SECTION A

Question 1 [18]

a) 1. $x = -2$ ① *a*

2. $x = \pm\sqrt{3}$ $\pm$ ① *a* $\sqrt{3}$ ① *a*

(3)

b) 1. $3x - 2x(x+1) = -2$

$3x - 2x^2 - 2x = -2$ ① *a*

$-2x^2 + x + 2 = 0$ ① *ca*

$2x^2 - x - 2 = 0$

$x = \dfrac{1 \pm \sqrt{17}}{4}$ *formula* ① *m*

Could use calculator only – no penalty

$x = 1{,}28$ or $x = -0{,}78$

① *ca* ① *ca*

(5)

2. $2\log x = \log 100$

$2\log x = 2\log 10$ ① *a*

$\log x = \log 10$ ① *ca*

$\therefore x = 10$ ① *ca*

OR

$2\log x = \log 100$

$\log x^2 = \log 100$ ① *a*

$x^2 = 100$ ① *ca*

$x = 10$ or $x \neq -10$ ① *ca*

Must show restriction (3)

3. $(x-3)(x-4) \geq 12$

$x^2 - 7x + 12 \geq 12$ ① *a*

$x^2 - 7x \geq 0$

$x(x-7) \geq 0$ ① *ca*

$x \leq 0$ or $x \geq 7$

① *ca* ① *ca*

(4)

c) $\dfrac{6^{x+1}.3^{2x-1}}{54^{x+1}}$

$= \dfrac{(2\times 3)^{x+1}.3^{2x-1}}{(3^3\times 2)^{x+1}}$ *prime* ① *a*

$= \dfrac{2^{x+1}.3^{x+1}.3^{2x-1}}{3^{3x+3}.2^{x+1}}$

$= 3^{-3}$ ① *ca*

$= \dfrac{1}{27}$ *must be real* ① *ca*

(3)

Question 2 [17]

a) $\sum_{k=0}^{2}\left(\dfrac{2k}{2^k}\right)$

$= \dfrac{2(0)}{2^0} + \dfrac{2(1)}{2^1} + \dfrac{2(2)}{2^2}$ *subst* ① *m*

$= 0 + 1 + 1$ ① *a*

$= 2$ ① *ca*

(3)

b) 1. $T_{19} = a + 18d = 11$ ① *a*

$T_{31} = a + 30d = 5$ ① *a*

$12d = -6$

$d = -\dfrac{1}{2}$ ① *ca*

$a + 18(\dfrac{-1}{2}) = 11$

$a = 20$ ① *ca*

$\therefore 20; 19\dfrac{1}{2}; 19...$ ① *ca*

(5)

2. $S_{81} = \dfrac{81}{2}\left[2(20) + 80(\dfrac{-1}{2})\right]$

formula ① *a* *subst* ① *ca*

$= 0$ ① *ca*

(3)

c) 1. $r = \dfrac{x-3}{2}$ ① *a*

$-1 < \dfrac{x-3}{2} < 1$ ① *a*

$1 < x < 5 \quad x \neq 3$ ① *ca*

(3)

2. $a = 2$ ① *a* $r = \dfrac{1}{4}$ ① *a*

$S_\infty = \dfrac{2}{1 - \dfrac{1}{4}}$

$= \dfrac{8}{3}$ ① *ca*

(3)

Question 3 [9]

a) $30 \le x \le 100$

↑ ① *a*

$y \le 2x$ *identify* ① *a*

↑ ① *a*

P1 for extra constraint (3)

b) *Time line* ① *m*

$$A = 12000\left[1+\frac{0{,}14}{2}\right]^{6}\left[1+\frac{0{,}12}{12}\right]^{48} + 8000\left[1+\frac{0{,}14}{2}\right]^{2}\left[1+\frac{0{,}12}{12}\right]^{48}$$

↑ ① *a* ↑ ① *a* ↑ ① *a* ↑ ① *a*

$= R\,43\,800{,}90$ ① *a*

(6)

Question 4 [15]

PIN on this question

a) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ ① *m*

$= \lim\limits_{h\to 0} \dfrac{4-(x+h)^2-(4-x^2)}{h}$ ① *a*

$= \lim\limits_{h\to 0} \dfrac{-2xh-h^2}{h}$ ① *ca*

$= \lim\limits_{h\to 0} (-2x-h)$

$= -2x$ ① *a*

(4)

b) 1. $y = 2\sqrt{x} + \dfrac{8}{x}$

$y = 2x^{\frac{1}{2}} + 8x^{-1}$

① *a* ① *a*

$\dfrac{dy}{dx} = x^{-\frac{1}{2}} - 8x^{-2}$

$= \dfrac{1}{\sqrt{x}} - \dfrac{8}{x^2}$

① *a* ① *a*

P1 neg exp (4)

b) 2. $D_p\left[\dfrac{2p^2+3p-2}{p+2}\right]$

$= D_p\left[\dfrac{(2p-1)(p+2)}{p+2}\right]$ ① *a*

$= D_p[2p-1]$ ① *ca*

$= 2$ ① *ca*

(3)

c) $g'(x) = 6x + k$ ① *a*

$6x + k = 8$ ① *ca*

$6(2) + k = 8$ ① *ca*

$\therefore k = -4$ ① *ca*

OR

$y = 8(2) - 8$ *substitute* ① *a*

$y = 8$ ① *ca*

$8 = 3(2)^2 + k(2) + 4$ ① *ca*

$\therefore k = -4$ ① *ca*

(4)

Question 5 **[22]**

a) 1. $h(x) = a^x$

$\frac{9}{4} = a^2$ ① *a*

$a = \frac{3}{2}$ ① *ca*

$\therefore h(x) = \left(\frac{3}{2}\right)^x$ ① *ca*

(3)

a) 2. $h^{-1}(x) = \log_{\frac{3}{2}} x$

log ① *m* ① *ca*

OR

$x = \left(\frac{3}{2}\right)^y$ ① *m*

$h^{-1}(x) = \log_{\frac{3}{2}} x$ ① *ca*

(2)

a) 3. $g(x) = \left(\frac{3}{2}\right)^{-x} = \left(\frac{2}{3}\right)^x$ ① *ca*

(1)

b) 1. $y = a(x+p)^2 + q$

$y = a(x-2)^2 + 7$ *subst* ① *a*

$3 = a(4-2)^2 + 7$ *subst* ① *a*

$\therefore a = -1$ ① *ca*

$y = -(x-2)^2 + 7$ ① *ca*

(4)

b) 2. $x \geq 2$ or $x > 2$ or $x \geq 3$ etc

$x \leq 2$ or $x < 2$ or $x \leq 1$ etc ① *a*

(1)

b) 3. $(1;5)$ ② *a*

(2)

c) 1. $y \in (-7;4]$ ② *a*

2. $y \in [-4;5)$ ② *a*

3. many-to-one ① *a*

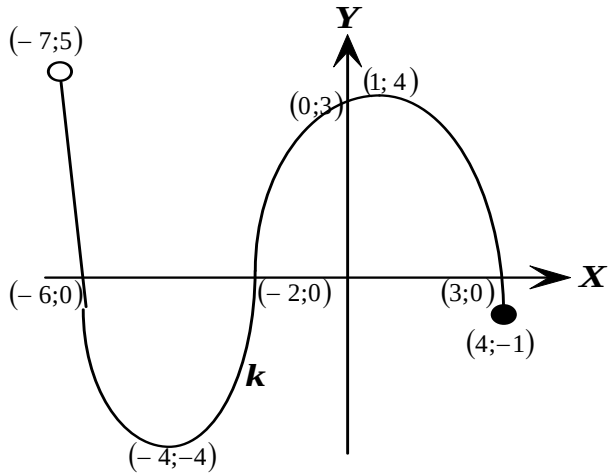
(5)

c) 4. Shape ① *a*

Intercepts (4 of) ① *a*

Stationary points ① *a*

End points ① *a*



(4)

SECTION B

Question 6 [10]

a) 30 ① *a*

(1)

b) ***Must show all calculations***

$T_n = an^2 + bn + c$

First and second difference ① *m*

$a = -1$ *half second diff* ① *a*

$c = -10$ T_0 ① *a*

$2 = -1 + b - 10$

$b = 13$ ① *ca*

$\therefore T_n = -n^2 + 13n - 10$

(4)

c) $\therefore T_n = -n^2 + 13n - 10$

$= -\left(n^2 - 13n + \frac{169}{4}\right) - 10 + \frac{169}{4}$

↑ ① *a* ↑ ① *a*

$= -\left(n - \frac{13}{2}\right)^2 + \frac{129}{4}$

↑ ① *ca* ↑ ① *ca*

$\therefore \text{Max} = \frac{129}{4} = 32{,}25$ ① *a*

(5)

Question 7 [15]

PIU / P1R / P1D on this question

a) 1. $F_V = x\left[\frac{(1+i)^n - 1}{i}\right]$ ① *a*

$250\,000 = 5000\left[\frac{\left(1 + \frac{0{,}09}{12}\right)^n - 1}{\frac{0{,}09}{12}}\right]$

i-value ① *a* *F and x-values* ① *ca*

$n = 42{,}619...$ ① *a*

$n = 43$ ① *ca*

must round up even if $< ...,5$ (5)

a) 2. $1 + i = \left[1 + \frac{i^{(m)}}{m}\right]^m$ ① *a*

$1 + i = \left[1 + \frac{0{,}09}{12}\right]^{12}$ *subst* ① *a*

$i = 9{,}38\%$ p.a. ① *ca*

(3)

b) 1. $P_V = x\left[\frac{1 - (1+i)^{-n}}{i}\right]$ ① *a*

$250\,000 = x\left[\frac{1 - (1 + \frac{0{,}185}{12})^{-60}}{\frac{0{,}185}{12}}\right]$

i-value ① *a* *P and n-values* ① *ca*

$x = R\,6\,416{,}55$ per month ① *a*

(4)

b) 2. $P_V = 6416{,}55\left[\frac{1 - (1 + \frac{0{,}185}{12})^{-35}}{\frac{0{,}185}{12}}\right]$

x-value ① *ca* *n-value* ① *ca*

$P_v = R\,17\,2561{,}67$ ① *a*

(3)

Question 8 [12]

a) $40 \le x \le 150$ ① *a*

$10 \le y \le 120$ ① *a*

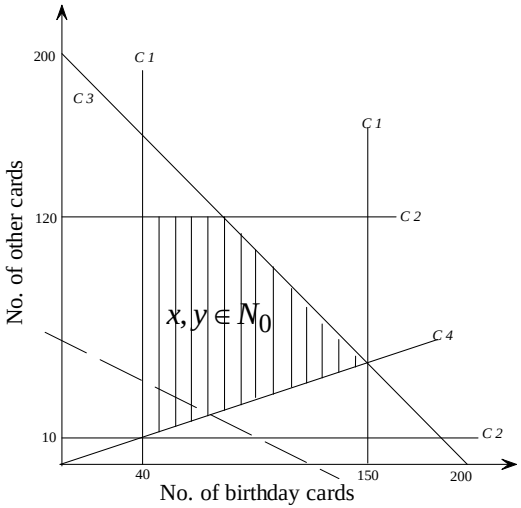
$x + y \le 200$ ① *a*

$x \le 3y$ ① *a*

$x, y \in N_0$ ***need not give here, but must then be on feasible region***

(4)

b)



① *ca per line*

① *ca feasible region – show* $x, y \in N_0$

P1 no labels on axes (5)

c) $P = 5x + 10y$ ① *a*

$m = -\frac{1}{2}$ *or vértices* ① *ca*

x = 80 birthday cards

y = 120 other cards ① *ca*

(3)

Question 9 [17]

PIU on this question

a) 1. $m(t) = 50t^3 - 200t + 3000$

$m(0) = 3000$ magazines

↑ ① *a* ↑ ① *ca*

(2)

a) 2. $m'(t) = 150t^2 - 200$ ① *a*

$m'(3) = 150(3)^2 - 200$ *subst* ① *a*

= 1 150 magazines per year

↑ ① *ca*

(3)

a) 3. $m'(t) = 0$ ① *a*

$150t^2 - 200 = 0$

$t^2 = \frac{4}{3}$

$t = \frac{2}{\sqrt{3}} = 1{,}15....$ ① *ca*

$m(1{,}15...)$

$= 50(1{,}15...)^3 - 200(1{,}15...) + 3000$ ① *m*

= 2846 magazines

(3)

b) 1. $h(x) = x^3 - 14x^2 + 59x - 70$

$x^3 - 14x^2 + 59x - 70 = 0$ ① *a*

$x = 2$ or $x = 5$ or $x = 7$

① *a* ① *a* ① *a*

∴ Width = 2 *km* ① *ca*

(5)

b) 2. $h'(x) = 3x^2 - 28x + 59$ ① *a*

$h'(5) = -6\ m/km$

↑ ① *ca* ↑ ① *ca*

(3)

b) 3. $h(x) = x^3 - 14x^2 + 59x - 68$ ① *a*

(1)

Question 10 [15]

a) $\log\frac{4}{5}+\log\frac{5}{6}+\log\frac{6}{7}+\ldots$

$T_{36}=\log\frac{39}{40}$ ① *a*

$\log\frac{4}{5}\times\frac{5}{6}\times\frac{6}{7}....\times\frac{39}{40}$ ① *ca*

$=\log\frac{4}{40}$ ① *ca*

$=\log\frac{1}{10}$

$=-1$ ① *a*

(4)

b) $\frac{dy}{dx}=2x$ ① *a*

$m=2\left(\frac{1}{2}\right)=1$ ① *ca*

$\therefore y-\frac{1}{4}=1\left(x-\frac{1}{2}\right)$ ① *m*

$\therefore y=x-\frac{1}{4}$ ① *ca*

$y=1-\frac{1}{4}=\frac{3}{4}$ *subst* ① *ca*

$\therefore$ Car will hit the tree ① *ca*

(6)

c) 1. Line 2: Multiply both sides of the equation by $(a-b)$ ① *a*

Line 5: Factorise by common factor on each side ① *a*

2. Division of Line 5 by ① *m*

$(a-b-c)\Rightarrow$ division by 0 ① *a*

$\because (a-b-c)=0$ from Line 1 ① *a*

OR

$a=b\Rightarrow c=0$ but $c\neq 0$ ① *m*

(5)