XT - MATHS Grade 12

Subject: Euclidean Geometry: Circles **Date:** 2010/06/29

Total Marks: 67

1. FALSE 2

Explanation: The quadrilateral formed must satisfy at least one of the following criteria for it to be cyclic:

- one pair of opposite angles of the quadrilateral must be supplementary;
- any exterior angle of the quadrilateral must be equal to the opposite interior angle;
- any one of the lines forming a side of the quadrilateral must subtend equal angles at the opposite vertices formed by another side adjacent to the first and one of the diagonals of the quadrilateral.

2. FALSE 7

Explanation:

$\hat{O}_1 = 360° - 176°$ [revolution]

$= 184°$

$\hat{Q} = \frac{1}{2} \times \hat{O}_2$ [$\angle$ at centre $= 2 \times \angle$ on circumference]

$= \frac{1}{2} \times 176°$

$= 88°$

Now …

$\hat{P} + \hat{Q} + \hat{R} + \hat{O}_1 = 360°$ [int. $\angle$'s of any quadrilateral]

$y + 88° + 3y + 184° = 360°$

$4y + 272° = 360°$

$4y = 88°$

$y = 22°$

Therefore …

$\hat{R} = 3y$

$= 3 \times 22°$

$= 66°$

3. C 6

Explanation: Let $Q\hat{P}R = x$.

Therefore:

$\hat{T} = x$ [tangent QP-chord RP]

$Q\hat{P}R = V\hat{P}S$ [vert. opp. ∠'s]

Therefore:

$\hat{M} = x$ [tangent PV-chord PS]

Therefore:

$\hat{T} = \hat{M}$

Therefore, the alternate angles are equal which means that they must lie between parallel lines.

Thus:

$TR \parallel SM$

If $RT \parallel QPV$, then $\hat{R}$ and $Q\hat{P}R$ would need to be equal. There is no way of proving these angles equal.

If $MS \parallel QPV$, then $\hat{S}$ and $V\hat{P}S$ would need to be equal. There is no way of proving these angles equal.

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4. D 5

Explanation: $CE = DE$ [given]

$\therefore \hat{C} = \frac{1}{2}\left[180° - C\hat{E}D\right]$ [int. ∠'s of isosc. Δ CED]

$= \frac{1}{2}\left[180° - 28°\right]$

$= \frac{1}{2} \times 152°$

$= 76°$

Now ...

$E\hat{B}D = \hat{C}$ [both subtended by chord DE]

$= 76°$

Therefore:

$A\hat{B}E = 180° - E\hat{B}D$ [supplementary ∠'s]

$= 180° - 76°$

$= 104°$

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5. TRUE 4

Explanation: $C\hat{D}B = D\hat{B}F$ [alt. ∠'s; $BF \parallel CDE$]

But ...

$C\hat{D}B = \hat{F}$ [Theorem: tangent CDE-chord BD]

Therefore ...

$D\hat{B}F = \hat{F}$

This means that $BD = FD$.

$\Delta\ BDF$ will therefore be an isosceles triangle.

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6. D 2

Explanation: $\hat{C}_1$: $\hat{A} = \hat{C}_1$ [int. ∠ equal to opp. ext. ∠ of cyc. quad. $ABCD$]

$\hat{C}_2$: $\hat{A} + \hat{C}_2 = 180°$ [opp. ∠'s of cyc. quad. $ABCD$ are suppl., not equal]

$\hat{B}_1$: $\hat{B}_1$ is not an opposite exterior angle to $\hat{A}$, as they share the common arm ABF.

$\hat{D}_1$: $\hat{D}_1$ is not an opposite exterior angle to $\hat{A}$, as they share the common arm ADH.

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7. 96°
96 6

Explanation: $\hat{O}_2 = 360° - 3x$ [revolution]

And ...

$\hat{O}_2 = 2\hat{A}$ [major arc BC subtends centre $\hat{O}_2$ and circumference $\hat{A}$]

Therefore:

$$360° - 3x = 2(x + 40°)$$
$$\therefore 360° - 3x = 2x + 80°$$
$$\therefore 280° = 5x$$
$$\therefore 56° = x$$

Now:

$$\hat{A} = (56° + 40°)$$
$$= 96°$$

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8. 72°
72 degrees 2

Explanation: As $D\hat{B}C$ is the angle between the tangent and the chord and $\hat{E}$ is the angle in the alternate segment of the circle, $\hat{E}$ will be equal to $D\hat{B}C$.

Therefore:

$\hat{E} = 72°$

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9. ADC
CDA
ADG
GDA 2

Explanation: The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle of the cyclic quadrilateral.

$B\hat{C}D$, $B\hat{A}D$, $A\hat{B}C$ and $A\hat{D}C$ are all interior angles of $ABCD$. The only angle opposite $F\hat{B}A$ is $A\hat{D}C$.

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10. (1) $33°$ 7

(2) an isosceles triangle

(3) $66°$

Explanation: (1) $A\hat{C}B = 90°$ [subtended by diameter]

$\therefore A\hat{C}D = 90°$ [supplementary ∠'s]

Now ...

$C\hat{A}D = 180° - 90° - \hat{D}$ [int. ∠'s of right-angled triangle]

$= 180° - 90° - 57°$

$= 33°$

(2) $EA = EC$ [tangents from same point (E)]

Therefore:

$\Delta\ AEC$ is isosceles [base ∠'s are equal]

(3) $C\hat{E}D = A\hat{C}E + C\hat{A}E$ [ext. ∠ of $\Delta\ AEC$]

$= 33° + 33°$

$= 66°$

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11. (1) $WU = UT$ 8

(2) UT subtends equal ∠'s

(3) converse of tangent-chord theorem

Explanation: (1) $\hat{P}_1 = \hat{P}_2$ [subtended by equal chords WU and UT]

(2) $\hat{P}_2 = \hat{W}_2$ [UT subtends equal ∠'s on circumference]

(3) $\hat{W}_2 = \hat{U}_1$ [alternate ∠'s; $WT \parallel SU$]

From all the above: $\hat{U}_1 = \hat{P}_1$

Therefore:

SU is a tangent. [converse of tan-chord theorem]

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12. (1) ext. ∠ equal to opp. int. ∠ of cyclic quadrilateral $DCEP$ 6

(2) alt. ∠'s; $CD \parallel AB$

(3) $\hat{F}_2$

(4) ext. ∠ of $ABFE$ is equal to opp. int. ∠ of $ABFE$

Explanation: (1) $\hat{P}_1 = \hat{C}_1 = x$ [ext. ∠ of cyc. quad. DCEP is equal to opp. int. ∠ of DCEP]

(2) $\hat{A}_2 = \hat{C}_1 = x$ [alternate ∠'s; DC || AB]

(3) $\hat{P}_1 = \hat{F}_2 = x$ [alternate ∠'s; AD || BC]

(4) From (1), (2) and (3): $\hat{A}_2 = \hat{F}_2$
Thus, the exterior angle of ABFE is equal to the opposite interior angle of ABFE.
Therefore, ABFE is a cyclic quadrilateral.

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13. TRUE 5

Explanation:

$R\hat{S}U = 90°$ [∠ subtended by diameter RU]

$U\hat{S}T = \hat{R}$ [Theorem: tangent ST-chord SU]

$= 36°$

$T\hat{U}S = 126°$

$\hat{T} = 180° - 126° - 36°$ [sum int angles of Δ RST]

$= 18°$

As with all geometry riders, there are many ways of answering the question. This is probably the shortest.

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14. B 4

Explanation: $CB = CD$ [tangents from same point C]

$\therefore C\hat{D}B = \frac{1}{2}\left[180° - \hat{C}\right]$ [int. ∠'s of isosc. Δ CDB]

$= \frac{1}{2}\left[180° - 42°\right]$

$= \frac{1}{2} \times 138°$

$= 69°$

Now:

$\hat{F} = C\hat{D}B$ [∠ between tangent CDE and chord BD]

$= 69°$

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15. perpendicular 1

Explanation: The converse of this theorem states:
If the radius and a chord of a circle are perpendicular to each other,
then the radius will bisect that chord.

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15 Questions, 5 Pages