GR 12

JULY 2010

MEMO (P1)

SECTION A

Question 1

a) $x^2 - 6x = 10 - 30x$

$\therefore x^2 + 24x - 10 = 0$ ✓

$\therefore x = \dfrac{-24 \pm \sqrt{(24)^2 - 4(1)(-10)}}{2(1)}$ ✓✓

$= \dfrac{-24 \pm \sqrt{616}}{2}$

$= -12 \pm \sqrt{154}$ ✓✓ (5)

b) $x^2 - 3x + 2 \leq 6$

$\therefore x^2 - 3x - 4 \leq 0$ ✓

$\therefore (x-4)(x+1) \leq 0$ ✓

$-1 \leq x \leq 4$ ✓✓ (4)

c) $3(4^{2x}) = 14,2$

$\therefore 4^{2x} = \dfrac{14,2}{3}$ ✓

$\therefore \log_4\left(\dfrac{14,2}{3}\right) = 2x$ ✓

$\therefore 0,56 = x$ ✓ (3)

[12]

Question 2

a) (1) $\log_2 4 + \log_2 \frac{1}{2}$

$= (2)$ ✓ $+ (-1)$ ✓

$= 1$ (2)

(2) $\dfrac{\log 3 - 2\log 5}{4\log 5 - 2\log 3}$ ✓✓

$= \dfrac{(\log 3 - 2\log 5)}{-2(\log 3 - 2\log 5)}$ ✓

$= -\frac{1}{2}$ ✓ (4)

b) $\therefore \log \dfrac{(x+3)}{x} = 1$ ✓

$\therefore \dfrac{(x+3)}{x} = 10$ ✓

$\therefore x + 3 = 10x$ ✓

$\therefore 3 = 9x$ ✓

$\therefore \frac{1}{3} = x$ ✓ (5)

[11]

Question 3

a) Hyperbola ✓ (1)

b) $y = -1$ ✓ ; $x = 3$ ✓ (2)

c) X-intercept:

$0 = \dfrac{5}{x-3} - 1$

$\therefore 1 = \dfrac{5}{x-3}$ ✓

$\therefore x - 3 = 5$

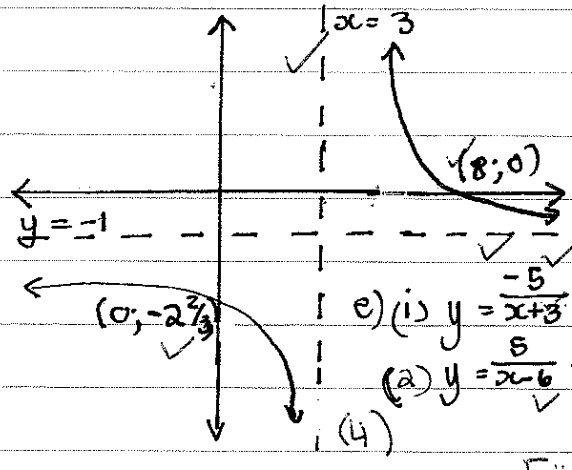
$\therefore x = 8$ ✓ (3)

y-intercept:

$y = \dfrac{5}{0-3} - 1$

$= -2\frac{2}{3}$ ✓

d)



(4)

e) (i) $y = \dfrac{-5}{x+3} - 1$ ✓✓ (2)

(2) $y = \dfrac{5}{x-6} + 1$ ✓✓ (2)

[14]

Question 4

a) 1) $\lim_{h \to 2} \frac{h(h+3)(h-2)}{(h+2)(h-2)}$

$= \frac{2(2+3)}{(2+2)}$

$= \frac{5}{2}$ ✓ (4)

2) $f'(x) = \lim_{h \to 0} \frac{[2(x+h)-4]-[2x-4]}{h}$

$= \lim_{h \to 0} \frac{2x+2h-4-2x+4}{h}$

$= \lim_{h \to 0} \frac{2h}{h}$ ✓

$= 2$ ✓ (4)

b) 1) $f(x) = 4x^2 - 12x + 9$

$f'(x) = 8x - 12$ (2)

2) $y = x^{3/2} + 2.x^{-2}$ ✓

$\frac{dy}{dx} = \frac{3}{2}x^{1/2} - 4x^{-3}$

$= \frac{3\sqrt{x}}{2} - \frac{4}{x^3}$ ✓ ✓ (3)

[13]

Question 5

a) 1) $T_{20} = 4 + 19(5)$ ✓ ✓

$= 99$ ✓ ( )

2) $S_{20} = \frac{20}{2}[2(4) + 19(5)]$ ✓ ✓ ✓

$= 1030$ ✓ (4)

b) $x - 2 = y - x$ ✓ ①

$\frac{y}{x} = \frac{9}{y}$ ✓ ②

From ①:

$y = 2x - 2$ ✓ ③

Sub ③ in ②:

$9x = y^2$ ✓

$\therefore 9x = (2x-2)^2$

$\therefore 9x = 4x^2 - 8x + 4$

$\therefore 0 = 4x^2 - 17x + 4$

$\therefore 0 = (4x-1)(x-4)$

$x = \frac{1}{4}$ ✓ or $x = 4$ ✓

Sub back in ③

$y = 2(\frac{1}{4}) - 2$ or $y = 2(4) - 2$

$= -3/2$ ✓ $= 6$ ✓

(6)

c) $4(2)^{4-1}; 4(2)^{5-1}; 4(2)^{6-1}; \ldots$

$\therefore 32; 64; 128; \ldots$ ✓

$\therefore r = 2$ ✓

$S_{17} = \frac{32(2^{17}-1)}{(2-1)}$ ✓

$= 4\ 194\ 272$ ✓ (4)

d) $(8x)^2; (4\sqrt{2}x)^2; (4x)^2;$ ✓✓

$r = \frac{(4\sqrt{2})^2}{(8)^2} = \frac{1}{2}$ ✓

$S_\infty = \frac{64x^2}{1/2}$ ✓ ✓

$= 128x^2$ ✓ (6)

[23]

Question 6

ANSWER SHEET

Question 7

a) CD = 4 ✓

X-intercept: $(y = 0)$

$-(x+1)^2 + 4 = 0$ ✓

$\therefore -(x^2 + 2x + 1) + 4 = 0$

$\therefore -x^2 - 2x - 1 + 4 = 0$

$\therefore x^2 + 2x - 3 = 0$

$\therefore (x+3)(x-1) = 0$

$\therefore x = -3$ ✓ of $x = 1$ ✓ (4)

b) Max value: 4 ✓ (1)

c) Max distance $= (-x^2 - 2x + 3) - (x^2 - 4x)$ ✓

$= -2x^2 + 2x + 3$

$= -2(x^2 - x + \frac{1}{4} - \frac{1}{4}) + 3$

$= -2(x - \frac{1}{2})^2 + \frac{1}{2} + 3$

$= -2(x - \frac{1}{2})^2 + 3\frac{1}{2}$

Max length $3\frac{1}{2}$ where $x = \frac{1}{2}$ ✓

OR $d(\text{Distance}) = 0$ ✓

$\therefore -4x + 2 = 0$

$\therefore -4x = -2$

$\therefore x = \frac{1}{2}$ ✓ (2)

d) $x \in (-3; 0) \cup (1; 4)$ ✓✓✓✓ (4)

e) $f'(x) = g'(x)$ ✓

$-2x - 2 = 2x - 4$ ✓

$\therefore -4x = -2$

$\therefore x = \frac{1}{2}$ ✓ (3)

[14]

Question 8

ANSWER SHEET

Question 9

a) $y = -(x-2)(x-2)(x+3)$ ✓✓✓

$= -(x^2 - 4x + 4)(x+3)$

$= -(x^3 - 4x^2 + 4x + 3x^2 - 12x + 12)$

$= -x^3 + x^2 + 8x - 12$

$a = 1$ ✓; $b = 8$ ✓; $c = -12$ ✓ (6)

b) $DE = 14 - (-12) = 26$ ✓✓ (2)

c) $f'(x) = 0$ ✓

$-3x^2 + 2x + 8 = 0$ ✓

$\therefore 3x^2 - 2x - 8 = 0$

$\therefore (x-2)(3x+4) = 0$

$x = -\frac{4}{3}$ ✓

$y = -(-\frac{4}{3})^3 + (-\frac{4}{3})^2 + 8(-\frac{4}{3}) - 12$ ✓

$= -\frac{500}{27}$ ✓

$A(-\frac{4}{3}; -\frac{500}{27})$ ✓ (6)

d) $f''(x) = 0$ ✓

$-6x + 2 = 0$ ✓

$x = \frac{1}{3}$ ✓

$y = -(\frac{1}{3})^3 + (\frac{1}{3})^2 + 8(\frac{1}{3}) - 12 = \frac{-250}{27}$ ✓

$(\frac{1}{3}; -\frac{250}{27})$

(4)

e) $E(0; -12)$

$m = f'(0)$ ✓

$= -3(0)^2 + 2(0) + 8$

$= 8$ ✓

$\underline{y = 8x - 12}$ ✓ (3)

f) $f(x) + k = 0$

$\therefore f(x) = -k$ ✓

$\therefore \underline{k < 0 \text{ ✓ or } k > \frac{500}{27}}$ (2)

[23]

Question 10

a) $\underline{l = 3x}$ ✓ (1)

b) $SA = 2(3x \times x) + 2(3x \times h) + 2(x \times h)$ ✓

$\therefore 450 - 6x^2 = h(6x + 2x)$ ✓ ✓

$\therefore \frac{450}{8x} - \frac{6x^2}{8x} = h$

$\therefore \underline{\frac{225}{4x} - \frac{3x}{4} = h}$ ✓ (3)

c) Volume = Area of base x height

$= (3x \times x) \times \left(\frac{225}{4x} - \frac{3x}{4}\right)$ ✓ ✓

$= \frac{675x}{4} - \frac{9x^3}{4}$

$= \underline{\frac{1}{4}(675x - 9x^3)}$

(2)

d) $d(V) = 0$ ✓

$\therefore \frac{675}{4}$ ✓ $- \frac{27x^2}{4}$ ✓ $= 0$

$\therefore 675 - 27x^2 = 0$

$\therefore 25 - x^2 = 0$

$\therefore (5 - x)(5 + x) = 0$

$\therefore \underline{x = 5}$ ✓ or $\underline{x = -5}$ ✓ (5)

[11]

+1

Question 3	[13 marks]

Given: $f(x) = \frac{5}{x-3} - 1$

a) What kind of graph does f represent? Hyperbola ✓ (1)

b) Write down the equations of the asymptotes of f. $y = -1$ ✓

$x = 3$ ✓ (2)

c) Determine the intercepts with the axes.

Y-intercept: $(x=0)$

$y = \frac{5}{0-3} - 1$

$= -2\,^{2}/_{3}$ ✓

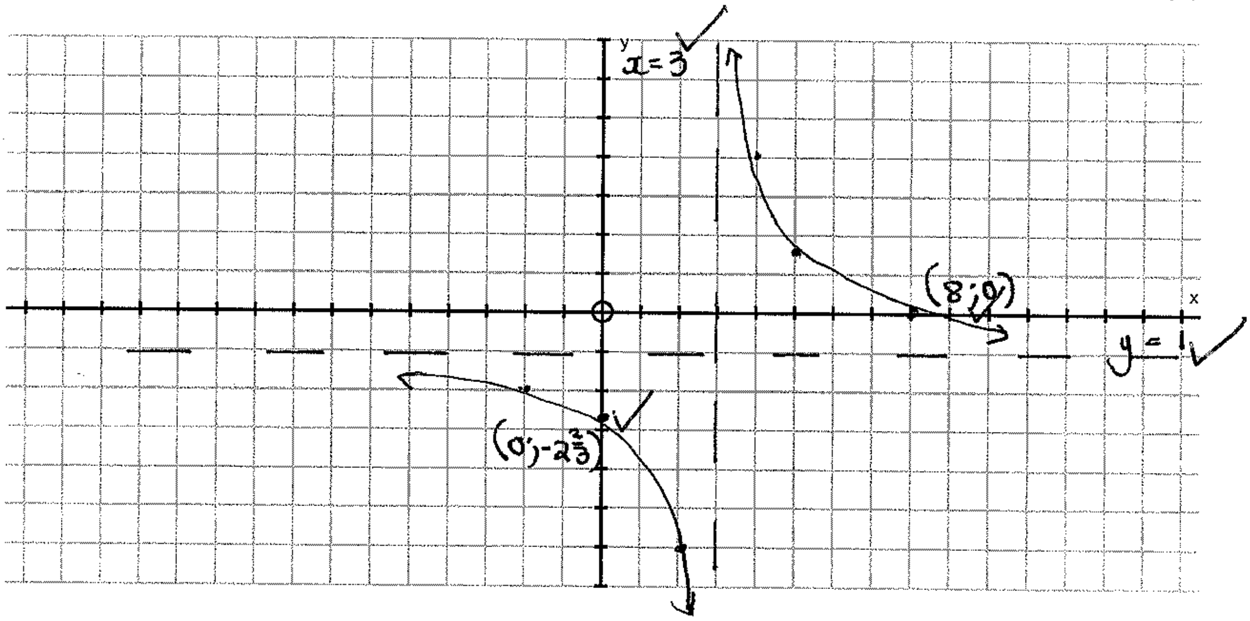
X-intercept: $(y=0)$

$0 = \frac{5}{x-3} - 1$

$\therefore x-3 = 5$

$\therefore x = 8$ ✓ (2)

d) Sketch the graph of f, clearly showing all relevant features of this graph. (4)



e) Give the new equations after the following transformations:

1) $f(x)$ reflected about the y-axis

$y = \frac{-5}{x+3} - 1$ ✓ ✓ ✓

(2) +1

2) $f(x)$ translated 3 units to the right and 2 units up

$y = \frac{5}{x-6} + 1$ ✓ ✓

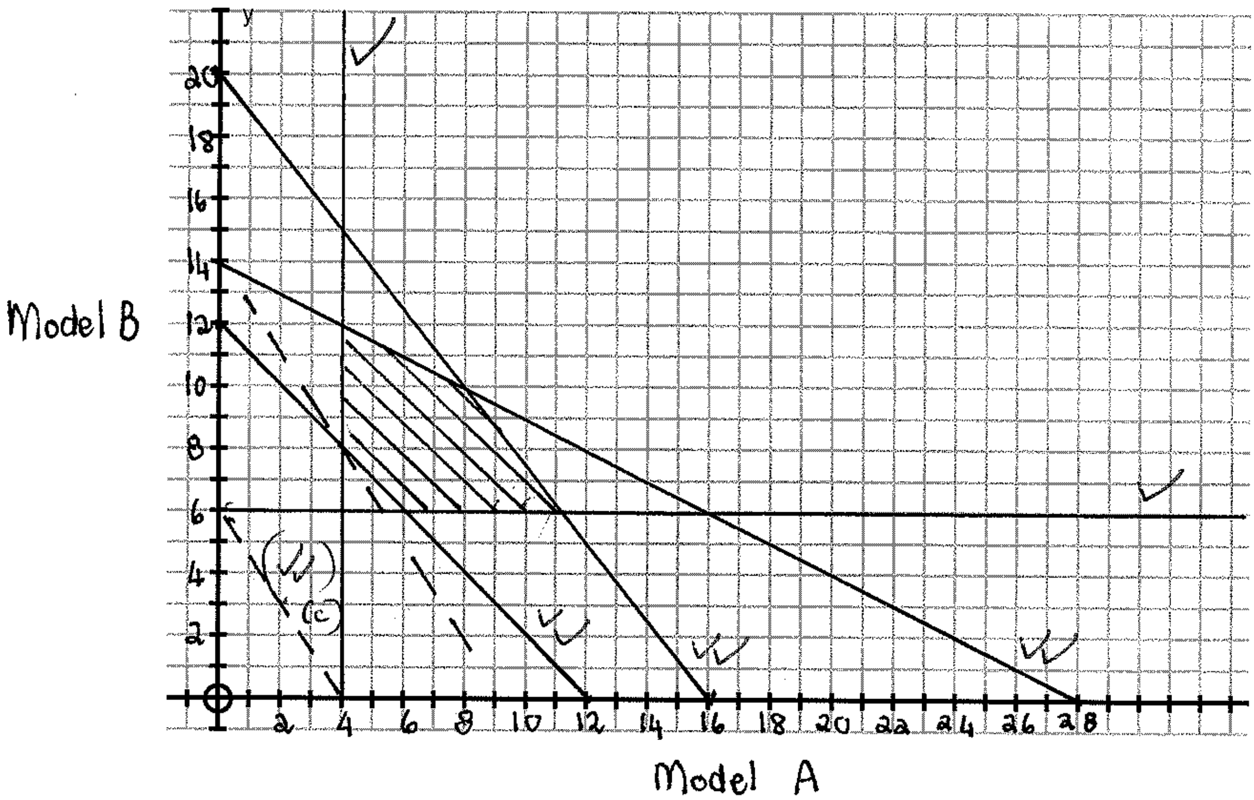
(2)

Question 6	[19 marks]

Let x be the number of articles of model A and y be the number of articles of model B which can be manufactured daily by a factory, subject to the following constraints:

$$x \geq 4$$
$$y \geq 6$$
$$x + y \geq 12$$
$$5x + 4y \leq 80$$
$$2y + x \leq 28$$

a) Represent all the constraints on the graph paper provided. Clearly indicate the feasible region. (8)



b) If it costs R300 to make each article of model A and R200 to make each article of model B, write down an equation to represent the total cost, T, to manufacture x articles of model A and y articles of model B. $T = 300x + 200y$

$m = -3/2$

(2)

c) Draw on the graph a straight line that you would use to minimize the total production cost.

(2)

d) Give the number of articles of each model that should be manufactured to ensure a minimum cost, and determine the minimum cost.

Model A - 4 articles ; Model B - 8 articles

$T = 300(4) + 200(8)$

$= R\,2800$

(3)

e) If the manufacturing cost is adjusted and it now costs the same to manufacture models A and B, but it is not desirable to make more of model A than of model B, determine how many of each should be manufactured to ensure minimum expenditure.

$T = Ax + Ay$; $m = -1$

Line will lie on the line $x + y = 12$

Any point between (4;8) and (6;6), including them would minimize the cost.

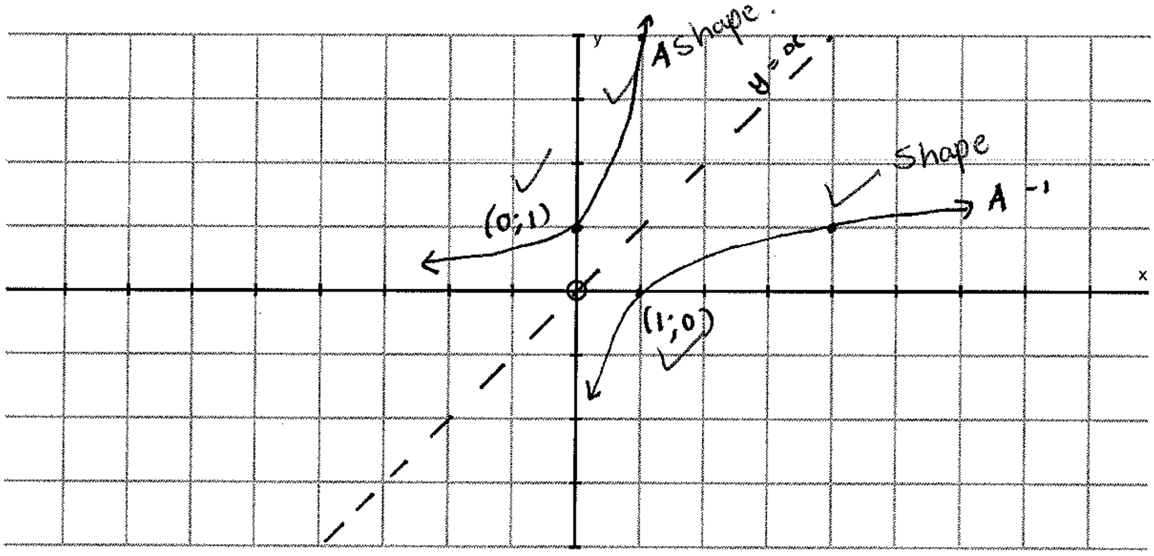
(4)

[19]

Question 8	[10 marks]

Given: A: $f(x) = 4^x$

a) Sketch A and A^{-1} on the same set of axes. Label all relevant points. (4)



b) Determine a if $f(a) = 8$.

$8 = 4^a$

$\therefore 2^3 = 2^{2a}$

$\therefore 3 = 2a$

$\therefore \frac{3}{2} = a$

(3)

c) Explain how you can use coordinates and transformation rules to determine b if $f^{-1}(8) = b$?

f^{-1} is the reflection of f in the line $y = x$.

$\therefore f^{-1}(8) = b$ and $f(a) = 8$ refers to the same point $(3/2 ; 8)$

(3)

[10]