GRADE 11 EXAMINATION
NOVEMBER 2007

MATHEMATICS: PAPER II
MARKING GUIDELINES

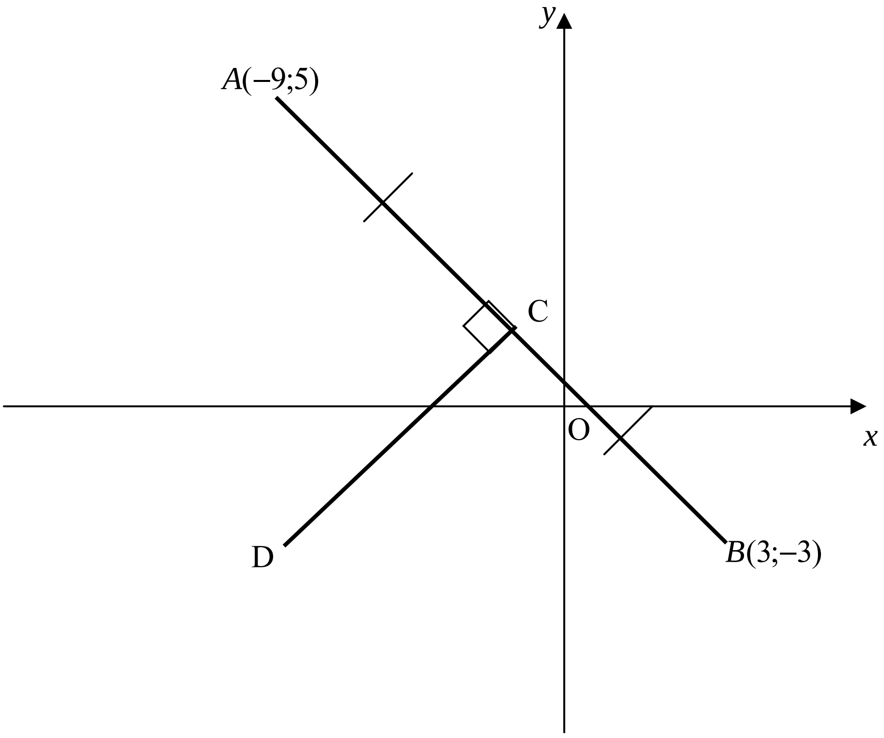
Time: 3 hours 150 marks

The marking guide is a working document prepared for use by teachers as they assess the Grade 11 externally set examinations.

There may be different interpretations of the marking guidelines but the teacher should keep as closely as possible to the suggested way of assessing. When in doubt, a teacher should check with another member of the cluster or with the relevant Assessment Specialist.

QUESTION 1

(a) In the diagram below, CD is the perpendicular bisector of AB.



1. Determine the co-ordinates of C, the midpoint of AB. (2)

$$C(\overset{a}{-3};\overset{a}{1})$$

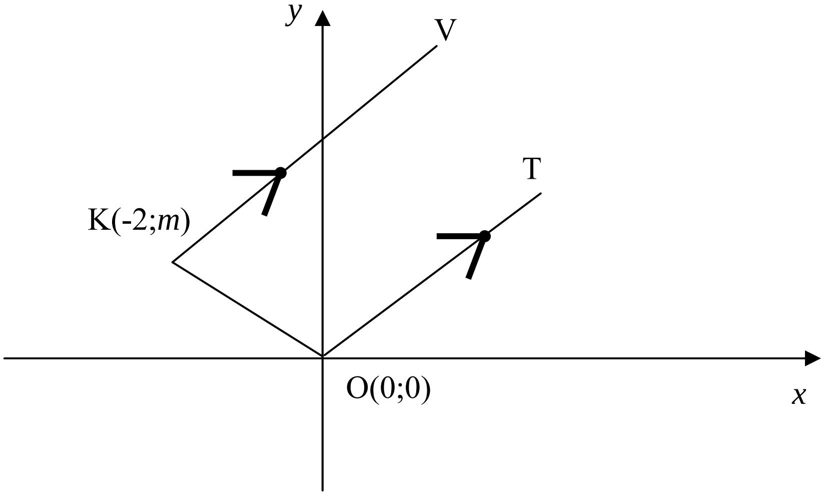
2. Determine the gradient of CD. (3)

$$m_{AB} = \overset{m}{\frac{5-(-3)}{-9-3}} = -\frac{8}{12} = \overset{a}{-\frac{2}{3}} \therefore m_{CD} = \overset{ca}{\frac{3}{2}}$$

3. Determine the equation of CD in the form $y = mx + c$. (2)

$$\overset{ca}{y-1=\frac{3}{2}(x+3)} \qquad \therefore \overset{ca}{y=\frac{3}{2}x+\frac{11}{2}}$$

(b) In the diagram below, lines KV and OT are parallel.
Line OT has equation $3y-4x=0$.
$K(-2;m)$



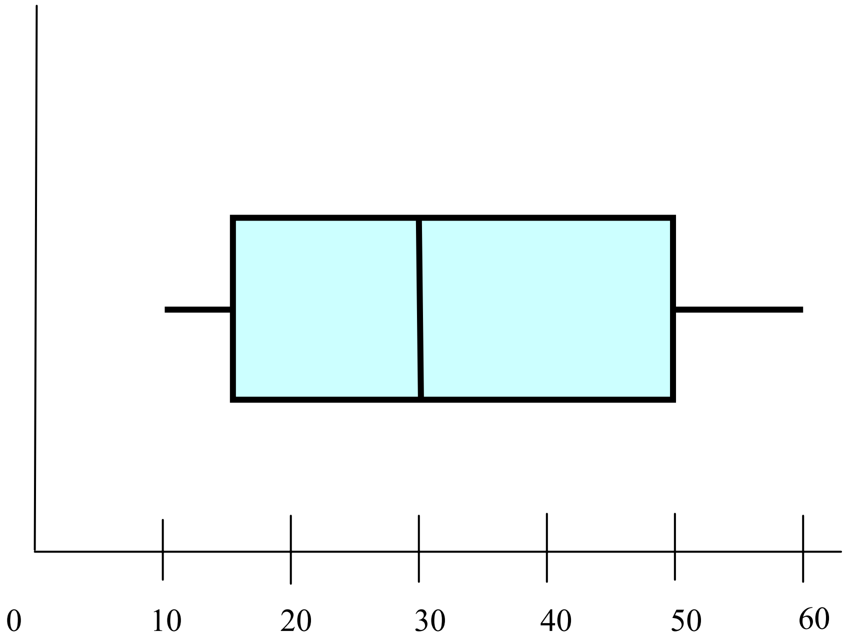
1. Write down the gradient of the line passing through K and V. (2)

$$3y = \overset{\textbf{a}}{4}x \qquad \therefore y = \frac{4}{3}x \qquad m_{KV} \overset{\textbf{ca}}{=} \frac{4}{3}$$

2. If the distance from K to the origin is 2,5 units, find the value of *m*. (3)

$$KO^2 = 2{,}5^2 \quad \therefore (0+2)^2 \overset{\textbf{a}}{+} (0-m)^2 = \overset{\textbf{a}}{6{,}25} \quad \therefore m^2 = 2{,}25 \quad \therefore m = \overset{\textbf{ca}}{1{,}5}$$

(c) Study the given box and whisker plot below and then answer the questions that follow.



For the set of data represented estimate:

(1) the minimum
10 a

(2) the lower quartile
15 a

(3) the median
30 a

(4) the upper quartile
50 a

(5) the maximum (5)
60 a

(d) Brian Lara is a record breaking cricketer who is regarded as one of the greatest batsmen of all time.

The following stem and leaf plot shows his batting scores for 19 of the one day internationals that he played.

10	2	3	4						
11	0	1	1	3	4	6	6	6	7
12	8								
13	1	9							
14	6								
15	3	6							
16	9								
17									

Key: 15 | 2 =152 runs

For the data above, determine:

1. the mean number of runs, correct to the nearest run (3)

$\dfrac{\overset{\mathbf{a}}{2355}}{19_{\mathbf{a}}} = \overset{\mathbf{ca}}{123{,}947}$, or 124

2. the mode number of runs (1)
116 $\mathbf{a}$

3. the median number of runs (1)
116 $\mathbf{a}$

4. the interquartile range (3)
$Q_1 = \underset{\mathbf{a}}{111}$ $\quad Q_3 = \underset{\mathbf{a}}{139}$ $\quad \therefore \underset{\mathbf{ca}}{IQR = 28}$

5. the standard deviation of the number of runs scored
$\sigma =$ 9,17.

$\mathbf{a}$
$\mathbf{a}$

$\mathbf{a}$ table showing x_i and $(x_i - \bar{x})^2$

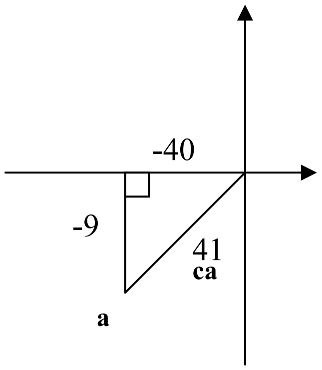
$\mathbf{a}$ correct values for $x_i - \bar{x}$

$\mathbf{a}$ correct value for $\sum\left(x_i - \bar{x}\right)^2$

(5)

(e) If $\tan A=\frac{9}{40}$ and $\cos A<0$, find using a sketch and without determining the value of *A*, the value of $\sin A+\cos A$. (5)

$$\sin A+\cos A$$
$$=-\frac{9}{41}^{\mathbf{a}}-\frac{40}{41}^{\mathbf{a}}$$
$$=-\frac{49}{41}\ \mathbf{ca}$$



(f) Simplify the following: (4)

1. $$(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2$$
$$=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\overset{\mathbf{a}}{\theta}+\sin^2\theta-2\sin\theta\cos\theta+\cos^2\overset{\mathbf{a}}{\theta}$$
$$=1+1\ \ \mathbf{a}$$
$$=2\ \ \mathbf{a}$$

2. $$\tan(180°-\theta).\sin(90°+\theta)$$ (3)
$$=-\overset{\mathbf{a}}{\tan}\theta\times\overset{\mathbf{a}}{\cos}\theta$$
$$=-\frac{\sin\theta}{\cos\theta}\times\cos\theta$$
$$=-\sin\theta_{\mathbf{a}}$$

(g) Find the general solution of

$$\sin\theta = -0,6$$

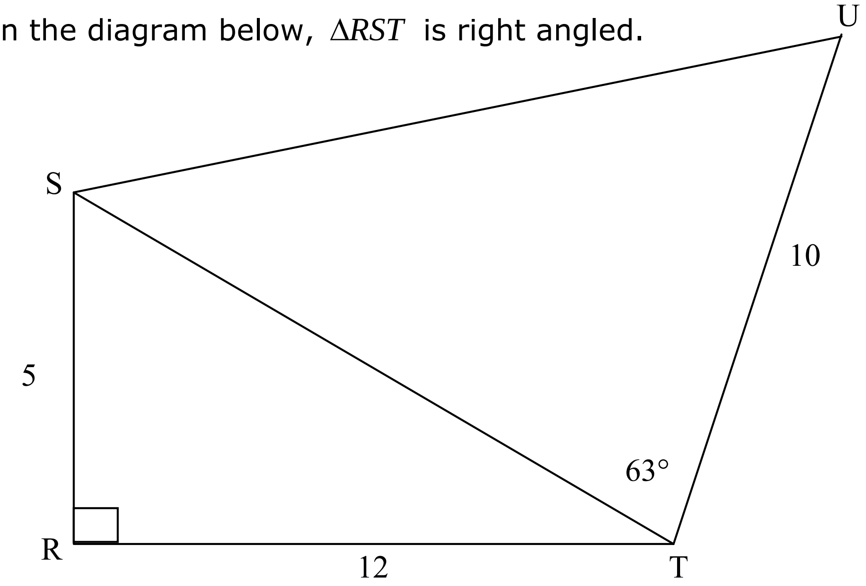
Give answers correct to one decimal digit. (5)

$$\text{III}^{\mathbf{a}} \qquad\qquad\qquad\qquad \text{IV}^{\mathbf{a}}$$

$$\theta = 180° + 36,9° + 360k \qquad or \qquad \theta = 360° - 36,9° + 360k$$

$$\therefore \qquad \theta = \underset{\mathbf{ca}}{216,9°} + 360k \qquad or \qquad \theta = \underset{\mathbf{ca}}{323,1°} + 360k \qquad k \underset{\mathbf{a}}{\in} Z$$

(h) In the diagram below, ΔRST is right angled.



Determine:

1. the length of ST. (2)

$$ST^2 = \overset{\mathbf{a}}{5^2 + 12^2} = 169 \qquad \therefore ST = \overset{\mathbf{ca}}{13}$$

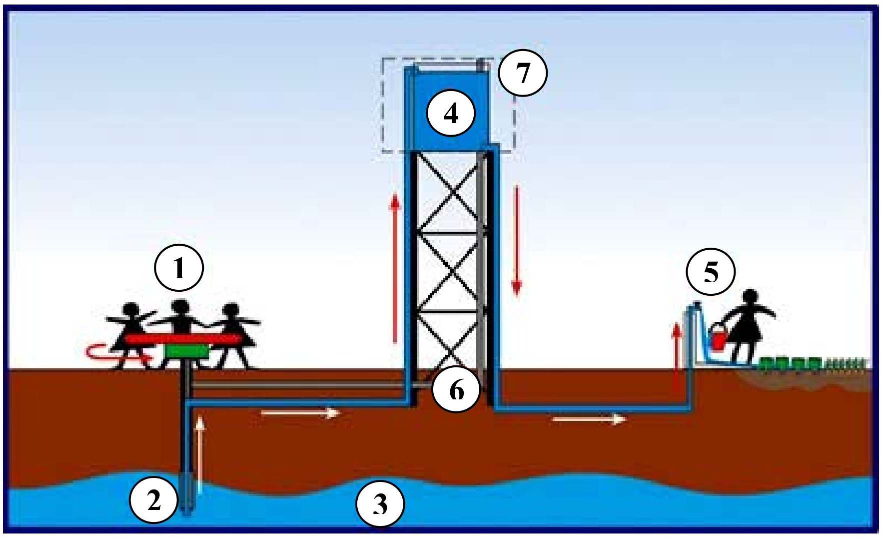
2. the area of ΔSTU correct to two decimal digits. (3)

Area of $\Delta STU = \frac{1}{2} \times \overset{\mathbf{ca}}{13} \times \overset{\mathbf{a}}{10 \times \sin 63°} = \overset{\mathbf{ca}}{57,9}\ units^{2}$

(i) Today, more than 1,1 billion people around the world have no access to clean drinking water.

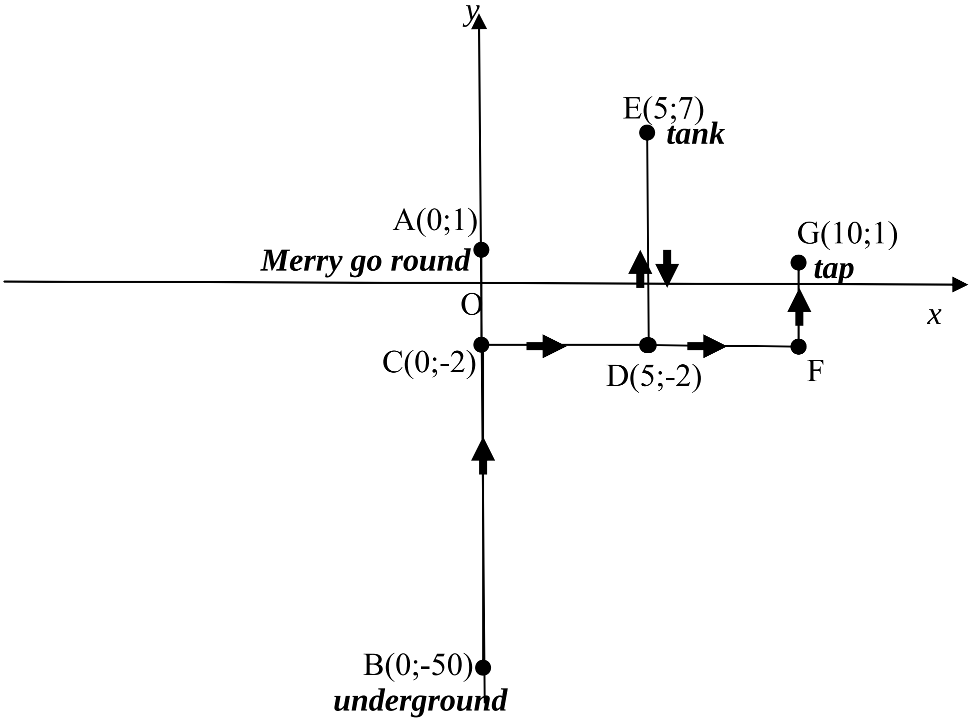
The PlayPump^{TM} water system provides easy access to clean, safe drinking water bringing about improvements in health, education, gender equality, and economic development.

HOW DOES THE PLAY PUMP WORK?



While children have fun spinning on the Play Pump merry-go-round (1), clean water is pumped (2) from underground (3) into a 2,500-liter tank (4), standing seven meters above the ground. A simple tap (5) makes it easy to draw water. Excess water is diverted from the storage tank back down into the borehole (6).

The path of the water from below the earth's surface to the tap is depicted in the Cartesian plane below



1. Write down a chain of transformations which describe the path taken by the water from B to G. The first two of six translations have been done for you.

$$(x;y)\to(x;y+48)\to(x+5;y)\to(x;\overset{\mathbf{a}}{y}+9)\to(x;\overset{\mathbf{ca}}{y}-9)\to(x\overset{\mathbf{ca}}{+}5;y)\to(x;\overset{\mathbf{ca}}{y+3})$$

(8)

2. The chain of transformations in (1) transforming point B to G, can be replaced by one equivalent transformation. State this single transformation as a general rule.

$$(x;y)\to(x\overset{\mathbf{a}}{+}10;y\overset{\mathbf{a}}{+}51\quad)$$

(2)

3. (i) Write down the co-ordinates of G', the reflection of G. (1)

$G'(-10;1)_{\mathbf{a}}$

(ii) What general rule applies to this transformation? (1)

$(x;y)\to(-x;y\quad)\ \mathbf{a}$

4. (i) Write down the co-ordinates of E', the reflection of E in the line $y = x$.

$\underset{\mathbf{a}}{E}(7;5)_{\mathbf{a}}$ (2)

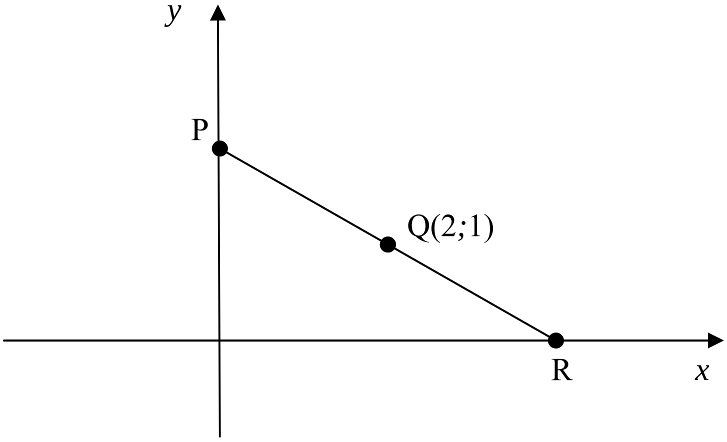
(ii) What general rule applies to this transformation? (1)

$(x;y)\to(y;x)^{\quad\mathbf{a}}$

$\boxed{\textbf{67 marks}}$

QUESTION 2

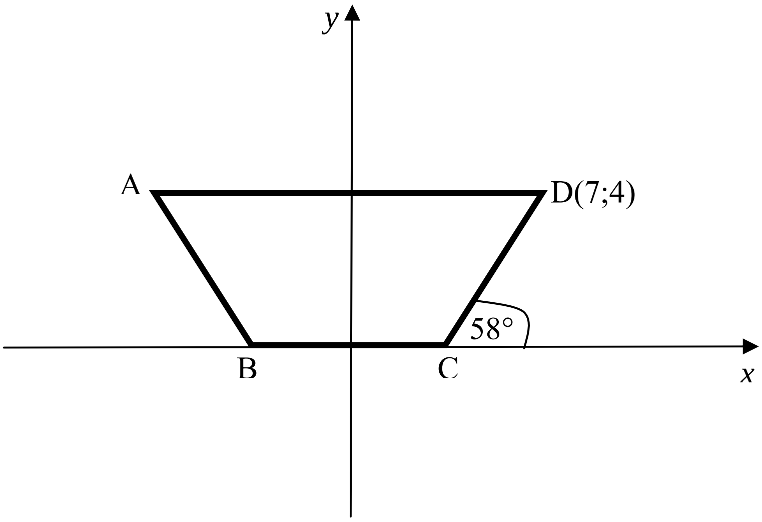
(a) In the diagram, $Q(2;1)$ is the midpoint of line segment PR. P is a point on the y axis and R is a point on the x axis.



Find the co-ordinates of P and R. (4)

$\overset{\textbf{a}\ \textbf{a}}{P(0;2)}$ $\overset{\textbf{a}\ \textbf{a}}{R(4;0)}$

(b) In the diagram below, isosceles trapezium ABCD (i.e, $AB = CD$ and $AB // CD$) is drawn with BC on the *x* axis. CD makes an angle of $58°$ with the *x* axis. The y axis is a line of symmetry of the trapezium.



1. Show that the equation of CD is given by $y = 1{,}6x - 7{,}2$. (3)

$m = \overset{\textbf{a}}{\tan 58°} = \overset{\textbf{a}}{1{,}6}$

$\therefore y - 4 \underset{\textbf{a}}{=} 1{,}6(x - 7) \quad \therefore y = 1{,}6x - 7{,}2$

2. Show that $BC = 9$ correct to the nearest integer. (3)

Let $\overset{\mathbf{a}}{y} = 0$

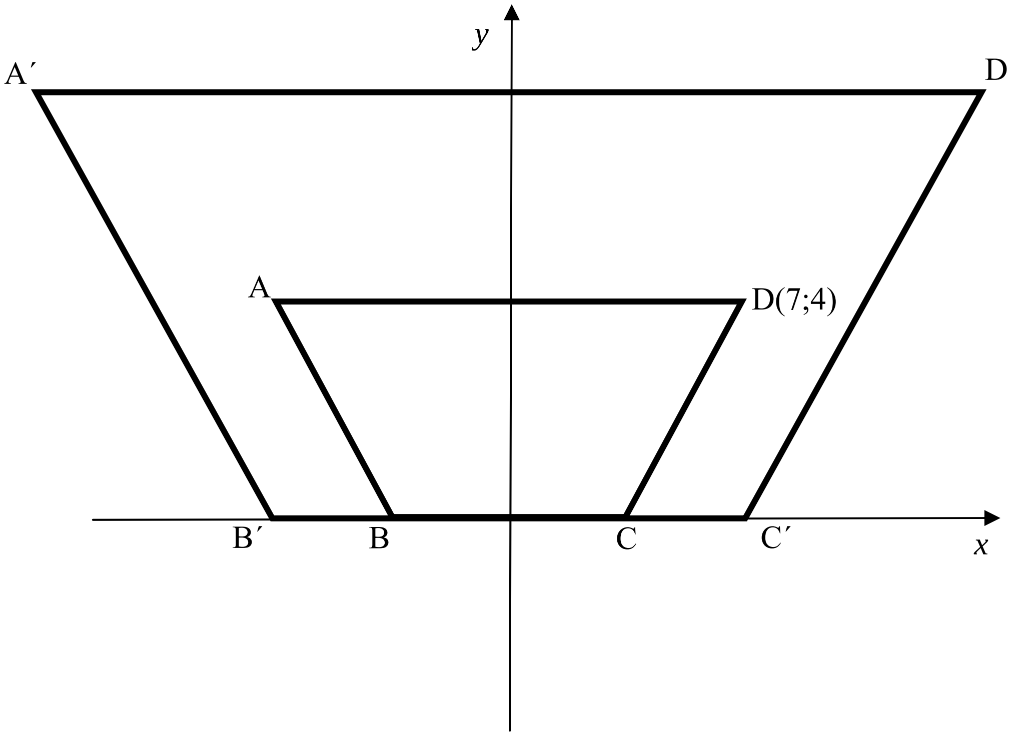
$\therefore \qquad \overset{\mathbf{a}}{0 = 1{,}6x - 7{,}2} \quad \therefore \overset{\mathbf{ca}}{x = 4{,}5}$

Therefore by symmetry $BC = 9$

3. Write down the equation of AB. (2)

$y = -\overset{\mathbf{a}}{1{,}6}x - \overset{\mathbf{a}}{7{,}2}$

4. If the trapezium is enlarged by a factor of 2 through the origin.



5. (i) find the equation of $A'B'$ (4)

$m_{A'B'} = \overset{\mathbf{a}\ \mathbf{a}}{-1{,}6}$

$\therefore \qquad \overset{\mathbf{a}}{y - 0} = -1{,}6(x + 9) \quad \mathbf{a}$

$\therefore \qquad y = -1{,}6x - 14{,}4$

(ii) find the length of $A'B'$ (4)

$A'(\overset{\mathbf{a}}{-14;8}) \qquad B'(\overset{\mathbf{a}}{-9;0})$

$(A'.B')^2 = (-14+9)^2 + (8-0)^2 = \overset{\mathbf{ca}}{89}$

$\therefore A'B' = \sqrt{89} \quad \mathbf{ca}$

(c) Simplify without using a calculator. (4)

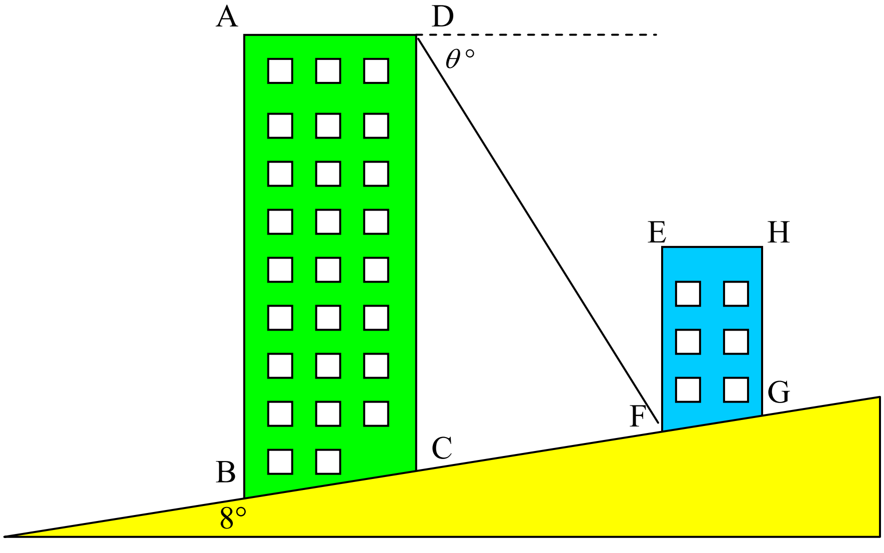
$$\sin 12°.\cos 102° - \cos^2(-12)°$$
$$= \sin 12°.(-\overset{a}{\cos} 78°) - \cos^2 12°^{a}$$
$$= \sin 12°.(-\sin 12°) - \cos^2 12°^{a}$$
$$= -(\sin^2 12° + \cos^2 12°)$$
$$= -1 \quad a$$

(d) Solve for θ correct to one decimal digit if $\theta \in (0°;360°)$ and (6)

$$\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{3}{2}$$

$$2\sin\theta \overset{a}{+} 2\cos\theta = 3\sin\theta - 3\overset{a}{\cos}\theta$$
$$\therefore \quad -\sin\theta = -5\cos\theta \quad a$$
$$\therefore \quad \tan\theta = 5 \quad a$$
$$\therefore \quad \underset{a}{\theta = 78{,}7°} \quad or \quad \underset{a}{\theta = 258{,}7°}$$

(e) In the diagram below, ABCD and EFGH represent vertical buildings on a slanting ground that has a constant inclination of $8°$.



If $DC = 31\,m$ and $CF = 20\,m$,

1. write down the size of $D\hat{C}F$. (2)

$$D\hat{C}F = 90° \overset{a}{-} 8° = \overset{a}{82°}$$

2. show that $DF = 34{,}5m$. (4)

$$DF^2 = 31^2 + 20^2 - 2\times 31\times \overset{\mathbf{m}}{20}\times \cos \overset{\mathbf{ca}}{82°} = 1188{,}425355^{\;\mathbf{ca}}$$

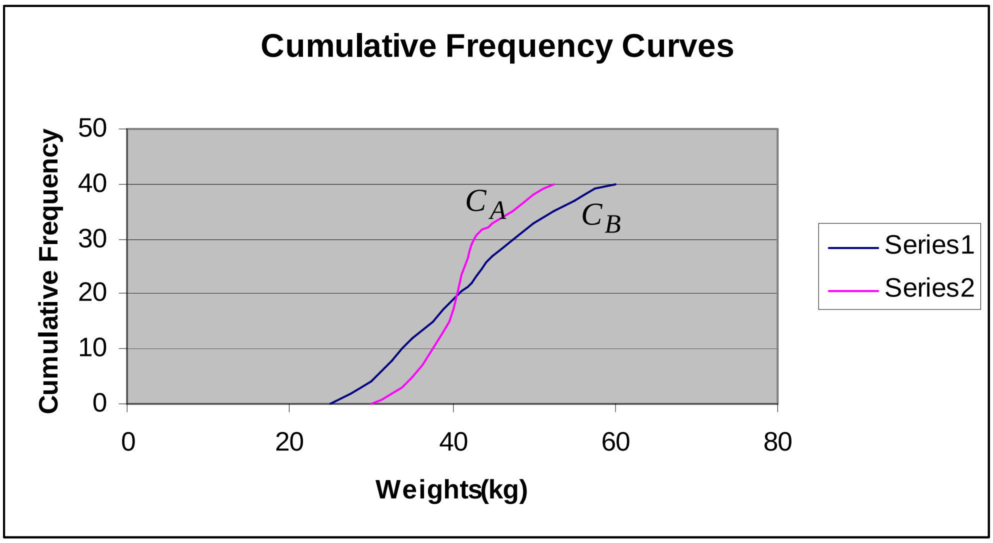
$$\therefore DF = \sqrt{1188{,}425355} = 34{,}5 \quad \mathbf{a}$$

3. find the value of θ, the angle of depression of F from D. Give answer correct to one decimal digit. (4)

$$\frac{\sin C\hat{D}F}{20} = \frac{\sin 82°}{34{,}5}^{\;\mathbf{a}} \qquad \therefore \sin C\hat{D}F = \overset{\mathbf{a}}{0{,}57406,,,} \qquad \therefore C\hat{D}F = \overset{\mathbf{a}}{35{,}0°}$$

$$\therefore \theta = 55° \quad \mathbf{a}$$

(f) In the diagram below, C_A and C_B are the cumulative frequency curves of two distributions of weights respectively.



1. Write down the median of C_A. (1)

40 **a**

2. In each case circle the statement that is correct.

(i) Median C_A > Median C_B

[✓ Median C_A = Median C_B] **a** **a**

Median C_A < Median C_B (2)

(ii) Range $C_A >$ Range C_B

Range $C_A =$ Range C_B

[✓ Range $C_A <$ Range C_B] a a (2)

(iii) [✓ Lower quartile $C_A >$ lower quartile C_B] a a

Lower quartile $C_A =$ lower quartile C_B

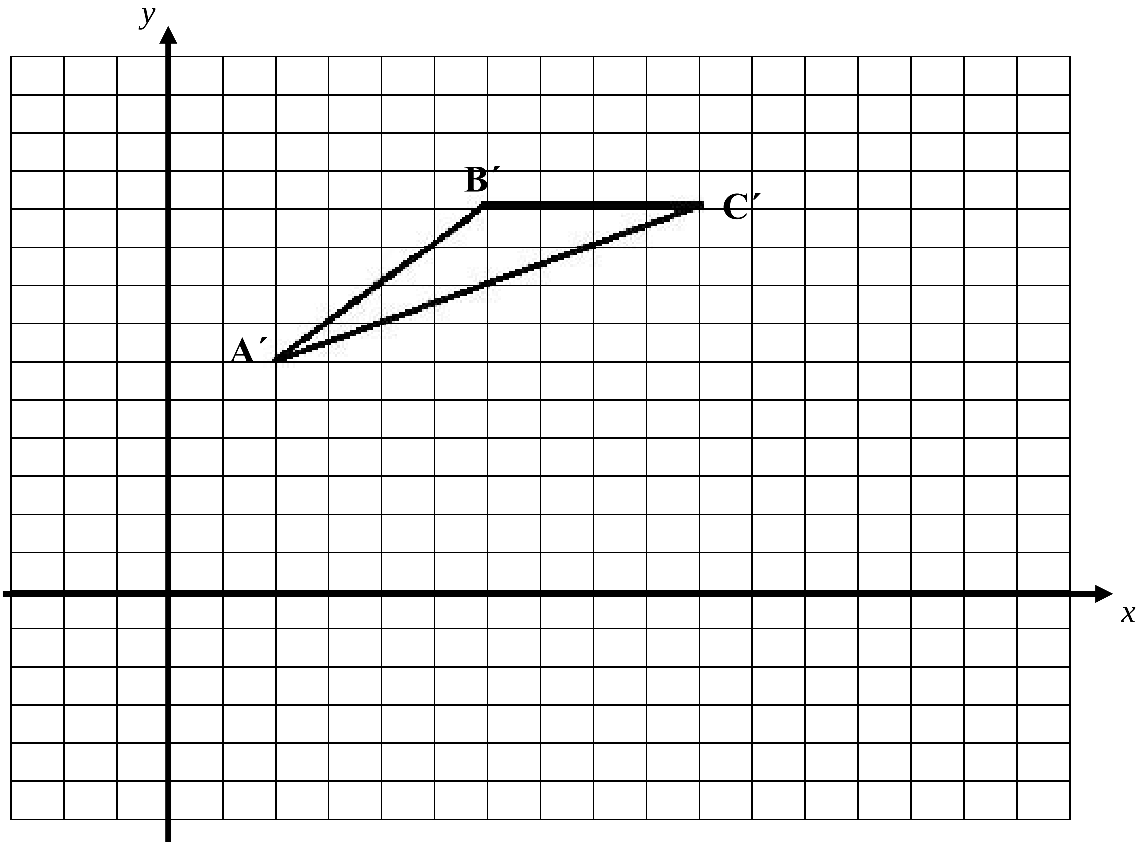
Lower quartile $C_A <$ Lower quartile C_B (2)

(iv) Inter-quartile Range $C_A >$ Inter-quartile Range C_B

Inter-quartile Range $C_A =$ Inter-quartile Range C_B

[✓ Inter-quartile Range $C_A <$ Inter-quartile Range C_B] a a (2)

(g) In the diagram below $\Delta A'B'C'$ is an enlargement of ΔABC through the origin by a factor of 2. $A'(2;6)$, $B'(6;10)$ and $C'(10;10)$.



1. Determine the co-ordinates of A, B and C. (3)

   $A(1;3)$ a $B(3;5)$ a and $C(5;5)$ a

2. If $\Delta A'B'C'$ is rotated $90°$ in an anticlockwise direction through the origin, give the co-ordinates of the A'', B'' and C'' the vertices of $\Delta A''B''C''$. State the general rule used. (4)

   $A''(-6;2)$ a $B''(-10;6)$ a $C''(-10;10)$ a

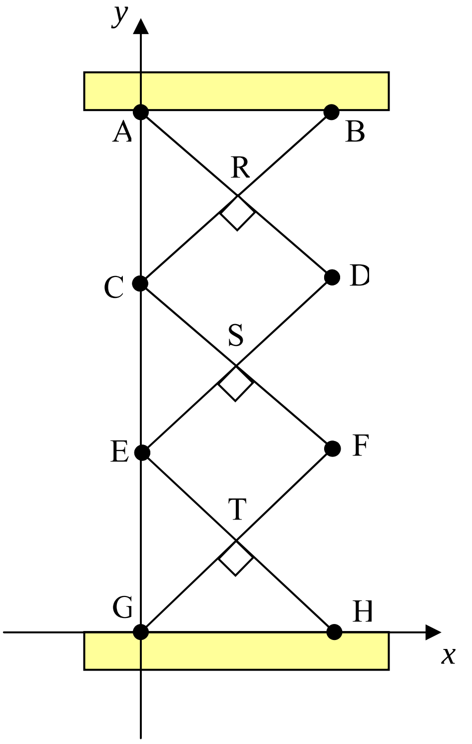
   $(x;y)\rightarrow(-y;x)$ a

56 marks

QUESTION 3

(a)

The diagram below shows a vertical cross-section of a scissors-type elevating platform which can be used to bring workers to a greater height.



AB and GH are horizontal lines.

The platform is raised to a height so that $C\hat{R}D = E\hat{S}F = G\hat{T}H = 90^\circ$.

The arms of the scissors are 2 units in length, i.e.
$GT = TH = ET = TF = ES = SF = CS = SD = CR = RD = AR = RB = 2$.

1. Determine the equation of GF. (2)

a a

$y = x$

2. Determine the equation of AD. (6)

$$EG^2 = \overset{\mathbf{a}}{2^2} + 2^2 = 8 \quad \therefore EG = \overset{\mathbf{a}}{2\sqrt{2}} \quad \therefore AG = \overset{\mathbf{a}}{6\sqrt{2}}$$

$$Also,\ m_{CB} = 1^{\mathbf{a}} \qquad \therefore \overset{\mathbf{a}}{m}_{AD} = -1$$

$$\therefore eqn: \qquad y = -x \overset{\mathbf{a}}{+} 6\sqrt{2}$$

(b) The following are all trig identities:

$$\frac{\sin\theta - \cos\theta}{1-\tan\theta} = -\cos\theta \qquad \frac{\sin^2\theta - \cos^2\theta}{1-\tan^2\theta} = -\cos^2\theta$$

$$\frac{\sin^3\theta - \cos^3\theta}{1-\tan^3\theta} = -\cos^3\theta \qquad \frac{\sin^4\theta - \cos^4\theta}{1-\tan^4\theta} = -\cos^4\theta$$

1. Use the pattern illustrated above to simplify. (2)

$$\frac{\sin^{2007}\theta - \cos^{2007}\theta}{1-\tan^{2007}\theta}$$

$$= -\cos^{2007}\theta \quad \mathbf{a\ a}$$

2. Write down a general statement which can be conjectured from the listed identities. (2)

$$\frac{\sin^n\theta - \cos^n\theta}{1-\tan^n\theta} = \overset{\mathbf{a\ a}}{-\cos^n\theta}$$

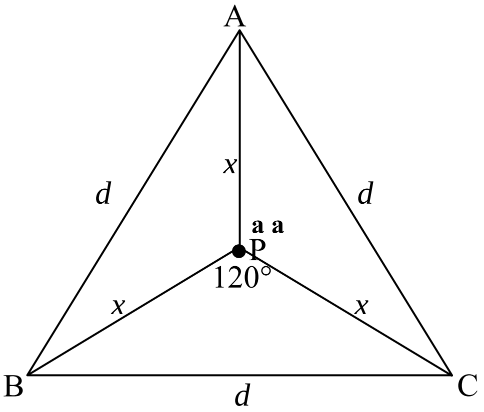
3. Prove your statement in (2). (6)

$$LHS =$$

$$\frac{\sin^n\theta - \cos^n\theta}{1-\tan^n\theta} = \frac{\sin^n\theta - \cos^n\theta}{1-\frac{\sin^n\theta}{\cos^n\theta}} \ \mathbf{a\ a}$$

$$= \frac{\sin^n\theta - \cos^n\theta}{\frac{\cos^n\theta - \sin^n\theta}{\cos^n\theta}} \mathbf{a\ a} = \frac{\sin^n\theta - \cos^n\theta}{1} \times \frac{\cos^n\theta \ \ \mathbf{aa}}{\cos^n\theta - \sin^n\theta} = -\cos^n\theta = RHS$$

(c) Three rural informal settlements (A,B and C) form an equilateral triangle as shown below.



Currently access to clean, safe water is only by moving vehicle as it is very far from the communities.

Finances have been approved to install a "Water Pump" which would be equidistant from all three settlements allowing the communities to access clean water regularly.

1. Show on the diagram where you would position the water pump so that it is equidistant from all three settlements. Use the letter P. (2)

2. If $AB = AC = BC = d$ units, and the distance from any settlement to the pump is x units, show that $d = \sqrt{3}x$. (7)

$$\text{In } \Delta BPC, \quad d^2 \overset{\mathbf{aa}}{=} x^2 + x^2 - 2x.x.\cos 120°$$

$$\therefore \quad d^2 = 2x^2 \overset{\mathbf{aa}}{-} 2x^2\left(-\cos 60°\right)$$

$$\therefore \quad d^2 = 2x^2 - 2x^2\left(-\frac{\overset{\mathbf{a\ a}}{1}}{2}\right) = 2x^2 + x^2$$

$$\therefore \quad \mathbf{a}d^2 = 3x^2$$

$$\therefore \quad d = \sqrt{3}x$$

27 marks