education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

EXEMPLAR 2008

MEMORANDUM

This memorandum consists of 12 pages.

QUESTION 1

1.1.1	$x^2 - 10x = 24$ $x^2 - 10x - 24 = 0$ $(x-12)(x+2) = 0$ $x = 12 \quad \text{or} \quad x = -2$ OR $(x-5)^2 = 49 = 7^2$ $\therefore \quad x - 5 = 7 \text{ or } -7$ $\therefore x = 12 \text{ or } -2$	✓ standard form ✓ factors ✓ answers (3)
1.1.2	$x^2 - 6x = 10(1-3x)$ $x^2 - 6x = 10 - 30x$ $x^2 + 24x - 10 = 0$ $x = \dfrac{-24 \pm \sqrt{(24)^2 - 4(1)(-10)}}{2(1)}$ $x = \dfrac{-24 \pm \sqrt{616}}{2}$ $x = 0,41 \quad \text{or} \quad x = -24,41$	✓ standard form ✓ substitution ✓ 616 ✓✓ answers (5)
1.1.3	$(x-1)(x-2) \le 6$ $x^2 - 3x - 4 \le 0$ $(x-4)(x+1) \le 0$ $+ \quad 0 \quad - \quad 0 \quad +$ $-1 \qquad 4$ $-1 \le x \le 4$	✓ standard form ✓ factors ✓✓ answer (4)
1.2	$x + 3y = 5$ $x = 5 - 3y$ $(5-3y)y + y^2 = 3$ $5y - 3y^2 + y^2 = 3$ $0 = 3 - 5y + 2y^2$ $0 = (3-2y)(1-y)$ $y = \dfrac{3}{2} \quad \text{or} \quad y = 1$ $x = \dfrac{1}{2} \quad \text{or} \quad x = 2$	✓ solve for x ✓ substitution ✓ standard form ✓ factors ✓✓ y-answers ✓ x-answers (7) **[19]**

QUESTION 2

2.1	$F_V = P_V(1-i)^n$ $0,25P = P(1-0,21)^n$ $n\log 0,79 = \log 0,25$ $n = \dfrac{\log 0,25}{\log 0,79}$ $n = 5,88$ years	✓ formula ✓ substitution ✓ simplification ✓ n = ✓ answer (5)
2.2.1	$Fv = Pv(1-i)^n$ $491520 = 1200000(1-i)^4$ $(1-i)^4 = 0,4096$ $i = 0,2$ $r = 20,00\%$	✓✓ substitution ✓ simplification ✓ answer (4)
2.2.2	$Fv_{(\text{sinkingfund})} = 1,2000000(1,15)^4 - 491520$ $= R1607287,50$	✓substitution ✓ answer (2)
2.2.3	$(1,15)^4 = 1,74900625$ $\therefore$ an increase of 74,90 %	✓ substitution ✓ increase (2)
2.2.4	Let x be the monthly repayment $i = \dfrac{9}{1200} = 0,0075$ $1607287,50 = x\left[\dfrac{(1,0075)^{48}-1}{0,0075}\right]$ $1607287,50 = x[57,5207111]$ $x = R27\ 942,76$	✓ i ✓1607287,50 ✓ 1,0075 ✓ substitution ✓ 57,5207111 ✓ answer (6) **[19]**

QUESTION 3

3.1	2 ; 21	✓ 2 ✓ 21 (2)
3.2	$\underbrace{(2+2+...+2)}_{\text{for 50 terms}}+\underbrace{(5+9+13+...)}_{\text{for 50 terms}}$ $=\sum_{i=1}^{50}2+\sum_{i=1}^{50}(4i+1)$ $=2(50)+\left[\frac{50}{2}(2(5)+49(4))\right]$ $=100+25(10+196)$ $=100+5150$ $=5250$	✓ $\underbrace{2+2+...+2}_{\text{for 50 terms}}$ ✓ $\underbrace{5+9+13...}_{\text{for 50 terms}}$ ✓ 100 ✓ substitution ✓ answer (5) **[7]**

QUESTION 4

4.1	130 ; 173	✓ ✓ (2)
4.2	5 18 37 62 93 13 19 25 31 6 6 6 The second difference is constant $\therefore$ T_n is quadratic $\therefore an^2+bn+c=T_n$ $2a=6$ $a=3$ $T_n=3n^2+bn+c$ $5=3(1)^2+b(1)+c$ $b+c=2$ ... (i) $18=3(2)^2+b(2)+c$ $2b+c=6$ ... (ii) (ii) – (i): $b=4$ $c=-2$ $T_n=3n^2+4n-2$	✓ $a=3$ ✓ substitution ✓ equation (ii) ✓ $b=4$ ✓ $c=-2$ (5)

	OR $a(1)^2 + b(1) + c = 5$ (i) $a + b + c = 5$ $a(2)^2 + b(2) + c = 18$ (ii) $4a + 2b + c = 18$ $a(3)^2 + b(3) + c = 37$ (iii) $9a + 3b + c = 37$ (ii) – (i): $3a + b = 13$ $b = 13 - 3a$ Substitute $b = 13 - 3a$ into (iii) $9a + 3(13 - 3a) + c = 37$ $9a + 39 - 9a + c = 37$ $c = -2$ Substitute $b = 13 - 3a$ and $c = -2$ into (ii) $4a + 2(13 - 3a) + (-2) = 18$ $-2a = -6$ $a = 3$ $\therefore\ b = 4$ OR Let T_n be the nth term of the sequence Then $T_2 - T_1 = 13$ $T_3 - T_2 = 19$ $T_4 - T_3 = 25$ } *add both sides* $T_5 - T_4 = 31$ $T_n - T_{n-1} = ...$ $T_n - T_1 = 13 + 19 + 25 + ...(to\ n-1\ terms)$ $T_n - 5 = \left(\frac{n-1}{2}\right)[2(13) + (n-2)6]$ $T_n = (n-1)(3n+7) + 5$ $T_n = 3n^2 + 4n - 2$	✓ subst. x = 1, 2, 3 ✓ Equations (ii) – (i): ✓ Substitution into (iii) ✓ *c = - 2* ✓ *b = 4* ✓ Let T_n ✓ subtracting terms ✓ $T_n - T$ ✓ $T_n - 5$ ✓ $T_n = 3n^2 + 4n - 2$
4.3	$3n^2 + 4n - 2 = 1278$ $3n^2 + 4n - 1280 = 0$ $(3n + 64)(n - 20) = 0$ $n = \frac{-64}{3}$ or $n = 20$ $n = \frac{-64}{3}$ is not valid $\therefore\ n = 20$	✓ substitution ✓ factors ✓ $n \neq \frac{-64}{3}$ ✓ $n = 20$ (4) **[11]**

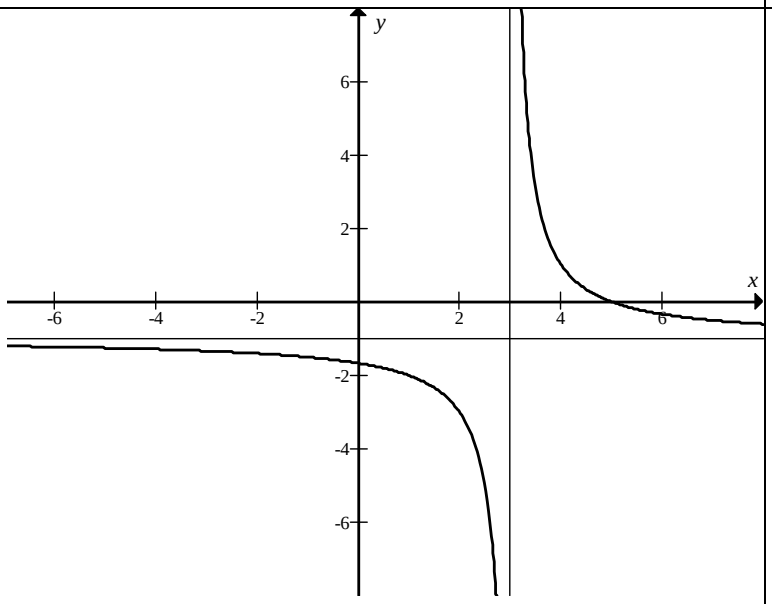
QUESTION 5

5.1	Pattern 3 $\frac{1}{4} + \frac{3}{16} + \frac{9}{64}$ Pattern 4 $\frac{1}{4} + \frac{3}{16} + \frac{9}{64} + \frac{27}{256}$	✓ sum ✓ $\frac{9}{64}$ ✓ sum ✓ $\frac{27}{256}$ (4)
5.2	$\frac{1}{4} + \frac{3}{16} + \frac{9}{64} + \ldots + \frac{3^{n-1}}{4^n}$ $\sum_{k=1}^{n} \frac{3^{k-1}}{4^k}$	✓✓ nth term ✓ sigma notation (3)
5.3	$a = \frac{1}{4} \qquad r = \frac{3}{4}$ $S = \frac{a}{1-r} = \frac{\frac{1}{4}}{1-\frac{3}{4}} = 1$	✓ r ✓ answer (2) **[9]**

QUESTION 6

6.1	$b(1)^2 = \frac{1}{2}$ $b = \frac{1}{2}$ $a^1 = \frac{1}{2}$ $a = \frac{1}{2}$	✓ answer ✓ answer (2)
6.2	$y = 2^{-x}$ $y = \left(\frac{1}{2}\right)^x$ $f^{-1} : x = \left(\frac{1}{2}\right)^y$ $y = \log_{\frac{1}{2}} x$	✓ substitution ✓ $f^{-1} : x = \left(\frac{1}{2}\right)^y$ ✓ answer (3)
6.3	The inverse is not a function because for example, $g^{-1}(\frac{1}{2}) = 1 \text{ or } -1$	✓✓ answer (2)
6.4	$x \in [0 ; \infty)$ or $x \in (-\infty ; 0]$	✓ $x \in [0 ; \infty)$ ✓ $x \in (-\infty ; 0]$ (2)
6.5.1	$0 < x < 1$	✓✓ answer (2)
6.5.2	$f(x) - 1 = g(x)$ $f(x) - g(x) = 1$ $x = 0$	✓ statement ✓ answer (2) **[13]**

QUESTION 7

7.1	$x = 3$ $y = -1$	✓ answer ✓ answer (2)
7.2	x-intercept: $0 = -1 + \frac{2}{x-3}$ $x - 3 = 2$ $x = 5$ x-intercept (5 ; 0) y-intercept $\left(0 ; -\frac{5}{3}\right)$	✓ substitution ✓ x-intercept ✓ y-intercept (3)
7.3		✓ shape ✓ intercepts ✓ asymptotes (3) **[8]**

QUESTION 8

8.1	Period = 360º	✓ answer (1)
8.2	The shift changes the range of *g* and will become now [-1 ; 3]	✓ – 1 ✓ 3 (2)
8.3	$h(x) = \cos(x + 30° - 30°)$ $h(x) = \cos x$	✓ answer (1) **[4]**

QUESTION 9

9.1	$f'(x) = \lim_{h\to 0} \dfrac{\dfrac{1}{x+h} - \dfrac{1}{x}}{h}$ $= \lim_{h\to 0} \dfrac{\dfrac{x-(x+h)}{x(x+h)}}{h}$ $= \lim_{h\to 0} \dfrac{x-(x+h)}{x(x+h)} \div h$ $= \lim_{h\to 0} \dfrac{-h}{x(x+h)} \times \dfrac{1}{h}$ $= \lim_{h\to 0} \dfrac{-1}{x(x+h)}$ $= \dfrac{-1}{x^2}$	✓ substitution ✓ common denominator ✓ simplification ✓ simplification ✓ answer (5)
9.2.1	$D_x\left[-5x^2 + 2x\right]$ $= -10x + 2$	✓✓ answer (2)
9.2.2	$y = \sqrt{x^3} + \dfrac{1}{3x^3}$ $y = x^{\frac{3}{2}} + \dfrac{1}{3}x^{-3}$ $\dfrac{dy}{dx} = \dfrac{3}{2}x^{\frac{1}{2}} - x^{-4}$	✓ $x^{\frac{3}{2}}$ ✓ $\dfrac{1}{3}x^{-3}$ ✓✓ answer (4) **[11]**

QUESTION 10

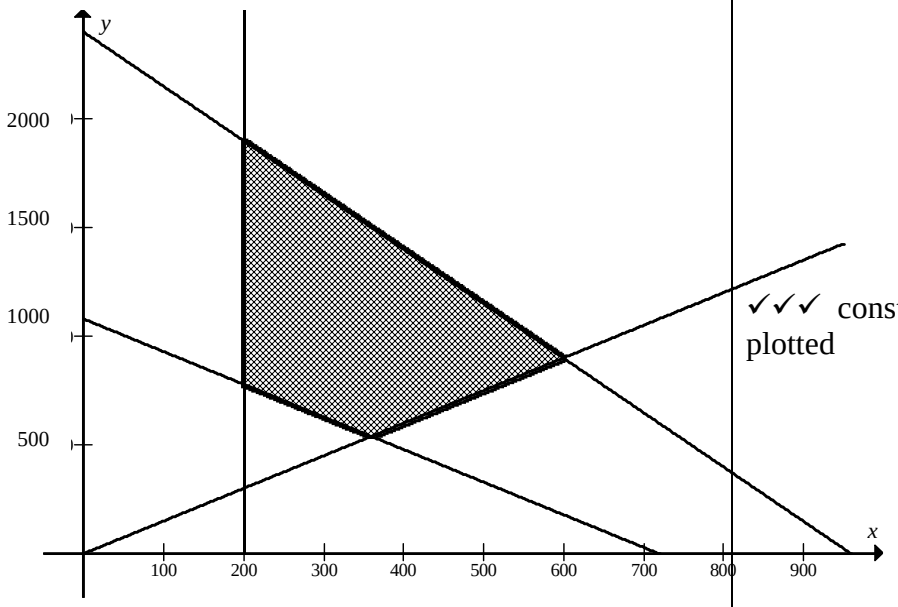
10.1	$f'(x) = 3x^2 - 8x - 11$ $0 = (3x - 11)(x + 1)$ $x = \frac{11}{3}$ or $x = -1$ $A(-1\,;\,36)$ and $B\left(\frac{11}{3}\,;\,-14{,}81\right)$ Or Some candidates may know Horners' method $f\left(\frac{11}{3}\right) = \left(\left(\left(\frac{11}{3} - 4\right) \times \frac{11}{3}\right) - 11\right) \times \frac{11}{3} + 30$ (this makes calculator work much easier) i.e. f(x) = ((x – 4)x – 11)x + 30	✓ derivative ✓ derivative = 0 ✓ factors ✓ x-values ✓ points (5)
10.2	$(1\,;\,36)$ $\left(\frac{17}{3}\,;\,-14{,}81\right)$	✓ x = 1 ✓ y-values remain the same (2)
10.3	Average rate of change $= \frac{36 - (-14{,}8)}{-1 - \frac{11}{3}}$ $= \frac{50{,}8}{-4{,}\dot{6}}$ $= -10{,}89$	✓ formula ✓ substitution ✓ answer (3)
10.4	$f'(1) = 3x^2 - 8x - 11$ $= 3(1)^2 - 8(1) - 11$ $= -16$ $f(1) = 1^3 - 4(1)^2 - 11(1) + 30$ $f(1) = 16$ $y - 16 = -16(x - 1)$ $y = -16x + 32$	✓ m = – 16 ✓ point (1 ; 16) ✓ substitution ✓ answer (4)
10.5	$-16x + 32 = x^3 - 4x^2 - 11x + 30$ $0 = x^3 - 4x^2 + 6x - 2$ $0 = (x - 1)(x^2 - 3x + 2)$ $0 = (x - 1)(x - 1)(x - 2)$ x = 1 or x = 2 the tangent cuts the graph again at x = 2	✓ substitution ✓ standard form ✓ factors ✓ answer (4)
10.6	$k > 36$ or $k < -14{,}81$	✓✓ answers (2)

10.7	$f'(x) = 3x^2 - 8x - 11$ $f''(x) = 6x - 8$ $0 = 6x - 8$ $x = \frac{4}{3}$ $f''\left(\frac{4}{3}\right) < 0$ $\quad$ $f''\left(\frac{4}{3}\right) > 0$ $\frac{4}{3}$ Point of inflection $\left(\frac{4}{3}; \frac{286}{27}\right)$ or (1,33 ; 10,59)	✓ $f''(x) = 6x - 8$ ✓ $f''(x) = 0$ ✓✓ argument ✓ x-value ✓ y-value (6) **[26]**

QUESTION 11

11.1	area $= \frac{\text{volume}}{\text{height}} = \frac{x^3 - 8x^2 + 5x + 50}{5 - x} = -x^2 + 3x + 10$	✓ division by 5 - x ✓✓ answer (3)
11.2	$f(x) = x^3 - 8x^2 + 5x + 50$ $f'(x) = 3x^2 - 16x + 5$ $0 = 3x^2 - 16x + 5$ $0 = (3x - 1)(x - 5)$ $x = \frac{1}{3}$ or $x \neq 5$ But $x = 5$ is not valid $\therefore x = \frac{1}{3}$	✓ $f'(x)$ ✓ = 0 ✓ factors ✓ answers ✓ reject $x = 5$ ✓ dimensions (6) **[9]**

QUESTION 12

12.1	$\frac{1}{2}x+\frac{1}{5}y \le 480$ $\therefore 5x+2y \le 4800$ $\frac{y}{x} \ge \frac{3}{2}$ $\therefore y \ge \frac{3x}{2}$	✓✓ inequality ✓✓ inequality (4)
12.2 and 12.3		✓✓✓ constraints correctly plotted (3) ✓ feasible region (1)
12.4	P = 12 000x + 4 000y	✓✓ equation (2)
12.5	x = 600 and y = 900	✓✓ answer (2)
12.6	P = 12 000(600) + 4 000(900) = R 10 800 000	✓ substitution ✓ answer (2) **[14]**

Total: 150