

DEPARTMENT OF MATHEMATICS

GRADE 12

PAPER 1: LO 1 and LO 2

EXAMINATION: Prelim 2010

TIME: 3 HOURS

MARKS: 150

INSTRUCTIONS:

- This examination paper has 6 pages in all.
- You will find a “**Mathematics Formula Sheet**” at the **end** of this exam.
- Answer all the questions logically and neatly. Show all your working. This will be to your advantage.
- Non-programmable calculators may be used, except in questions where this is expressly forbidden.
- Any given diagrams need **not** be redrawn.
- Answer all questions in your answer booklet. **Use both sides of each page.**
- Question **3, 6** and **8** is to be answered on the answer sheet provided in the answer booklet.
- If not specified, round all answers to two decimal places where necessary.

SECTION A

Question 1

[12 marks]

Solve for x:

- a) $x^2 - 6x = 10(1 - 3x)$ (5)
b) $(x - 1)(x - 2) \leq 6$ (4)
c) $3(4^{2x}) = 14,2$ (3)

Question 2

[11 marks]

- a) Simplify the following:
(1) $\log_2 4 + \log_2 \frac{1}{2}$ (2)
(2) $\frac{\log 3 - \log 25}{\log 625 - \log 9}$ (4)
- b) Solve for x, **without using a calculator**:
 $\log(x + 3) - \log x = 1$ (5)

Question 3

[14 marks]

Answer this question on the answer sheet provided in the answer booklet.

Given: $f(x) = \frac{5}{x - 3} - 1$

- a) What kind of graph does f represent? (1)
b) Write down the equations of the asymptotes of f . (2)
c) Determine the intercepts with the axes. (2)
d) Sketch the graph of f , clearly showing all relevant features of this graph. (4)
e) Give the new equations after the following transformations:
1) $f(x)$ reflected about the y-axis (3)
2) $f(x)$ translated 3 units to the right and 2 units up (2)

Question 4

[13 marks]

- a) Determine the following:
(1) $\lim_{h \rightarrow 2} \frac{h^3 + h^2 - 6h}{h^2 - 4}$ (4)
(2) $f'(x)$ from **first principles** if $f(x) = 2x - 4$ (4)

b) Use the rules of differentiation to determine the following:

(1) $f'(x)$ if $f(x) = (2x - 3)^2$ (2)

(2) $\frac{dy}{dx}$ if $y = \sqrt{x^3} + \frac{2}{x^2}$ (3)

Question 5

[23 marks]

a) The terms 4, 9 and 14 form a sequence.

Determine:

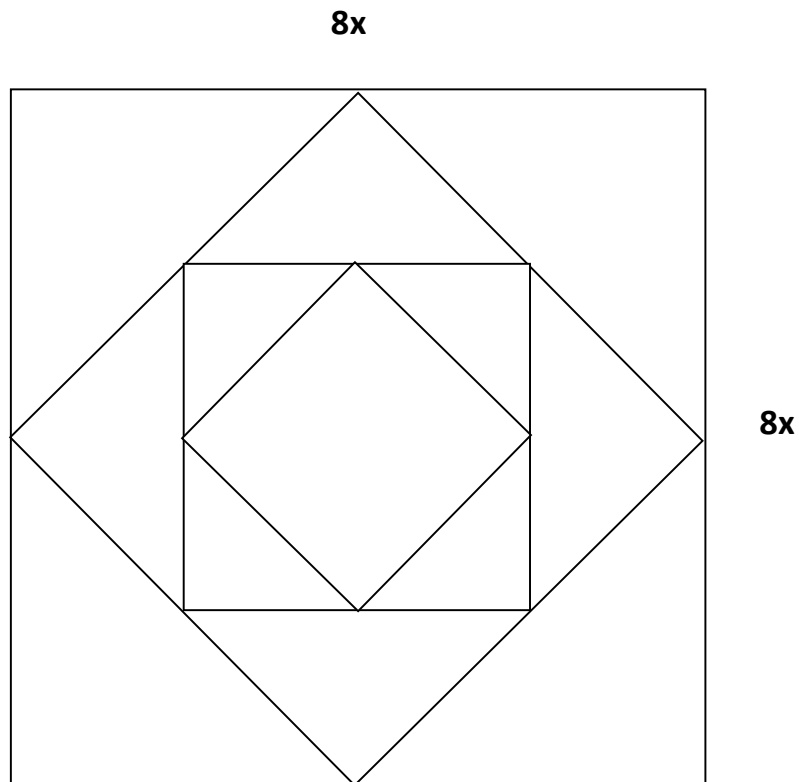
(1) the 20th term of the sequence (3)

(2) the sum of the first 20 terms (4)

b) Given that in the sequence 2 ; x ; y ; 9 the first three terms form an arithmetic progression and the last three terms form a geometric progression, determine the values of x and y. (6)

c) Calculate $\sum_{k=4}^{20} 4 \cdot 2^{k-1}$ (show all workings) (4)

d) The midpoints of the sides of a square with sides **8x cm** are joined to form another square. This process is repeated indefinitely. Calculate the sum of the areas of the squares that are found this way. (6)



SECTION B

Question 6

[19 marks]

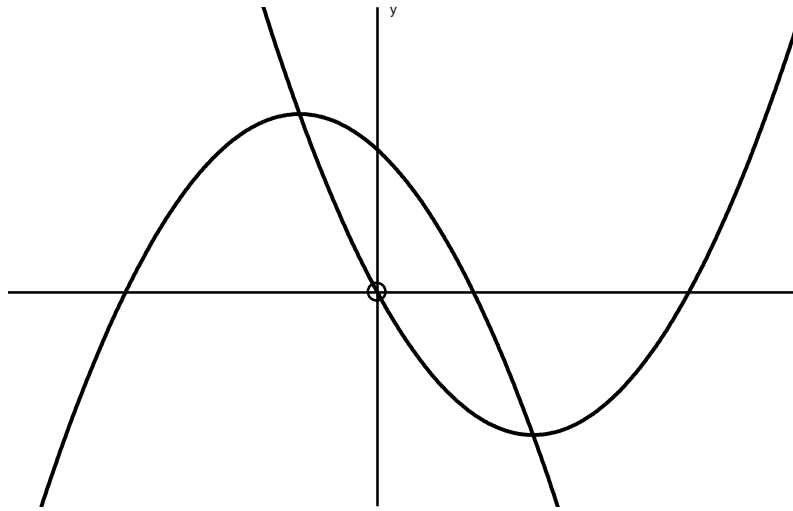
Answer this question on the answer sheet provided in the answer booklet.

Let x be the number of articles of model A and y be the number of articles of model B which can be manufactured daily by a factory, subject to the following constraints:

$$\begin{aligned}x &\geq 4 \\y &\geq 6 \\x + y &\geq 12 \\5x + 4y &\leq 80 \\2y + x &\leq 28\end{aligned}$$

- Represent all the constraints on the graph paper provided.
Clearly indicate the feasible region. (8)
- If it costs R300 to make each article of model A and R200 to make each article of model B, write down an equation to represent the total cost, T , to manufacture x articles of model A and y articles of model B. (2)
- Draw on the graph a straight line that you would use to minimize the total production cost. (2)
- Give the number of articles of each model that should be manufactured to ensure a minimum cost, and determine the minimum cost. (3)
- If the manufacturing cost is adjusted and it now costs the same to manufacture models A and B, but it is not desirable to make more of model A than of model B, determine how many of each should be manufactured to ensure minimum expenditure. (4)



Question 7**[14 marks]**

The graphs of $f(x) = -(x+1)^2 + 4$ and $g(x) = x^2 - 4x$ are shown. They intersect at A, the turning point of f , and B, the turning point of g .

- Determine the length of CD. (4)
- Write down the maximum value of $f(x)$. (1)
- Determine the value of x for the length between $f(x)$ and $g(x)$ to be a maximum. (2)
- For which values of x is $f(x) \cdot g(x) > 0$? (4)
- Give a point where $f(x)$ and $g(x)$ will have the same gradient. (3)

Question 8**[10 marks]**

Answer this question on the answer sheet provided in the answer booklet.

Given: A: $f(x) = 4^x$

- Sketch A and A^{-1} on the same set of axes. Label all relevant points. (4)
- Determine a if $f(a) = 8$. (3)
- Explain how you can use coordinates and transformation rules to determine b if $f^{-1}(8) = b$? (3)



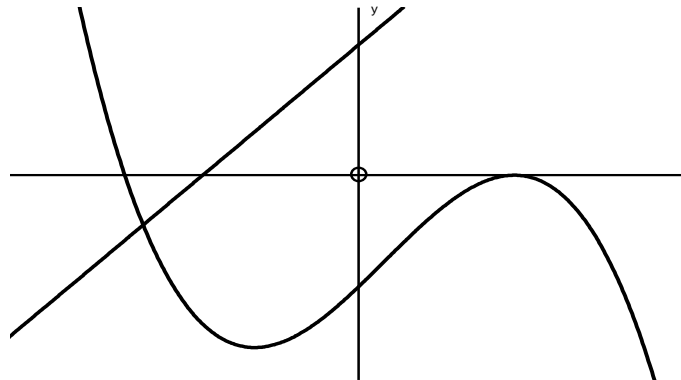
Question 9**[23 marks]**

The graph f is defined by $y = -x^3 + ax^2 + bx + c$.

The line g is defined by $y = 7x + 14$.

Turning point B has coordinates $(2 ; 0)$.

A sketch graph of f and g is given below.



Find the following:

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|----|--------------------------------------------------------------------------------|-----|
| a) | a, b and c | (6) |
| b) | the length of DE | (2) |
| c) | the coordinates of A | (6) |
| d) | any points of inflection | (4) |
| e) | the equation of the tangent to the curve f at E | (3) |
| f) | Determine the value(s) of k such that $f(x) + k = 0$ has one real root only. | (2) |

Question 10**[11 marks]**

The length of a rectangular storage basket with a drop-down lid is three times its width, x , and the total area is 450 square units.

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|----|--------------------------------------|-----|
| a) | Express the length in terms of x . | (1) |
|----|--------------------------------------|-----|

- | | | |
|----|-----------------------------------------------------------|-----|
| b) | Show that the height, $h = \frac{225}{4x} - \frac{3x}{4}$ | (3) |
|----|-----------------------------------------------------------|-----|

- | | | |
|----|------------------------------------------------------------------|-----|
| c) | Show that the volume is given by, $V = \frac{1}{4}(675x - 9x^3)$ | (2) |
|----|------------------------------------------------------------------|-----|

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| d) | Now, find the value of x that will give a maximum volume. | (5) |
|----|-------------------------------------------------------------|-----|

