XT - MATHS Grade 11

Subject: Equations 1: Basic, Formula, Inequalities **Date:** 2010/06/29

Total Marks: 64

---

1. TRUE 2

Explanation: $x^2 - 3x - 10 = 0$

Then ...

$$(x - 5)(x + 2) = 0$$

$$\therefore\ x = 5 \text{ or } x = -2$$

---

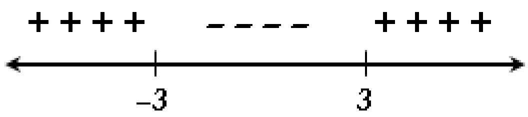
2. FALSE 3

Explanation: For quadratic inequalities, we must use a number line to determine whether the interval(s) we are looking for lies between or beyond the critical values.

$$x^2 \leq 9$$

$$x^2 - 9 \leq 0$$

$$(x - 3)(x + 3) \leq 0$$



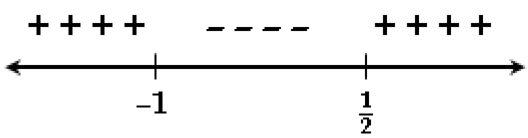
If we substitute a number less than ***-3*** into the given inequality, the product of the two brackets will be positive.
If we substitute a number between ***-3*** and ***3*** into the given inequality, the product of the two brackets will be negative.
If we substitute a number greater than ***3*** into the given inequality, the product of the two brackets will again be positive.
The product of the two brackets must be negative or zero; hence the solution is $-3 \leq x \leq 3$.

---

3. A 4

Explanation: $2x^2 + x - 1 < 0$

$$(2x - 1)(x + 1) < 0$$



If we substitute a number less than ***-1*** into the given inequality, the product of the two brackets will be positive.
If we substitute a number between ***-1*** and ½into the given inequality, the product of the two brackets will be negative.
If we substitute a number greater than ½into the given inequality, the product of the two brackets will again be positive.
The product of the two brackets must be negative; hence the solution is $-1 < x < \frac{1}{2}$

---

4. A 5

Explanation: p is a root, therefore: $x = p$

The roots of this equation are …

$$x = \frac{-(-2q) \pm \sqrt{(2q)^2 - 4(1)(q)}}{2(1)}$$

$$= \frac{2q \pm \sqrt{4q^2 - 4q}}{2}$$

Therefore …

$$p = \frac{2q \pm \sqrt{4q^2 - 4q}}{2}$$

$$= \frac{2q \pm \sqrt{4(q^2 - q)}}{2}$$

$$= \frac{2q \pm 2\sqrt{q^2 - q}}{2}$$

$$= q \pm \sqrt{q^2 - q}$$

---

5. A — 6

Explanation:

$$(x^2 - 4)^2 = 9x^2$$

Taking the square root on both sides :

$$\sqrt{(x^2 - 4)^2} = \pm\sqrt{9x^2}$$

$$x^2 - 4 = \pm 3x$$

Therefore:

$x^2 - 4 = 3x$ or $x^2 - 4 = -3x$

$x^2 - 3x - 4 = 0$ $x^2 + 3x - 4 = 0$

$(x + 1)(x - 4) = 0$ $(x - 1)(x + 4) = 0$

$x = -1;\ x = 4$ $x = 1;\ x = -4$

Here's another route you could have tried:

$(x^2 - 4)^2 = 9x^2$
$x^4 - 8x^2 + 16 = 9x^2$
$x^4 - 17x^2 + 16 = 0$
$(x^2 - 16)(x^2 - 1) = 0$
$(x - 4)(x + 4)(x - 1)(x + 1) = 0$
$x = 4,\ x = -4,\ x = 1,\ x = -1$

---

6. 0; 2 — 2
2; 0

Explanation: $x(x - 2) = 0$

$x = 0$ or $x - 2 = 0$

$x = 0$ $x = 2$

---

7. 0,19; -0,86 — 5
-0,86; 0,19

Explanation:

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-½)}}{2(3)}$$

$$= \frac{-2 \pm \sqrt{10}}{6}$$

$\approx 0{,}19$ or $-0{,}86$

8. 1; 0 4
0 ; 1

Explanation:

$$3x^3 - 6x^2 + 3x = 0$$

$$3x(x^2 - 2x + 1) = 0$$

$$3x(x - 1)(x - 1) = 0$$

Therefore …

$x = 0$ or $x = 1$

---

9. -1 3
- 1

Explanation: Rewrite the equation as:

$$(x - 2)^3 = -27$$

Taking the cube root :

$$x - 2 = -3$$

$$x = -1$$

---

10. (1) quadratic 4 (2 per answer)

(2) $2x^2 + 5x - 3 = 0$

Explanation:
- An equation in which the highest power of the variable is one, is called a linear equation.
- An equation in which the highest power of the variable is two, is called a quadratic equation.
- An equation in which the highest power of the variable is three, is called a cubic equation.

This equation has 2 as its highest power, therefore the equation is a quadratic equation.

Standard form for a quadratic equation is $ax^2 + bx + c = 0$.
The first step is to collect all terms on one side, with zero on the other side:

$$3 - 2x^2 - 5x = 0$$

The next step is to arrange the terms in descending powers of x:

$$-2x^2 - 5x + 3 = 0$$

This answer is acceptable for standard form, but it is not one of the options given.
Multiply every term through by -1: $2x^2 + 5x - 3 = 0$

---

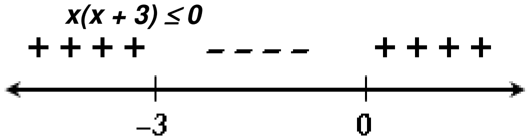
11. (1) $-3 \leq x \leq 0$ 8

(2) $-4 \leq x \leq 1$

(3) $x \leq -3$

Explanation: 1.

$x(x + 3) \leq 0$



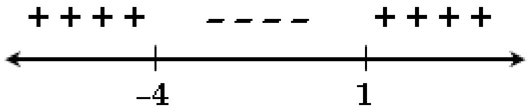
If we substitute a number less than -3 into the given inequality, the product of the two brackets will be positive.
If we substitute a number between -3 and 0 into the given inequality, the product of the two brackets will be negative.
If we substitute a number greater than 0 into the given inequality, the product of the two brackets will again be positive.
The product of the two brackets must be negative or zero; hence the solution is $-3 \leq x \leq 0$.

2.

$x(x + 3) \leq 4$

$x^2 + 3x - 4 \leq 0$ Standard form (zero on one side)

$(x + 4)(x - 1) \leq 0$ Factorise



If we substitute a number less than ***-4*** into the given inequality, the product of the two brackets will be positive.
If we substitute a number between ***-4*** and ***1*** into the given inequality, the product of the two brackets will be negative.
If we substitute a number greater than ***1*** into the given inequality, the product of the two brackets will again be positive.
The product of the two brackets must be negative or zero; hence the solution is $-4 \le x \le 1$.

3. $x^2(x + 3) \le 0$

x^2 is always positive or zero, as it is a perfect square.
That means that for a product in the given inequality that is negative or zero, the second bracket must be negative or zero.

$$(x + 3) \le 0$$
$$x \le -3$$

---

12. (1) 5 2

(2) -1

Explanation: Write the equation in the form $ax^2 + bx + c = 0$:

$$-3x^2 + 5x = 1$$

$$-3x^2 + 5x - 1 = 0$$

Then : $a = -3$, $b = 5$ and $c = -1$

But you could divide the equation by -1 :

$$3x^2 - 5x + 1 = 0$$

Then : $a = 3$, $b = -5$ and $c = 1$

---

13. (1) ± 3 6

(2) ± 5

(3) ± 4

Explanation: The solution of $px^2 + qx + r = 0$ will be:

$$x = \frac{-q \pm \sqrt{q^2 - 4pr}}{2p}$$

But it is given that $x = \dfrac{5 \pm \sqrt{25 - 4(12)}}{6}$,

which could also be written as $x = \dfrac{5 \pm \sqrt{25 - 4(12)}}{6}$ or as $x = \dfrac{-5 \pm \sqrt{25 - 4(12)}}{-6}$.

Therefore :

$$\frac{-q \pm \sqrt{q^2 - 4pr}}{2p} = \frac{5 \pm \sqrt{25 - 4(12)}}{6} \quad \text{or} \quad \frac{-q \pm \sqrt{q^2 - 4pr}}{2p} = \frac{-5 \pm \sqrt{25 - 4(12)}}{-6}$$

first term of the discriminant : $q^2 = 25$

$$\therefore \quad q = \pm 5$$

second term of the discriminant : $-4pr = -4(12)$

$$\therefore \quad pr = 12$$

If q = - 5:

Then the first term of the numerator will be: $-q = 5$

This will give the option $x = \dfrac{5 \pm \sqrt{25 - 4(12)}}{6}$.

Therefore, the denominator must be equal to 6:

$$2p = 6$$

$$p = 3$$

and together with the second term of the discriminant:

$$pr = 12$$

$$3r = 12$$

$$r = 4$$

Therefore, $p = 3$; $q = -5$; $r = 4$.

If q = 5:

Then the first term of the numerator will be: $-b = -5$

This will give the option $x = \dfrac{-5 \pm \sqrt{25 - 4(12)}}{-6}$.

Therefore, the denominator must be equal to -6:

$$2p = -6$$

$$p = -3$$

and together with the second term of the discriminant:

$$pr = 12$$

$$-3r = 12$$

$$r = -4$$

Therefore, $p = -3$; $q = 5$; $r = -4$.

All the possible values of the unknowns are therefore $p = \pm 3$, $q = \pm 5$ and $r = \pm 4$.

---

14. 0; 2 5
0 ; 2
2; 0
2 ; 0

Explanation:

$$(5x - 1)(x - 1) = (x + 1)^2$$

$$5x^2 - 6x + 1 = x^2 + 2x + 1$$

$$4x^2 - 8x = 0$$

$$4x(x - 2) = 0$$

$$x = 0 \quad \text{or} \quad x = 2$$

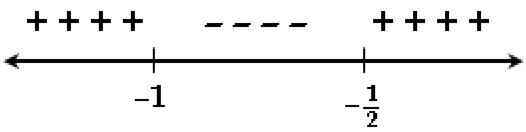
15. $-1 < x < -\frac{1}{2}$ 5
$-1 < x < -0.5$
$-1 < x < -0,5$
$-\frac{1}{2} > x > -1$
$-0.5 > x > -1$
$-0,5 > x > -1$

Explanation: $(2x - 1)(x + 2) < -3$

$2x^2 + 3x - 2 < -3$ remove brackets, as zero is not on one side

$2x^2 + 3x + 1 < 0$ standard form (zero on one side)

$(2x + 1)(x + 1) < 0$ factorise



If we substitute a number less than ***-1*** into the given inequality, the product of the two brackets will be positive.
If we substitute a number between ***-1*** and ***-½***into the given inequality, the product of the two brackets will be negative.
If we substitute a number greater than ***-½***into the given inequality, the product of the two brackets will again be positive.
The product of the two brackets must be negative; hence the solution is ***-1 < x < -½***