \frac{dy}{dx} = -\frac{1}{x^2} + \frac{3}{x^{\frac{5}{2}}} - \frac{2}{x^3}$ [4]

(c) $g(x) = \frac{x^2}{4}$ [1]

(d) $f(x+h) = -\frac{3}{2}(x+h)^2$

$= -\frac{3}{2}x^2 - 3xh - \frac{3}{2}h^2$

$f'(x) = \lim_{h \to 0} \frac{-\frac{3}{2}x^2 - 3xh - \frac{3}{2}h^2 + \frac{3}{2}x^2}{h}$

$= \lim_{h \to 0} \frac{h\left(-3x - \frac{3}{2}h\right)}{h}$

$= -3x$ [5]

**[13]**

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9. (a) $f'(x) = \frac{1}{300}x^2 - \frac{2}{5}x + 9 = 0$

$x^2 - 120x + 2700 = 0$

$(x-90)(x-30) = 0$

$x = 90 \qquad x = 30$

$y = 0 \qquad y = 120$ [6]

$\therefore B(90;\ 0) \qquad A(30;\ 120)$

(b) $\frac{600-120}{150-30} = 4$ [4]

(c) $f'(0) = 9$

$\therefore \quad y = 9x$ [2]

(d) Point of inflection: $A''(x) = \frac{1}{150}x - \frac{2}{5} = 0$

$x = 60$

$\therefore \quad y = 60$ [2]

**[14]**

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10. (a) $A = \frac{\pi r^2}{2} + 2rh$ [2]

(b) $2h + 4r = 20$

$h = 10 - 2r$

$$\therefore A = \frac{\pi r^2}{2} + 2r(10 - 2r)$$

$$= \frac{\pi r^2}{2} + 20r - 4r^2$$

$$A' = \pi r + 20 - 8r = 0$$

$$\therefore r = \frac{20}{8 - \pi}$$

$$\approx 4{,}1 \text{ cm}$$ [5]

(c) $A'' = \pi - 8 < 0$

$\therefore$ max [2]

**[9]**

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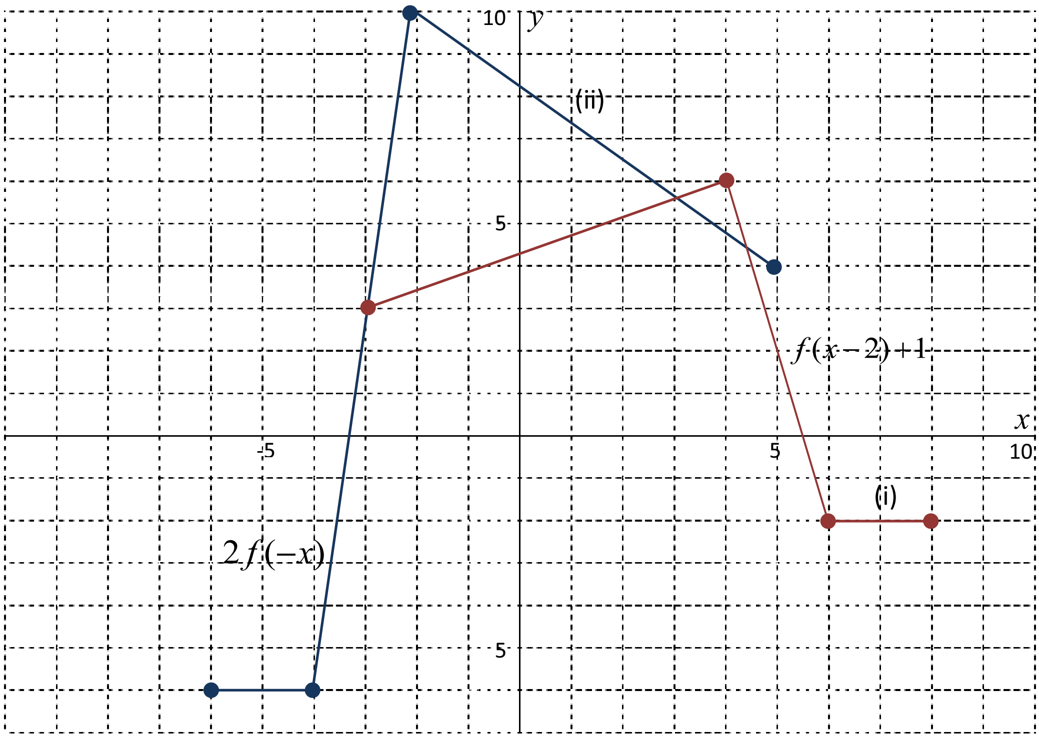
11. (a) [4]




(b) $f^{-1}(x)$: $x = \frac{y}{y-2}$

$$xy - 2x = y$$

$$y(x-1) = 2x$$

$$\therefore y = \frac{2x}{x-1}$$

$$\therefore \quad f^{-1}(x) = \frac{2x}{x-1} = g(x)$$

or can show $f(g(x)) = g(f(x)) = x$ [5]

**[9]**

12. (a) $1+4+9+16=30$ or $\frac{4(4+1)(8+1)}{6}=30$ [1]

(b) $40^2=1600$ [2]

(c) $(\sqrt{m}-1)^2$ [2]

(d) $S(40)=\frac{40(41)(81)}{6}=22140$

$S(80)=\frac{80(81)(161)}{6}=173880$

$\therefore$ between $41^{st}$ and $80^{th}$ layers

$S(80)-S(40)=173880-22140$

$=151740$ [4]

**[9]**

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**END** **[150]**