MATHEMATICS

PRELIMINARY EXAMINATION

PAPER 1

GRADE 12

MARKS: 150
TIME: 3 hours

LEARNING OUTCOMES:	LO 1	LO 2
ASSESSMENT STANDARDS:	AS 1, 2, 3, 4,5, 6	AS 1, 2, 3, 4, 5, 6, 7, 8

LEARNER'S NAME: _____   CLASS: ____

_____

INSTRUCTIONS

1. Show all necessary calculations.
2. Round off to **two** decimal places, unless specified otherwise.
3. Non-programmable calculators may be used, unless instructed otherwise.
4. Diagrams are not necessarily drawn to scale.
5. This question paper consists of 7 typed pages.
6. An information sheet is provided.

%

SECTION	**A**					**B**					
QUESTION	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**TOTAL**
MAXIMUM	18	17	9	15	22	10	15	12	17	15	150
MARK											

SECTION A

QUESTION 1

a) There are four solutions to the equation $(x^2 - 3)(3x + 1)(x + 2) = 0$. Write down the value of x if the solution is:

    1)   an integer

    2)   an irrational number   (3)

b) Solve for x:

    1) $3x - 2x(x + 1) = -2$ (5)

    2) $2\log x = \log 100$ (3)

    3) $(x - 3)(x - 4) \geq 12$ (4)

c) Simplify: $\dfrac{6^{x+1}.3^{2x-1}}{54^{x+1}}$ (3)

| **18 marks** |

QUESTION 2

a) Calculate $\displaystyle\sum_{k=0}^{2}\left(\dfrac{2k}{2^k}\right)$ (3)

b) The 19th term of an arithmetic sequence is 11, while the 31st term is 5.

    1)   Determine the first three terms of the sequence.   (5)

    2)   Calculate the sum of the first 81 terms of the sequence.   (3)

c) Consider the sequence $8(x - 3)^2; 4(x - 3)^3; 2(x - 3)^4; ...$ $x \neq 3$

    1)   Determine the values of x for which the sequence will converge?   (3)

    2)   Calculate the sum to infinity if $x = 3,5$ (3)

| **17 marks** |

QUESTION 3

a) The "Hot Stuff" factory manufactures x mini and y maxi barbecue sets each month. The factory cannot manufacture more than 100 of the mini sets or more than 60 of the maxi sets per month. At most 140 barbecue sets can be produced each month. The retailers require at least 10 maxi and 30 mini barbecue sets per month. The number of maxi sets manufactured each month is not more than twice the number of mini sets.

Billy wrote down the following inequalities to represent the constraints:

$$10 \leq y \leq 60 \quad ; \quad 30 \leq x \geq 100 \quad ; \quad x + y \leq 140 \quad ; \quad 2y \leq x$$

He has made **two** errors. Write down the inequalities that are incorrect and then correct each of them. (3)

b) Prior to opening the "Hot Stuff" factory Billy saved money in order to have the necessary capital to purchase the necessary tools and material. He deposited R12 000 seven years ago and two years later he deposited another R8 000. The interest rate for the first three years was 14% per annum compounded semi-annually and thereafter 12% per annum compounded monthly. How much money had he saved at the end of the seventh year? (6)
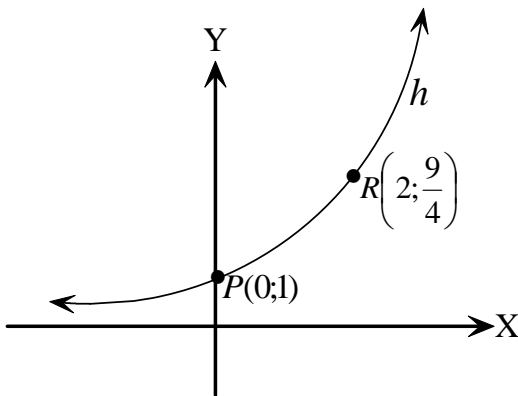
> **9 marks**

QUESTION 4

a) Determine $f'(x)$ from first principles if $f(x) = 4 - x^2$ (4)

b) Determine:

1) $\dfrac{dy}{dx}$ if $y = 2\sqrt{x} + \dfrac{8}{x}$ (4)

2) $D_p \left[\dfrac{2p^2 + 3p - 2}{p + 2} \right]$ (3)

c) Determine the value of k for which the equation of the tangent to $g(x) = 3x^2 + kx + 4$ at the point where $x = 2$ is $y = 8x - 8$ (4)

> **15 marks**

QUESTION 5

a) The function $h(x) = a^x$ cuts the Y-axis at the point $P(0;1)$ and $R\left(2;\dfrac{9}{4}\right)$ is a point on the curve.

1) Determine the equation of h. (3)

2) Write down the equation of h^{-1}. (2)

3) Write down the equation of g, where g is a reflection of h in the Y-axis. (1)

b)

This poster is being used to promote the 2010 Fifa World Cup in Bloemfontein. The arch has a parabolic shape. The highest point of the arch has coordinates $(2;7)$ and the player's hand touches the arch at the point $(4;3)$.

1) Determine the equation f of the arch. (4)

2) Restrict the domain of f in one specific way so that the inverse of the parabola will also be a function. (1)

3) Write down the coordinates of the turning point of the transformed parabola g if $g(x) = f(x+1) - 2$ (2)

c) The sketch shows the graph of the **inverse of k**.
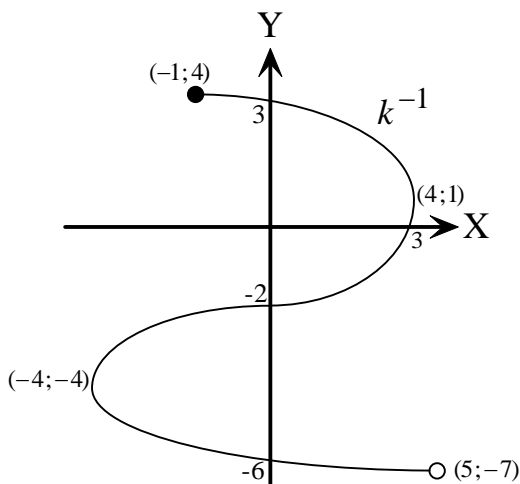
1) Write down the range of $k^{-1}(x)$ (2)

2) Write down the range of k. (2)

3) Choose one of the following to describe the function k:
    - one-to-one
    - one-to-many
    - many-to-one (1)

4) Sketch the graph of k. (4)

22 marks

SECTION B

QUESTION 6

The following sequence is given: 2 ; 12 ; 20 ; 26 ; …

a) Write down the next term in the sequence. (1)

b) Determine a formula for the n-th term of the sequence, showing all calculations. (4)

c) If it is given that $T_n = -n^2 + 13n - 10$ show by completing the square that no term in the sequence will have an integral value greater than 32. (5)

> **10 marks**

QUESTION 7

a) Kate wants to save R250 000 to buy a new car. She has been paying R5 000 into a savings account at the end of every month. The bank pays interest at 9% per annum compounded monthly.
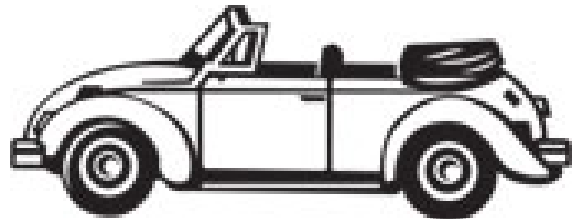
   1) How many payments of R5 000 will have to be made into the account for Kate to achieve her goal? (5)

   2) What is the effective interest rate that the bank is paying? (3)

b) Kevin wants to buy the same car as Kate, but he opts to take out a bank loan for R250 000. The interest rate charged by the bank is 18,5% per annum compounded monthly.

   1) What will his monthly repayment be if he pays the loan back over five years, starting one month after the granting of the loan? (4)

   2) Calculate the balance outstanding after the 25th repayment. (3)

> **15 marks**

QUESTION 8

CARDS MANIA is a small company that makes two types of cards, birthday cards and cards for other special occasions. With the amount of labour and material that is available, the company cannot make more than 150 birthday cards and or more than 120 other cards per week; they can also not make more than 200 cards in total.

A local gift shop requires a weekly order of at least 40 birthday cards and at least 10 other cards. Sales records show that the number of birthday cards made should not be more than three times the number of other cards. A profit of R5 is made from the sale of each birthday card and R10 from the sale of any other card.

Let x represent the number of birthday cards and y represent the number of other cards made per week

a) Set up a system of inequalities to illustrate the constraints. (4)

b) Using the graph paper provided, determine the feasible region. (5)

c) Determine the number of each type of card that should be sold in order for maximum profit to be made. (3)

12 marks

QUESTION 9

a)   According to an estimate, the predicted circulation in t years' time of the *TROUT-fishing* magazine will be given by $m(t) = 50t^3 - 200t + 3000$.

   1)   What was the circulation of the magazine at the time of first publication?   (2)

   2)   At what rate will the circulation be changing three years' from now?   (3)

   3)   What will the minimum circulation of the magazine be to the nearest integer?   (3)
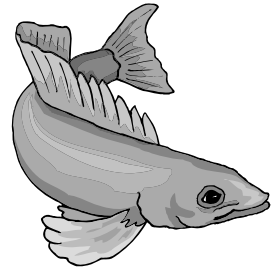
b)   Simon has a trout farm located in a hilly section of the country, where there are many natural dams. He has drawn a cross-section of part of the farm where he would like to open a lodge for guests to spend a few days while trout fishing. The equation representing the cross section is given by $h(x) = x^3 - 14x^2 + 59x - 70$ where $h(x)$ represents the height (in metres) above or below the level of the water in the dam and x represents the horizontal cross-sectional distance (in kilometres) at the level of the water.
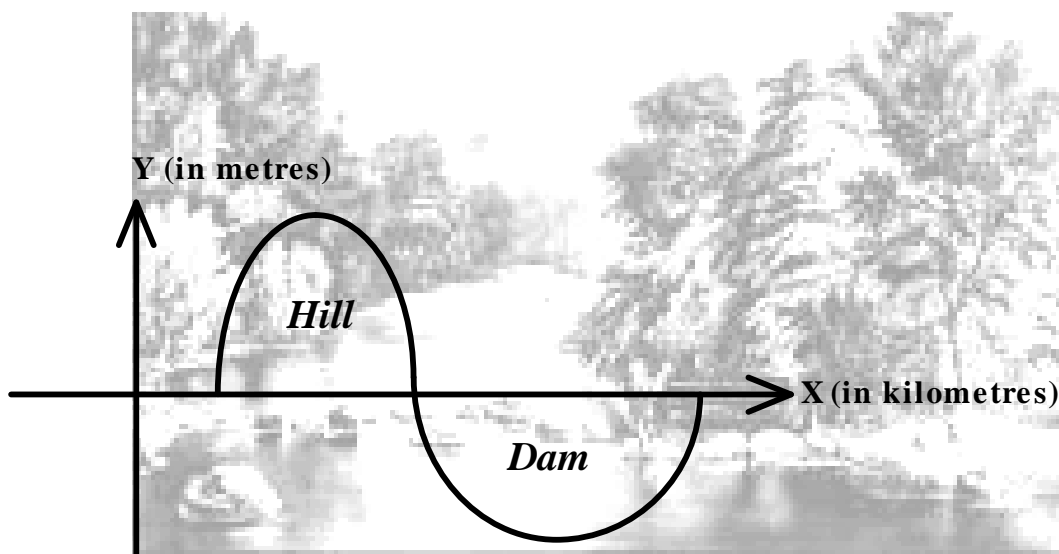

Help Simon to calculate the following:

1)   The width of the dam at the level of the water.   (5)

2)   The slope of the hill as it reaches the dam.   (3)

3)   The new cross-sectional equation if the level of the water in the dam were to drop by 2
       metres.   (1)

17 marks

QUESTION 10

a) Calculate $\log\dfrac{4}{5} + \log\dfrac{5}{6} + \log\dfrac{6}{7} +$ … to 36 terms without using a calculator. (4)

b) A car speeding around a curve in the shape of $y = x^2$, moving from left to right, skids off at the point $\left(\dfrac{1}{2}; \dfrac{1}{4}\right)$.
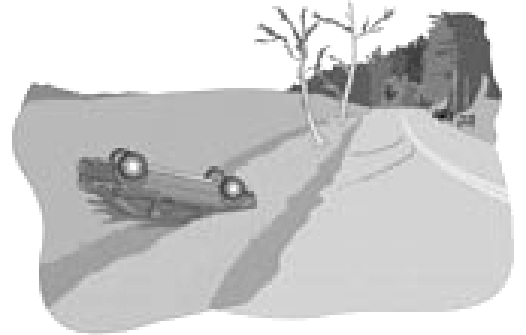
If the car continues skidding in a **straight** path, will it hit a tree that is located at $\left(1; \dfrac{3}{4}\right)$?

(6)

c) A <u>paradox</u> is defined as "a seemingly absurd or contradictory statement".

Investigate the following Mathematical paradox:

$$a = b + c \text{, where } c \neq 0 \qquad \text{Line 1}$$
$$(a - b)a = (a - b)(b + c) \qquad \text{Line 2}$$
$$a^2 - ab = ab + ac - b^2 - bc \qquad \text{Line 3}$$
$$a^2 - ab - ac = ab + ac - b^2 - bc - ac \qquad \text{Line 4}$$
$$a(a - b - c) = b(a - b - c) \qquad \text{Line 5}$$
$$a = b \qquad \text{Line 6}$$

1) Explain the Mathematical process that was used in each of Lines 2 and 5.

2) There is a contradiction between the statement in Line 1 and the statement in Line 6. Explain what resulted in this contradiction. (5)

15 marks

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