GRADE 11 EXEMPLAR PAPERS

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

QUESTION 1

(a) $\dfrac{4x^2+2}{2} = \dfrac{2(2x^2+1)}{2} = 2x^2+1$ ✓ (1) k

(b) $\dfrac{x^2+5x+6}{3x+6} = \dfrac{(x+2)(x+3)}{3(x+2)}$ ✓✓ $= \dfrac{(x+3)}{3}$ ✓ (3) R

4 marks

QUESTION 2

(a) $x^2 - 5x = 6$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x = 6$ or $x = -1$ ✓✓ (2) K

(b) $x + 2 = \dfrac{6}{x}$

$x^2 + 2x - 6 = 0$ ✓

$x = \dfrac{-(2) \pm \sqrt{(2)^2 - 4(1)(-6)}}{2(1)}$ ✓

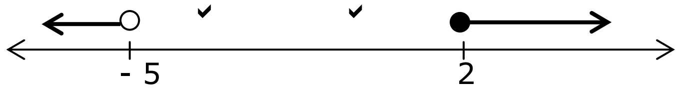
$x = 1{,}65$ ✓ or $x = -3{,}65$ ✓ ✓ **a** (correct to 2 decimal places) (5) R

(c) $\dfrac{2-x}{x+5} \leq 0$ and represent your solution graphically.

$\dfrac{x-2}{x+5} \geq 0$

$x < -5$ ✓**a** or $x \geq 2$ ✓**a** ✓**inequality** (5) R

✓ ✓



(d) $x^2 < 4x$

$x^2 - 4x < 0$ ✓

$x(x-4) < 0$

$0 < x < 4$ ✓✓ (3) R

(e) $x^2 - px - 4 = 0$ (by completing the square).

$x^2 - px + \left(\frac{p}{2}\right)^2 - \left(\frac{p}{2}\right)^2 - 4 = 0$

$\left(x - \frac{p}{2}\right)^2 - \frac{p^2}{4} - 4 = 0$ ✓**m** ✓**a**

$\left(x - \frac{p}{2}\right)^2 = \dfrac{p^2+16}{4}$ ✓

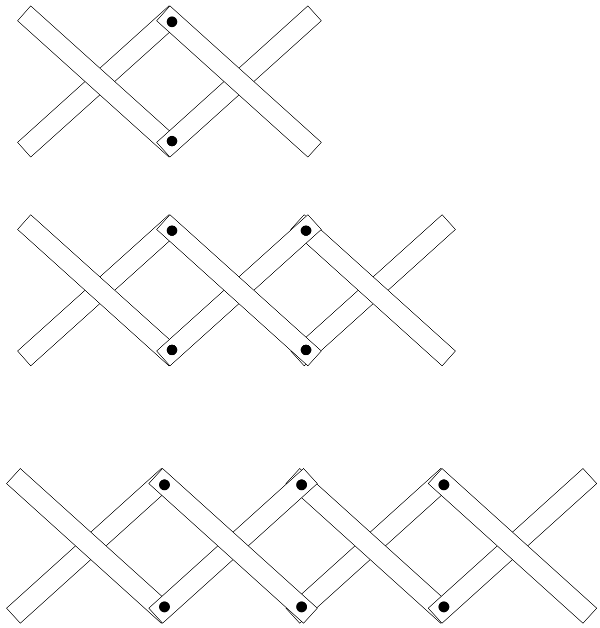
$x - \dfrac{p}{2} = \pm\sqrt{\dfrac{p^2+16}{4}}$ ✓

$x = \dfrac{p \pm \sqrt{p^2+16}}{2}$ ✓ (5) C

20 marks

QUESTION 3

Wooden ice-cream sticks are joined using split-pins to create the following pattern:



Complete the table by filling in the missing values.

Number of diamond shapes formed	***1***	***2***	***3***	***4***		***20***	***n***
Number of split pins	2	4	6	**(1) 8** ✓K		**(2) 40** ✓K	**(3) 2n** ✓R
Number of ice-cream sticks	4	6	8	**(4) 10** ✓K		**(5) 42** ✓K	**(6) 2n+2** ✓R

(4) K (2) R

6 marks

QUESTION 4

An advertisement advertises a mirror of dimensions 2,3 m by 1,5 m.

This means that the actual length of the mirror could be 2,25 m ≤ length < 2,35 m.

(a) Write down the possible measurements of the width of the mirror. (2)

$1,45\,\text{m} \le \text{width} < 1,55\,\text{m}$ ✓ ✓

(b) Hence calculate the minimum possible area of the mirror (correct to **2 decimal places**).

$145 \times 2,25$ ✓ ✓ $= 3,26\ \text{m}^2$ ✓ (3)

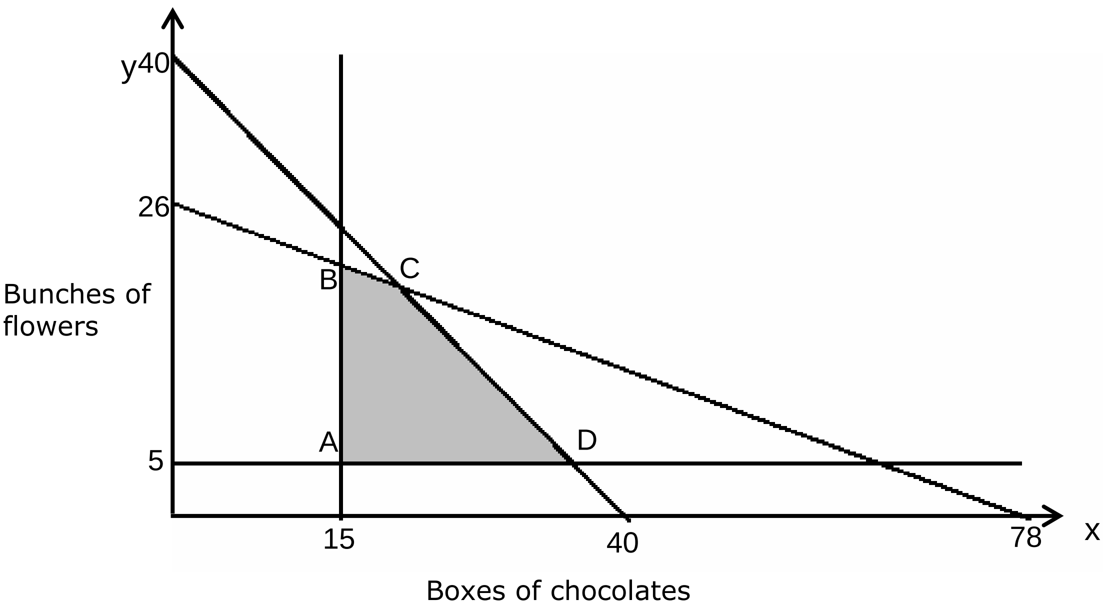
5 marks

QUESTION 5

A gift shop sells x boxes of chocolates and y bunches of flowers per day subject to the following constraints.

$y \le 40 - x$
$3y + x \le 78$
$y \ge 5$
$x \ge 15$

In the diagram a graphical representation with the feasible region shaded, is given.



(a) Determine the coordinates of vertices A, B, C and D. (8) R

A (15 ; 5) ✔ ✔
B lies on line $3y + x = 78$ and has x = 15, so y = 21 B (15 ;21) ✔ ✔
C lies on point of intersection of $y = 40 - x$ and $3y + x = 78$
$3(40 - x) + x = 78$
$120 - 3x + x = 78$
$120 - 2x = 78$
$-2x = -42$
$x = 21$
$y = 19$
C (21 ; 19) ✔ ✔
D lies on $y = 40 - x$ and has y = 5, so x = 35 D (35 ; 5) ✔ ✔

(b) The shopkeeper makes a profit of R10 on a box of chocolates and R20 on a bunch of flowers.

Write down an equation that will represent the profit for the day. (2) R

$P = 10x + 20y$ ✔ ✔

(c) How many boxes of chocolates and bunches of flowers should be sold every day to make a maximum profit? What is this maximum profit? (3) R

point C (21 ; 19) 21 boxes of chocolates and 19 bunches of flowers ✔ ✔
$P = 10(21) + 20(19) = 590$ max profit is R590 ✔

(d) Near Valentine's day flowers become very expensive and the shopkeeper finds that whilst he still makes R10 profit on a box of chocolates, he now makes no profit on a bunch of flowers.
Subject to the above constraints, how many boxes of chocolates and bunches of flowers should he now sell to make a maximum profit? (2) C

$P = 10x$
at D (35 ; 5) 35 boxes of chocolates and 5 bunches of flowers ✔ ✔

15 marks

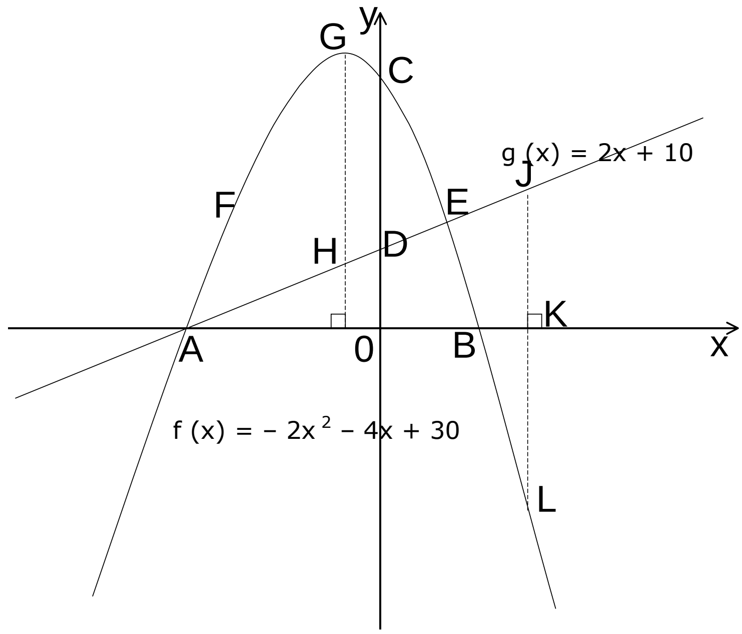
QUESTION 6

Refer to the figure.

The graphs of $f(x) = -2x^2 - 4x + 30$ and $g(x) = 2x + 10$ are drawn (not to scale).

A and B are the x-intercepts and C is the y-intercept of f (x). G is the turning point of f (x). A is the x-intercept and D is the y-intercept of g (x).

Use the sketch to answer the following questions:



(a) Determine the coordinates of A, B, C and D.

A and B are the x-int of f(x) so put $-2x^2 - 4x + 30 = 0$ ✓
$x^2 + 2x - 15 = 0$
$(x + 5)(x - 3) = 0$
$x = -5$ or $x = 3$ A (− 5;0) ✓ and B (3;0) ✓

C is y-int of f (x) so put x = 0 , y = 30 C (0 ; 30) ✓
D is y-int of f (x) so put x = 0 , y = 10 C (0 ; 10) ✓ (5) K

(b) Hence write down for which values of x, f (x) > 0.

$-5 < x < 3$ ✓ ✓ (2) R

(c) Determine the coordinates of E one of the points of intersection of f (x) and g (x).

$y = -2x^2 - 4x + 30$ ***1***
$y = 2x + 10$ **2**

$-2x^2 - 4x + 30 = 2x + 10$ ✓
$-2x^2 - 6x + 20 = 0$
$x^2 + 3x - 10 = 0$
$(x + 5)(x - 2) = 0$ ✓
$x = -5$ or $x = 2$ ✓
A (− 5 ; 0) and E (2 ; 14) ✓ (4) R

(d) Determine the equation of the axis of symmetry of f (x).

$x = \frac{-5+3}{2} = \frac{-2}{2} = -1$ ✔ ✔ (2) R

(e) Determine the length of GH if GH is parallel to the y-axis.

At G $x = -1$ so $y = -2(-1)^2 - 4(-1) + 30$ ✔ $= 32$ ✔
At H $x = -1$ so $y = 2(-1) + 10$ ✔ $= 8$ ✔
GH = 24 units ✔ (5) R

(f) If JL = 60 units, determine the length of OK.

$JL = g(x) - f(x) = 60$
$60 = (2x + 10) - (-2x^2 - 4x + 30)$ ✔ ✔
$60 = 2x + 10 + 2x^2 + 4x - 30$
$0 = 2x^2 + 6x - 80$
$0 = x^2 + 3x - 40$ ✔
$0 = (x + 8)(x - 5)$ ✔
$x = -8$ or $x = 5$
OK = 5 units ✔ (5) C

23 marks

QUESTION 7

Circle the number of the equation which best describes the sketch graph alongside:

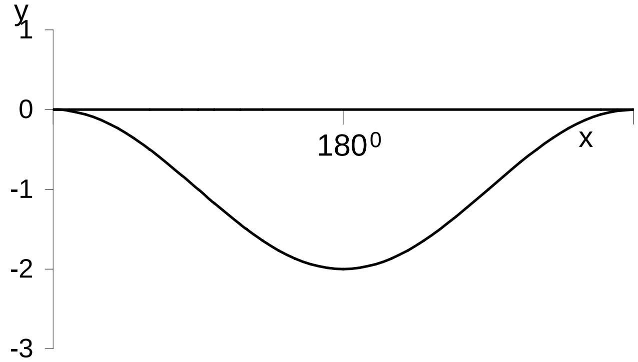
(a)

(1) $y = -2\sin x$

(2) $y = \cos x - 1$ ✓✓

(3) $y = -\cos x + 1$

(4) $y = -\sin x + 2$



(2) R

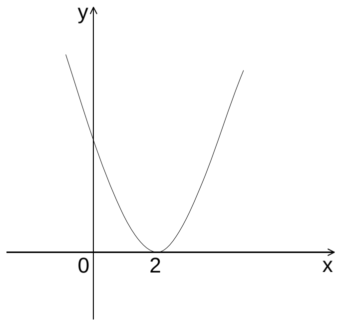
(b)

(1) $y = x^2 + 2$

(2) $y = x^2 - 2$

(3) $y = (x + 2)^2$

(4) $y = (x - 2)^2$ ✓✓



(2) R

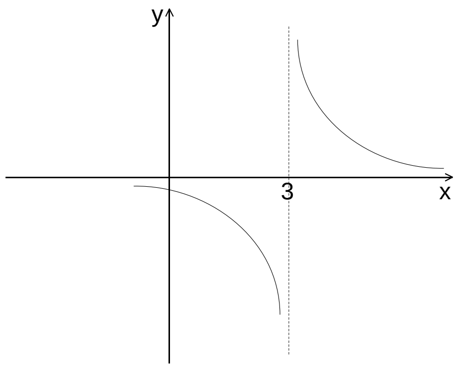
(c)

(1) $y = \frac{2}{x} - 3$

(2) $y = \frac{1}{x+2} - 3$

(3) $y = \frac{2}{x-3}$ ✓✓

(4) $y = \frac{3}{x} + 2$



(2) R

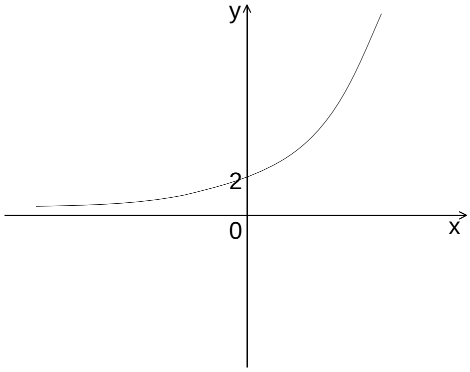
(d)

(1) $y = 2 \cdot 3^x$ ✓✓

(2)$y = 3^x + 2$

(3)$y = 3^{x+2}$

(4)$y = 3 \cdot 2^x$

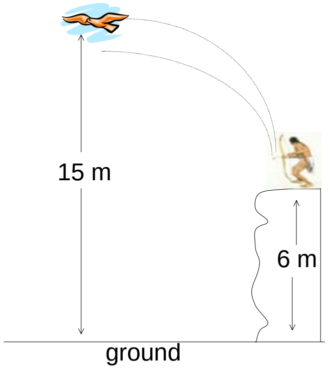


(2) R

8 marks

QUESTION 8

Refer to the diagram



A hunter is standing on a 6 m high cliff. He shoots an arrow at a bird flying 15 m above the ground.

The height of the arrow **above the ground** (in m) for the time that the arrow is in the air (in s) is given by the equation $h = -5t^2 + 13t + 6$.

Is it possible for the hunter to hit the bird? Show all working.

max height of arrow at maximum turning point of parabola

$t = -\frac{b}{2a} = -\frac{13}{2(-5)} = 1{,}3\text{ s}$ ✓ ✓

$h = -5(1{,}3)^2 + 13(1{,}3) + 6 = 14{,}45\text{ m}$ ✓ ✓

bird is 15 m above the ground therefore the arrow won't reach it. ✓

5 marks

QUESTION 9

Given $A(x) = \frac{\sqrt{3-x}}{x^2 - 4}$

Determine the value(s) of x for which :

(a) $A(x) = 0, \quad x = 3$ ✓ (1) K

(b) $A(x)$ undefined, $x^2 - 4 = 0$ ✓ $x = \pm 2$ ✓ **a** (2) K

(c) $A(x) \geq 0$ **and** Real Real if $3 - x \geq 0$ ✓ i.e. $-x \geq -3$, so $x \leq 3$ ✓ (2) R
and $x^2 - 4 > 0$ ✓, $x < -2$ or $x > 2$ ✓ **a**

$x < -2$ or $2 < x \leq 3$ ✓ ✓ (4) P

9 marks

QUESTION 10

(a) Michael invests R 3 500 in a savings account. The interest rate for the first 4 years is 8% p.a. compounded monthly, thereafter the interest rate is changed to 9% p.a. compounded semi-annually for the next 5 years. Determine the amount of money that Michael had in his savings account at the end of this period. (6) C

$$F = P\left(1+\frac{i^{(m)}}{m}\right)^{m \times n}$$

$$= 3500\left(1+\frac{8}{1200}\right)^{\overset{\checkmark\checkmark}{12 \times 4}}\left(1+\frac{9}{200}\right)^{\overset{\checkmark\checkmark}{2 \times 5}}$$

$$= 7\ 477.29 \checkmark\checkmark$$

(b) Leigh-Anne is saving for university and decides to put her money into a fixed deposit paying 10% per annum compounded annually. She starts her savings with R 1 000. After 3 years she deposits another R4 000. A final deposit of R 8 000 is made 8 years after the initial deposit. How much money is accumulated in the fixed deposit at the end of 10 years? (6) C

$$F = P\left(1+\frac{i^{(m)}}{m}\right)^{m \times n}$$

$$= \overset{\checkmark}{1000\left(1+\frac{10}{100}\right)^{10}} + \overset{\checkmark}{4000\left(1+\frac{10}{100}\right)^{7}} + \overset{\checkmark}{8000\left(1+\frac{10}{100}\right)^{2}} \checkmark$$

$$= 2\ 593.74 + 7794.87 + 9680$$

$$= 20\ 068.61 \checkmark\checkmark$$

(c) Our atmosphere contains ozone which protects the earth from the harmful effects of the sun's radiation. When chloroflurocarbons are released into the atmosphere they destroy the ozone in the atmosphere. It is estimated that 0,2% of the ozone layer is being lost each year. If we continue at this rate, it is predicted that in 300 – 400 years we will have destroyed half of the ozone layer.

Determine, to the **nearest decade**, how long it will take to have destroyed half of the ozone layer. (4) P

$$F = P\left(1-\frac{r}{100}\right)^{n}$$

$$x = 2x\left(1-\frac{0,2}{100}\right)^{n} \checkmark\checkmark$$

$$\frac{1}{2} = (0{,}998)^{n}$$

$0{,}998^{100} = 0{,}818$
$0{,}998^{200} = 0{,}670$
$0{,}998^{300} = 0{,}548$
$0{,}998^{350} = 0{,}496$
$0{,}998^{340} = 0{,}506$

350 years to the nearest decade by trial and improvement ✓ ✓

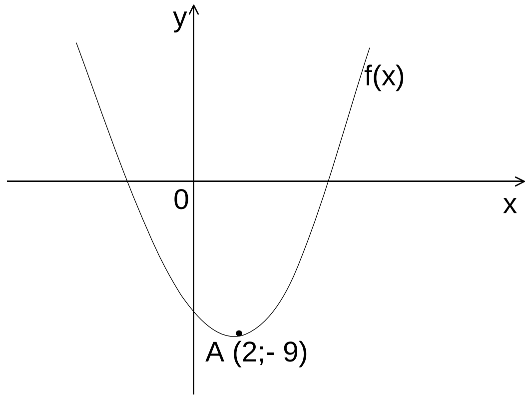
16 marks

QUESTION 11

Refer to the figure.

The graph of $y = f(x)$ with minimum turning point A (2 ; - 9) is drawn (not to scale).

Write down the co-ordinates of A if $y = f(x)$ becomes:



(a) $y = -f(x)$ (2) C
$(2 ; 9)$ ✓✓

(b) $y = f(x) + 3$ (2) C
$(2 ; -6)$ ✓✓

(c) $y = f(x+3)$ (2) C
$(-1 ; -9)$ ✓✓

(d) $y = f(x+3) + 3$ (2) C
$(-1 ; -6)$ ✓✓

8 marks

QUESTION 12

Simplify: (write all answers with positive exponents)

(a) $\sqrt{a^{3n}} \cdot \left(a^{-\frac{n}{4}}\right)^2 = a^{\frac{3n}{2}} \cdot a^{-\frac{n}{2}}$ ✓✓ $= a^n$ ✓ (3) R

(b)
$$\frac{54^n - 18^{n-1}.3^{n+1}}{(3^{n-1})^2.6^n} = \frac{3^{3n}.2^n - 3^{2n-2}.2^{n-1}.3^{n+1}}{3^{2n-2}.2^n.3^n}$$ ✓✓✓✓
$$= \frac{3^{3n}.2^n - 3^{3n-1}.2^{n-1}}{3^{3n-2}.2^n}$$
$$= \frac{3^{3n}.2^n\left(1 - \frac{1}{3}.\frac{1}{2}\right)}{3^{3n}.2^n.\frac{1}{9}}$$ ✓✓
$$= \frac{\frac{5}{6}}{\frac{1}{9}}$$
$$= \frac{15}{2}$$ ✓ (7) C

(c) $2\sqrt{8a}\left(\sqrt{2a} - 3\sqrt{18a}\right)$
$= 2.2\sqrt{2a}\left(\sqrt{2a} - 3.3\sqrt{2a}\right)$
$= 4\sqrt{2a}\left(\sqrt{2a} - 9\sqrt{2a}\right)$ ✓✓
$= 4\sqrt{2a}\left(-8\sqrt{2a}\right)$
$= -32.2a$ ✓
$= -64a$ ✓ (4) P

14 marks

QUESTION 13

Pete is driving his remote controlled car at the local track. Once he has set the throttle, he starts to take note of the how far the car is from the starting line (in cm) at a particular time (in seconds) and records his measurements.

His results are presented in the table below.

time (seconds)	0	1	2	3	4	5	6
distance from start line (in cm)	3	4	9	18			

Assume the pattern continues.

(a) If the throttle setting and all other conditions remain the same, complete the table for the next 3 seconds.

time (seconds)	0	1	2	3	4	5	6
distance (in cm)	3	4	9	18	**31**	**48**	**69**

first difference	1	5	9	13	17	21	25

2nd difference		4	4	4	4	4	4

(a) 31, 48, 69 ✓ ✓ ✓ ✓ ✓ ✓ (6) P

(b) If it is given that the relationship between the distance the car travels (d) in a particular time (t) is an equation of the form $d = at^2 + bt + c$

(1) Write down the value of c (the distance travelled when t = 0 seconds).

$c = 3$ ✓ (1) K

(2) Determine the value of a and b and hence the equation of motion of this remote controlled car.

$a = 2$ ✓ ✓
so $d = 2t^2 + bt + 3$
subt (1 ; 4) $4 = 2(1)^2 + b(1) + 3$ ✓
$4 = 5 + b$
$b = -1$ ✓

$d = 2t^2 - t + 3$ (4) C

(c) Hence (or otherwise) determine how far from the starting line the car is after 10 seconds.

$d = 2(10)^2 - (10) + 3 = 193$ cm. ✓ ✓ (2) R

(d) What is the average speed of the car in the first 3 seconds? Give your answer in **metres per minute (m / min).**

$$\text{ave speed} = \frac{\text{change in distance}}{\text{change in time}} = \frac{18-3}{3-0} = \frac{15}{3} = 5 \text{ cm/sec}$$ ✓ ✓

(2) K

$$\frac{5\text{ cm}}{\text{sec}} = \frac{300\text{ cm}}{60\text{ s}} = \frac{3\text{ m}}{\text{min}} = 3\text{ m/min}$$ ✓ ✓

✓ ✓ (2) R

17 marks