XT - MATHS Grade 12

Subject: Calculus 1: Factor Theorem **Date:** 2010/06/29

Total Marks: 76

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1. TRUE 8

Explanation: $f(x) = 2x^3 - 9x^2 + 16x - 12$

If $x = 2$ is a root of $f(x)$, then $f(2)$ must be equal to 0.

$$\begin{aligned} f(2) &= 2(2)^3 - 9(2)^2 + 16(2) - 12 \\ &= 2(8) - 9(4) + 32 - 12 \\ &= 16 - 36 + 32 - 12 \\ &= 0 \end{aligned}$$

Therefore, $x = 2$ is a root of $f(x)$.

Now …

$$\begin{aligned} f(x) &= 2x^3 - 9x^2 + 16x - 12 \\ &= (x - 2)(2x^2 - 5x + 6) \end{aligned}$$

$$\begin{array}{r|l} & 2x^2 \; - \; 5x \; + \; 6 \\ \hline x - 2 & 2x^3 - 9x^2 + 16x - 12 \end{array}$$

If $2x^2 - 5x + 6 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(6)}}{2(2)} \\ &= \frac{5 \pm \sqrt{-23}}{4} \end{aligned}$$

Therefore, $2x^2 - 5x + 6$ has no real roots, as $\sqrt{-23}$ is not real

$f(x)$ is therefore factorised fully and thus $x = 2$ is the only real root of $f(x)$.

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2. FALSE 8

Explanation: $f(x) = 0$

Therefore ...

$$x^3 - 8x^2 + 5x + 14 = 0$$

But …

$$\begin{aligned} f(2) &= (2)^3 - 8(2)^2 + 5(2) + 14 \\ &= 8 - 8(4) + 10 + 14 \\ &= 8 - 32 + 10 + 14 \\ &= 0 \end{aligned}$$

Therefore, $x - 2$ is a factor of $f(x)$.

Now …

$$x^{3} - 8x^{2} + 5x + 14 = 0$$

$$(x-2)(x^{2} - 6x - 7) = 0$$

$$(x-2)(x+1)(x-7) = 0$$

$$x = 2 \quad \text{or} \quad x = -1 \quad \text{or} \quad x = 7$$

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3. A 2

Explanation: To find the remainder, calculate $f(-1)$:

$f(-1) = (-1)^2 - (-1)$

$= 1 + 1$

$= 2$

The remainder is 2.

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4. A 4

Explanation:

$$f(x) = x^{3} + 2x^{2} - 3x - 3$$

$$\therefore\ f(x) + p = x^{3} + 2x^{2} - 3x - 3 + p$$

There is no remainder when $f(x) + p$ is divided by $(2x - 1)$.

Therefore ...

$$f(\tfrac{1}{2}) + p = 0$$

But ...

$$f(\tfrac{1}{2}) = (\tfrac{1}{2})^{3} + 2(\tfrac{1}{2})^{2} - 3(\tfrac{1}{2}) - 3$$

Therefore ...

$$(\tfrac{1}{2})^{3} + 2(\tfrac{1}{2})^{2} - 3(\tfrac{1}{2}) - 3 + p = 0$$

$$\tfrac{1}{8} + 2(\tfrac{1}{4}) - \tfrac{3}{2} - 3 + p = 0$$

$$\tfrac{1}{8} + \tfrac{1}{2} - \tfrac{3}{2} - 3 + p = 0$$

$$\tfrac{1}{8} + \tfrac{4}{8} - \tfrac{12}{8} - \tfrac{24}{8} + p = 0$$

$$-\tfrac{31}{8} + p = 0$$

$$p = \tfrac{31}{8}$$

$$p = 3\tfrac{7}{8}$$

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5. A 6

Explanation:

$$\begin{array}{rl} & x^{2} \quad - \; 10x \quad + \; 24 \\ \cline{2-2} x-2 \big) & x^{3} - 12x^{2} + 44x - 48 \end{array}$$

$$\underline{x^{3} - 2x^{2}}$$

$$-10x^{2} + 44x$$

$$\underline{-10x^{2} + 20x}$$

$$24x - 48$$

$$\underline{24x - 48}$$

$$0$$

$\therefore \; f(x) = (x - 2)(x^2 - 10x + 24)$
$= (x - 2)(x - 4)(x - 6)$
To solve: $(x - 2)(x - 4)(x - 6) = 0$
Therefore ... $x = 2; \; x = 4; \; x = 6$

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6. TRUE 2

Explanation: In general, ***f(x) = g(x). Q(x) + R***, where ***R*** is the remainder when ***f(x)*** is divided by ***g(x)***.

In this case, ***f(x) - 7 = (2x + 3).Q(x) + 7 - 7***
f(x) - 7 = (2x + 3).Q(x)
This new function does therefore not have a remainder.
Therefore, $(2x + 3)$ will be a factor of the new function.

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7. TRUE 3

Explanation: $f(-2)$

$= (-2)^3 + 8(-2)^2 + 17(-2) + 10$

$= -8 + 32 - 34 + 10$

$= 0$

Therefore, $(x + 2)$ is a factor of the expression.

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8. 0; -6 8

Explanation: If ***g(x)*** is a factor of ***f(x)***, then the factors of ***g(x)*** will also be factors of ***f(x).***

$g(x) = x^2 + x - 2 = (x + 2)(x - 1)$

$f(-2) = (-2)^3 + a(-2)^2 - 7(-2) + b = 0$ ***(x + 2)*** is a factor of ***f(x)***
$- 8 + 4a + 14 + b = 0$
$4a + b + 6 = 0$ eqtn 1

$f(1) = (1)^3 + a(1)^2 - 7(1) + b = 0$ ***(x - 1)*** is a factor of ***f(x)***
$1 + a - 7 + b = 0$
$a - b - 6 = 0$ eqtn 2
$4a - 4b - 24 = 0$ eqtn 2 x4

therefor $4a + b + 6 = 4a - 4b - 24$ eqtn 1 = eqtn 2
$5b = -30$
$b = -6$
$a = 0$ by substituting back into either eqtn 1 or eqtn 2

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9. 7 4

Explanation: Remainder : $f(2) = 12$

But …

$$f(2) = (2)^3 + (k - 4)(2)^2 + (k - 9)(2) - 4$$
$$= 8 + 4(k - 4) + 2(k - 9) - 4$$
$$= 8 + 4k - 16 + 2k - 18 - 4$$
$$= 6k - 30$$

Therefore …

$$12 = 6k - 30$$
$$6k = 42$$
$$k = 7$$

10. 58 4

Explanation: $2x + 1$ is a factor , then $f(-\frac{1}{2}) = 0$.

Therefore ...

$2x + 1$ is a factor, then $f(-\frac{1}{2}) = 0$:

$$12(-\tfrac{1}{2})^3 + m(-\tfrac{1}{2})^2 + 10(-\tfrac{1}{2}) - 8 = 0$$

$$12(-\tfrac{1}{8}) + m(\tfrac{1}{4}) - 5 - 8 = 0$$

$$-\tfrac{3}{2} + \tfrac{m}{4} - 13 = 0$$

$$-6 + m - 52 = 0 \qquad [\ \times 4\ ]$$

$$m = 58$$

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11. 2 3

two

Explanation:

$$f(-\tfrac{3}{2}) = 2(-\tfrac{3}{2})^3 - 7(-\tfrac{3}{2})^2 - 3(-\tfrac{3}{2}) + 20$$

$$= 2(-\tfrac{27}{8}) - 7(\tfrac{9}{4}) + \tfrac{9}{2} + 20$$

$$= -\tfrac{27}{4} - \tfrac{63}{4} + \tfrac{9}{2} + 20$$

$$= -\tfrac{27}{4} - \tfrac{63}{4} + \tfrac{18}{4} + \tfrac{80}{4}$$

$$= \tfrac{8}{4}$$

$$= 2$$

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12. (1) $f(a) = 0$ 4

(2) $f(2) = 3$

(3) $5\frac{5}{8}$

Explanation: (1) If $(x - a)$ is a factor of $f(x)$, then substituting a into $f(x)$ will give an answer of 0.

Therefore ...

$$f(a) = 0$$

(2) If $f(x) \div (x - 2)$ leaves a remainder of 3, then the remainder which is given by $f(2)$ will be equal to 3.

Therefore ...

$$f(2) = 3$$

(3) The remainder when $f(x)$ is divided by $(2x - 1)$ is given by $f(\frac{1}{2})$.

Therefore ...

$$\begin{aligned} f(\tfrac{1}{2}) &= (\tfrac{1}{2})^3 - 2(\tfrac{1}{2})^2 - 2(\tfrac{1}{2}) + 7 \\ &= \tfrac{1}{8} - 2(\tfrac{1}{4}) - 1 + 7 \\ &= \tfrac{1}{8} - \tfrac{1}{2} - 1 + 7 \\ &= \tfrac{1}{8} - \tfrac{4}{8} - \tfrac{8}{8} + \tfrac{56}{8} \\ &= \tfrac{45}{8} \\ &= 5\tfrac{5}{8} \end{aligned}$$

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13. (1) $y = -4$ 6

(2) $y = \frac{1}{4}$

(3) $y = \frac{1}{2}$

Explanation:

$$\begin{aligned} f(\tfrac{1}{2}) &= 8(\tfrac{1}{2})^3 + 26(\tfrac{1}{2})^2 - 23(\tfrac{1}{2}) + 4 \\ &= 8(\tfrac{1}{8}) + 26(\tfrac{1}{4}) - \tfrac{23}{2} + 4 \\ &= 1 + \tfrac{13}{2} - \tfrac{23}{2} + 4 \\ &= \tfrac{2}{2} + \tfrac{13}{2} - \tfrac{23}{2} + \tfrac{8}{2} \\ &= 0 \end{aligned}$$

$$\begin{array}{r|l} & \quad 4y^2 \quad + \quad 15y \quad - \quad 4 \\ \hline 2y - 1 & \quad 8y^3 + 26y^2 - 23y + 4 \\ & \underline{8y^3 - \quad 4y^2} \\ & \qquad 30y^2 - 23y \\ & \qquad \underline{30y^2 - 15y} \\ & \qquad\qquad -8y + 4 \\ & \qquad\qquad \underline{-8y + 4} \\ & \qquad\qquad\qquad 0 \end{array}$$

$$\begin{aligned} f(y) &= (2y - 1)(4y^2 + 15y - 4) \\ &= (2y - 1)(y + 4)(4y - 1) \end{aligned}$$

$$y = -4; \qquad y = \tfrac{1}{4}; \qquad y = \tfrac{1}{2}$$

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14. (1) $m = 2;\ m = 12$ 10

(2) $m = 2;\ m = -\frac{9}{8}$

(3) $m = 2$

Explanation: (1) $f(-1) = (-1)^3 + m^2(-1)^2 - 25(-1) - 14m$

$$0 = -1 + m^2 + 25 - 14m$$

$$0 = m^2 - 14m + 24$$

$$= (m - 2)(m - 12)$$

$$\therefore \quad m = 2; \; m = 12$$

(2) $f(4) = (4)^3 + m^2(4)^2 - 25(4) - 14m$

$$0 = 64 + 16m^2 - 100 - 14m$$

$$0 = 16m^2 - 14m - 36$$

$$= 8m^2 - 7m - 18 \qquad [\ \div 2\ ]$$

$$= (m - 2)(8m + 9)$$

$$\therefore \quad m = 2; \; m = -\frac{9}{8}$$

(3) The product $(x + 1)(x - 4)$ can only be a factor if both $(x + 1)$ and $(x - 4)$ are factors.

Both $(x + 1)$ and $(x - 4)$ are factors if the value of m is 2.

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15. Yes 4

Explanation:

$$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 + 2\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 2$$

$$= 4\left(\frac{1}{8}\right) + 2\left(\frac{1}{4}\right) + 1 - 2$$

$$= \frac{1}{2} + \frac{1}{2} + 1 - 2$$

$$= 0$$

Therefore, $2x - 1$ is a factor of $f(x)$.

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15 Questions, 6 Pages