education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

EXEMPLAR 2008

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, 2 diagram sheets and a formula sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Number the answers correctly according to the numbering system used in this question paper.

6. Diagrams are NOT necessarily drawn to scale.

7. It is in your own interest to write legibly and to present the work neatly.

8. TWO diagram sheets for answering QUESTION 7.3, QUESTION 12.2 and QUESTION 12.3 are included at the end of this question paper. Write your name/examination number in the spaces provided and hand them in together with your ANSWER BOOK.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 10x = 24$ (3)

1.1.2 $x^2 - 6x = 10(1 - 3x)$ (5)

1.1.3 $(x - 1)(x - 2) \leq 6$ (4)

1.2 Solve for x and y simultaneously:

$x + 3y = 5$ and $xy + y^2 = 3$ (7)

[19]

QUESTION 2

2.1 Determine how long, in years, it will take for the value of a motor vehicle to decrease to 25% of its original value if the rate of depreciation, based on the reducing-balance method, is 21% per annum. (5)

2.2 The cost of a bus is R1,2 million. It is expected that the value of this bus will depreciate on a reducing balance per annum to R491 520 in 4 years' time. The price of a new bus is expected to increase by 15% per annum.

2.2.1 Calculate the percentage annual rate of depreciation of the bus. (4)

2.2.2 If the bus needs to be replaced in 4 years' time, calculate the value of the sinking fund that needs to be set up to pay for the new bus. Assume that the old bus will be traded in at its depreciation value of R491 520. (2)

2.2.3 Calculate the percentage increase in the cost of the bus in 4 years. (2)

2.2.4 Calculate the amount that must be invested monthly into a sinking fund to cover the replacement cost of the bus in 4 years' time if the interest paid by the financial institution is 9% per annum compounded monthly. Payments are made at the end of each month. (6)

[19]

QUESTION 3

Consider the following sequence of numbers:

2 ; 5 ; 2 ; 9 ; 2 ; 13 ; 2 ; 17 ; …

3.1 Write down the next TWO terms of the sequence, given that the pattern continues. (2)

3.2 Calculate the sum of the first 100 terms of the sequence. (5)

[7]

QUESTION 4

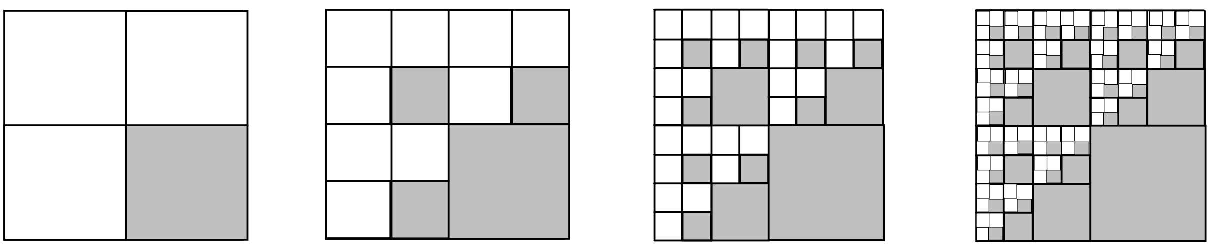
Consider the sequence: $5 ; 18 ; 37 ; 62 ; 93 ; \ldots$

4.1 If the sequence behaves consistently, determine the next TWO terms of the sequence. (2)

4.2 Calculate a formula for the n^{th} term of the sequence. (5)

4.3 Use your formula to calculate n if the n^{th} term in the sequence is 1 278. (4)
[11]

QUESTION 5



Pattern 1 **Pattern 2** **Pattern 3** **Pattern 4**

In the patterns above each consecutive pattern has more shaded squares than the previous one. The area of the shaded portion of the first pattern is $\frac{1}{4}$ square units. Assume that the pattern behaves consistently, as shown above.

5.1 The shaded area in Pattern 2 is $\frac{1}{4}+\frac{3}{16}$. Write down the area of the shaded portions of Patterns 3 and 4. (4)

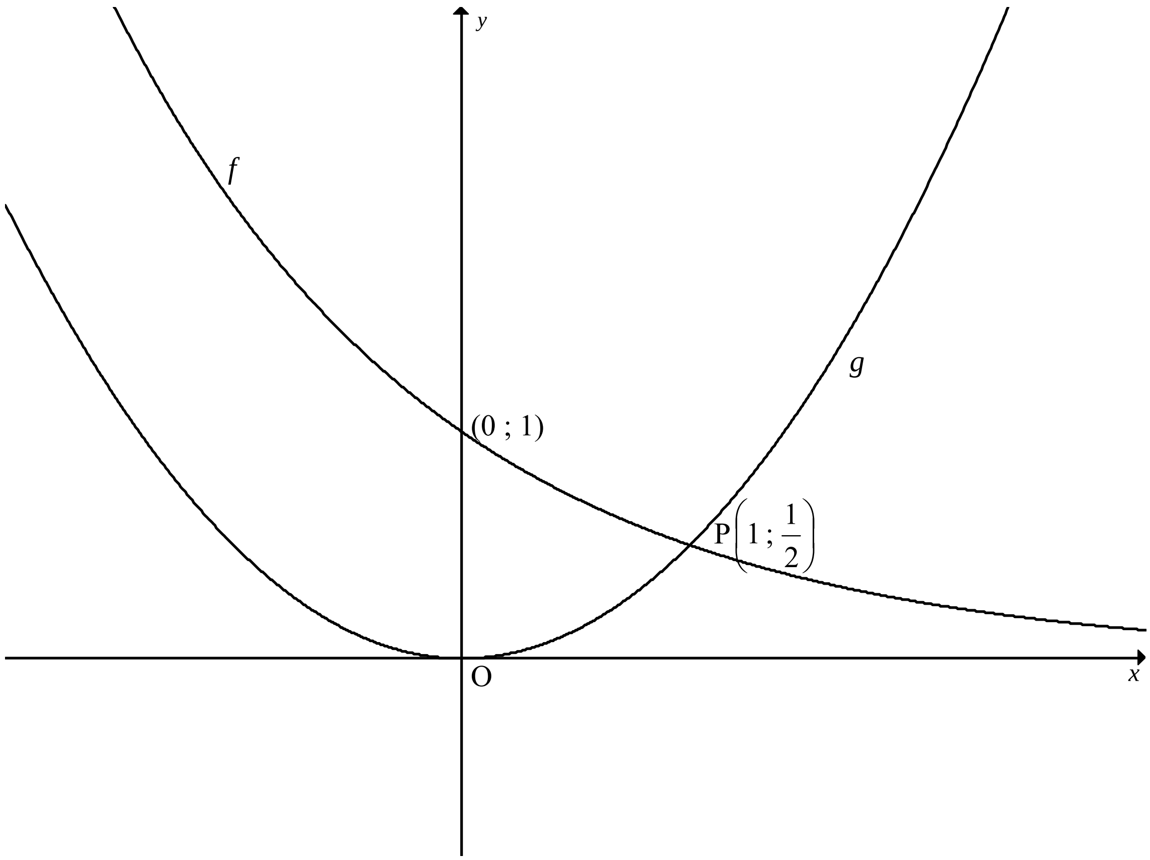
5.2 Write down the area of the shaded portion of the n^{th} pattern in sigma notation. (3)

5.3 If the pattern continues without end, what does the area in QUESTION 5.2 approach? (2)
[9]

QUESTION 6

The diagram below shows the graphs of $f(x) = a^x$ and $g(x) = bx^2$

The point $P\left(1\,;\frac{1}{2}\right)$ is the point of intersection of f and g.



6.1 Calculate the values of a and b. (2)

6.2 Write down the equation of f^{-1} in the form $y = ...$ (3)

6.3 Explain why the inverse of g is not a function. (2)

6.4 Write down TWO ways in which the domain of g could be restricted in order that g^{-1} is a function. (2)

6.5 Determine the *x*-values for which:

6.5.1 $f^{-1}(x) > 0$ (2)

6.5.2 $f(x) - 1 = g(x)$ (2)

[13]

QUESTION 7

Given: $g(x) = \dfrac{2}{x-3} - 1$

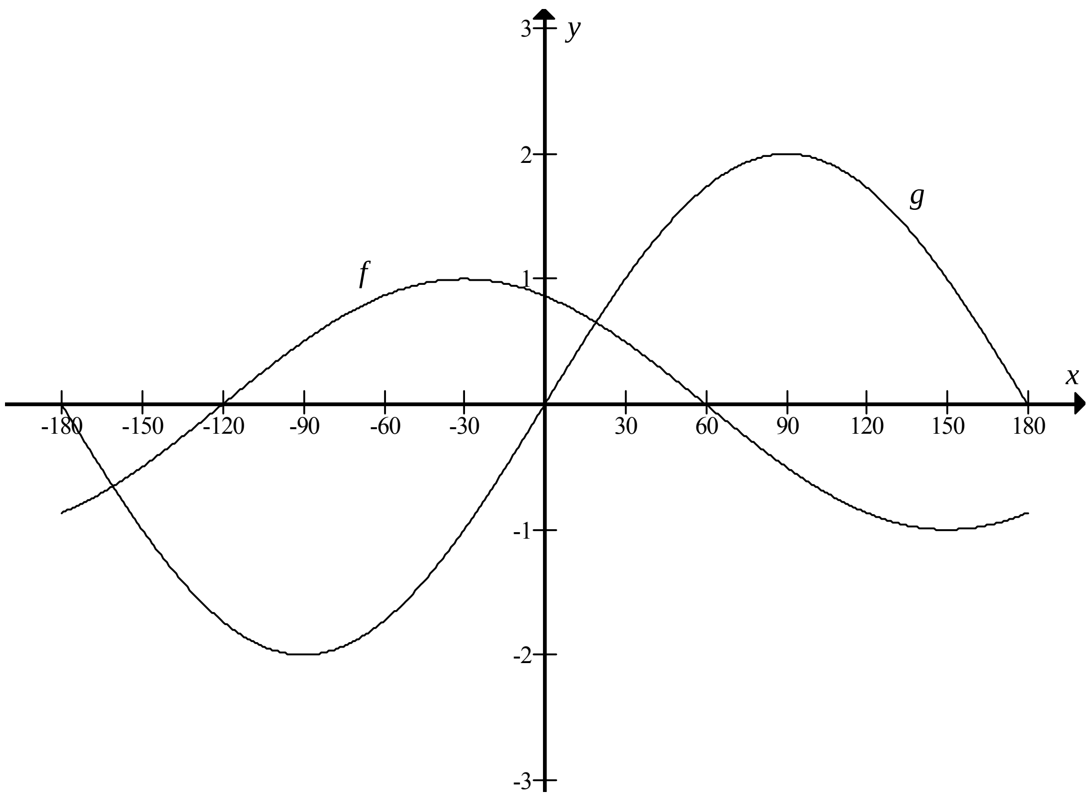
7.1 Write down the equations of the asymptotes of g. (2)

7.2 Calculate the intercepts of g with the axes. (3)

7.3 Use DIAGRAM SHEET 1 to draw the asymptotes and make a neat sketch of g. (3)
[8]

QUESTION 8

Sketched below are the functions $f(x) = \cos(x + 30°)$ and $g(x) = 2\sin x$ for $x \in [-180° \,;\, 180°]$



8.1 Write down the period of f. (1)

8.2 Give the new range of g if g undergoes a positive vertical shift of 1 unit. (2)

8.3 Write down the new equation of f if it is shifted 30º horizontally to the right. (1)
[4]

QUESTION 9

9.1 Determine $f'(x)$ from first principles if $f(x) = \frac{1}{x}$ (5)

9.2 Determine the derivatives of:

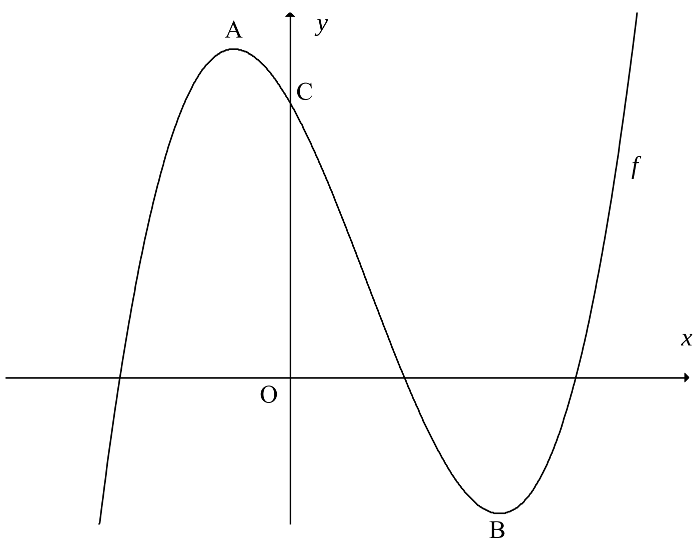
9.2.1 $f(x) = -5x^2 + 2x$ (2)

9.2.2 $y = \sqrt{x^3} + \frac{1}{3x^3}$ (4)

[11]

QUESTION 10

Sketched is the graph of $f(x) = x^3 - 4x^2 - 11x + 30$
A and B are the turning points of f.



10.1 Determine the coordinates of A and B. (5)

10.2 Determine the turning points of g if $g(x) = f(x-2)$ (2)

10.3 Determine the average rate of change of the function f from A to B. (3)

10.4 Determine the equation of the tangent to the graph of f at $x = 1$. (4)

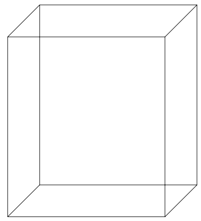
10.5 Determine the x-coordinate of the point at which the tangent in QUESTION 10.4 cuts the graph of f again. (4)

10.6 Determine the value(s) of k for which $x^3 - 4x^2 - 11x + 30 = k$ will have only one real root. (2)

10.7 Determine the point(s) of inflection of f. (6)

[26]

QUESTION 11



The volume of a certain rectangular box is given by the equation $f(x) = x^3 - 8x^2 + 5x + 50$

11.1 If the height of the box is $(5 - x)$ units, determine an algebraic expression for the area of the base of the box. (3)

11.2 Calculate the value of x for which the volume is a maximum. (6)
[9]

QUESTION 12

A BizBus motor assembly factory employs you as a production planner at the factory. Your job will be to advise the management on how many of each model should be produced per week in order to maximise the profit on the local production. The factory is producing two types of minibuses: Quadrant and Shosholoza.

Quadrant

Shosholoza

Two of the production processes that the minibuses must go through are: bodywork and engine work.

* The factory cannot operate for less than 360 hours on engine work for the minibuses.
* The factory has a maximum capacity of 480 hours for bodywork for the minibuses.
* $\frac{1}{2}$ hour of engine work and $\frac{1}{2}$ hour of bodywork is required to produce one Quadrant.
* $\frac{1}{3}$ hour of engine work and $\frac{1}{5}$ hour of bodywork is required to produce one Shosholoza.
* The ratio of Shosholoza minibuses to Quadrant minibuses produced per week must be at least 3:2.
* A minimum of 200 Quadrant minibuses must be produced per week.

Let the number of Quadrant mini buses manufactured in a week be x.
Let the number of Shosholoza minibuses manufactured in a week be y.

Two of the constraints are:

$$x \geq 200$$

$$3x + 2y \geq 2\,160$$

12.1 Write down the remaining constraints in terms of x and y to represent the above-mentioned information. (4)

12.2 Use the attached graph paper (DIAGRAM SHEET 2) to represent the constraints graphically. (3)

12.3 Clearly indicate the feasible region by shading it. (1)

12.4 If the profit on one Quadrant minibus is R12 000 and the profit on one Shosholoza minibus is R4 000, write down an equation that will represent the profit on the minibuses. (2)

12.5 Using a search line and your graph, determine the number of Quadrant and Shosholoza minibuses that will yield a maximum profit. (2)

12.6 Determine the maximum profit per week. (2)
[14]

TOTAL: 150

NAME/EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 7

7.3

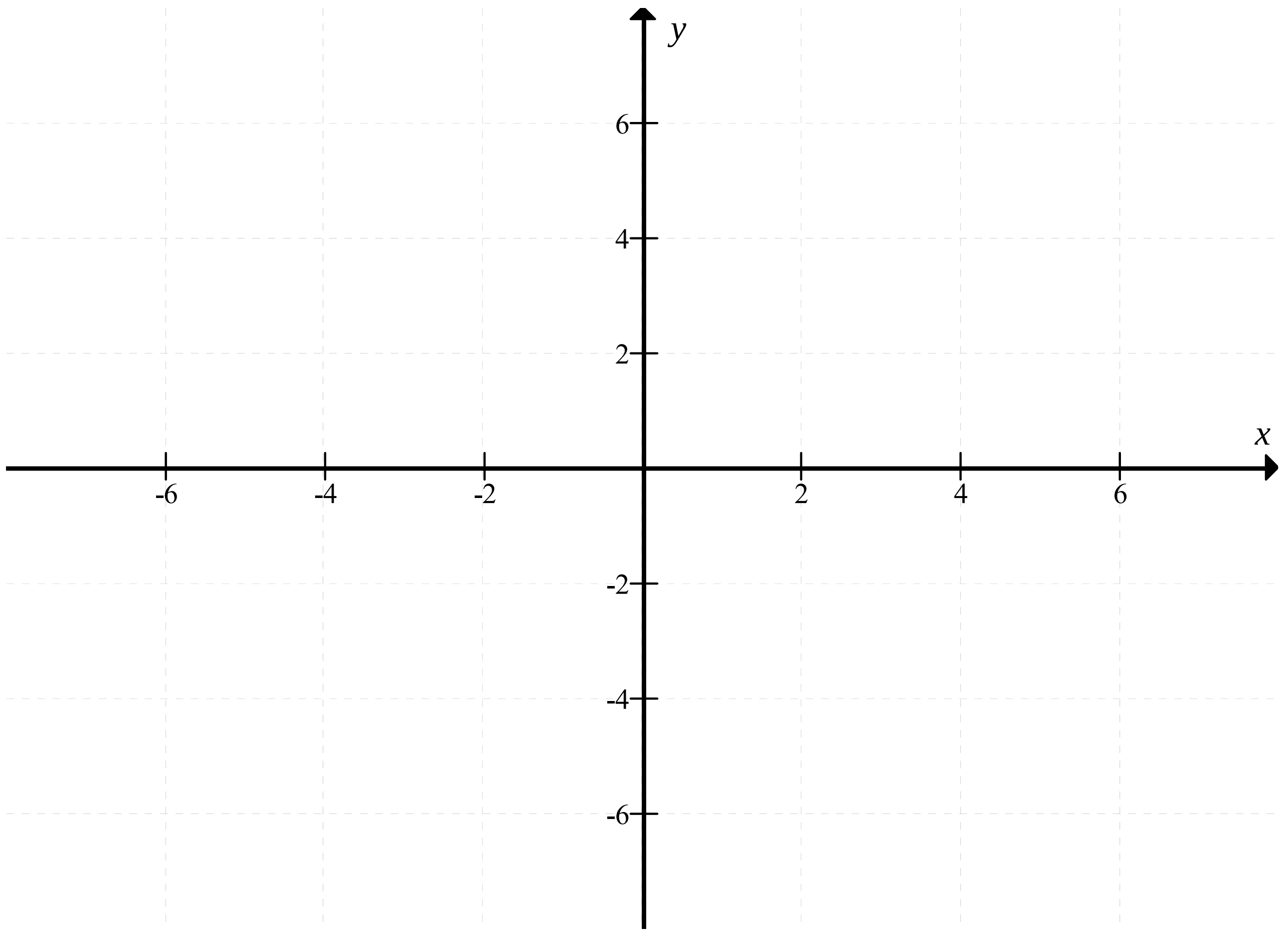


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DIAGRAM SHEET 2

QUESTION 12

12.2 & 12.3

FORMULA SHEET: MATHEMATICS
FORMULEBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\qquad$ $A = P(1 - ni)$

$A = P(1 - i)^n$ $\qquad$ $A = P(1 + i)^n$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\qquad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$\sum_{i=1}^{n} 1 = n$ $\qquad$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$$\sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a\left(r^n - 1\right)}{r-1} \ ; \quad r \neq 1$ $\qquad$ $\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r} \ ; \ -1 < r < 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\qquad$ $y - y_1 = m(x - x_1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad$ $m = \tan\theta$

$$\left(x - a\right)^2 + \left(y - b\right)^2 = r^2$$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\qquad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\qquad$ $area\ \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin\left(\alpha + \beta\right) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\qquad$ $\sin\left(\alpha - \beta\right) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos\left(\alpha + \beta\right) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\qquad$ $\cos\left(\alpha - \beta\right) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\qquad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\qquad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\qquad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$