ALGEBRA

Simplifying Expressions

Expressions

Expressions are groups of letters separated by + or – signs.

$3p + 2t$ is an **expression**.

$3p$ and $2t$ are called **terms**.

Like terms are the same letters.

Rules for Addition and Subtraction

Expressions can be simplified by adding or subtracting 'like terms' only.

See how the following expressions can be simplified:

$t + t + t = 3t$
$3t - t = 2t$
$4p + 3p = 7p$
$pq + pq = 2pq$
$q^2 + q^2 = 2q^2$

These all have 'like terms' and can therefore be simplified (note: the powers must also be the same).

The expressions below do not have like terms and so cannot be simplified:

$3y + 2t = 3y + 2t$
$4y + 3 = 4y + 3$
$y^2 + y^3 = y^2 + y^3$
$5x - 3y = 5x - 3y$

This can be applied to more difficult expressions, as in the following examples.

Example 1: Simplify $3t + 4p + 2t - 3p$.

$3t + 2t = 5t$ (note: a term has a sign 'in front of it' which must stay with it)

$4p - 3p = p$

Therefore $3t + 4p + 2t - 3p = 5t + p$.

Example 2: Simplify $5y + 6x - 3y - 8x$.

$5y - 3y = 2y$

$6x - 8x = -2x$

Therefore $5y + 6x - 3y - 8x = 2y - 2x$.

Multiplication of Terms

a. Like terms

$$y \times y \times y = y^3$$
$$y \times y \times y \times y = y^4$$

The small number is an index, commonly called a 'power' — it tells us how many times to multiply a term by itself.

Example: $p^5 = p \times p \times p \times p \times p$

$$p^5 \times p^2 = p \times p \times p \times p \times p \times p \times p = p^7$$

Note: This can be achieved by simply adding the powers as follows.

$p^5 \times p^2 = p^{5+2} = p^7$

See how these expressions have been simplified:

$$3p^2 \times 5p^3 = 15p^5$$
$$2y^3 \times 4y^4 = 8y^7$$

b. Unlike terms

See how the following expressions have been simplified:

$p \times q = pq$ (note: we leave out the times sign)

$3p \times 2q = 6pq$ (multiply the numbers first and then the letters)

$p^2 \times q^3 = p^2 q^3$

$5 \times q = 5q$

Rules for multiplying in algebra

Rule 1: For 'like terms' we add the powers.

Rule 2: For 'unlike terms' we leave out the multiply sign.

Dividing Terms

a. Like terms

Simplify the following: $t^5/t^2 = \frac{t^5}{t^2}$ (/ is used as a divide sign in algebra)

$$= \frac{t \times t \times t \times t \times t}{t \times t}$$

$$= t^3$$

Therefore $t^5/t^2 = t^3$

This can be done by subtracting the powers, as in the example below.

$6p^7/3p^2 = 2p^5$
The numbers are divided first and then the letters.

b. Unlike terms

Example 1: Simplify the following $p^5/y^3 = \dfrac{p^5}{y^3}$

In this case, we cannot subtract the powers.

Example 2: Simplify the following $6q^3/2t^5 = \dfrac{6q^3}{2t^5}$

$$= \frac{3q^3}{t^5}$$

In this case, we can divide the numbers.

Making Expressions from Statements

Writing Expressions from Statements

We can use the rules of simplifying to write down an expression from a statement.

Examples

John has *m* sweets and Vusi has *y* sweets. How many sweets do they have altogether?

They have $m + y$ sweets.

These are unlike terms, so we cannot simplify this answer. It is written as an **expression**.

Lee has 5 more sweets than Parvati. How many sweets does Lee have?

Lee has $m + 5$ sweets (unlike terms).

Bill has twice as many sweets as Charlize. How many sweets does Bill have?

Bill has $2 \times y = 2y$ sweets (can be simplified to $2y$ by leaving out the multiply sign).

Janet has half the number of sweets that Thandi has. How many does Janet have?

Janet has $m / 2 = \frac{m}{2}$ sweets (division)

Jill has 4 less sweets than John.

Jill has $m - 4$ sweets (unlike terms).

Making Equations

More information can be given in order to make an equation.

Example 1: I multiply a number by 3 and then add 5. If the answer is 32, what was the number?

Let the number be *y*.

Three times the number = $3y$

Add 5 = $3y + 5$

So $3y + 5 = 32$. We can now solve this equation to find *y*.

$3y = 32 - 5$ (– 5)
$3y = 27$ (÷ 3)
$y = 27/3$
$y = 9$

The number is 9.

Example 2: The length of a rectangle is $(m + 3)$ cm and the width is 5 cm. If the area is 50 cm^2, find the length of the rectangle.

Area = Length x Width
$50 = (m + 3) \times 5$

We need the brackets to show that the 3 and the *m* together make the length.

The equation is $5(m + 3) = 50$.

We can solve this equation to find m.

$$
\begin{aligned}
5m + 15 &= 50 \qquad (-15)\\
5m &= 50 - 15\\
5m &= 35 \qquad (\div 5)\\
m &= 35/5\\
m &= 7\\
\text{Length} &= m + 3\\
&= 7 + 3\\
&= 10 \text{ cm}
\end{aligned}
$$

Example 3: The length of a rectangle is t cm. The width is 5 cm less than the length. If the perimeter is 50 cm, make an equation in t and solve to find the length and width of the rectangle.

Length = a
Width = $a - 5$
Perimeter = $a + a + (a - 5) + (a - 5)$

$$
\begin{aligned}
50 &= 2a + 2a - 10\\
50 &= 4a - 10 \qquad (+10)\\
50 + 10 &= 4a\\
60 &= 4a \qquad (\div 4)\\
60/4 &= a\\
15 &= a
\end{aligned}
$$

Length = 15 cm
Width = 15 – 5 = 10 cm

Substitution and Formula

Substitution

We can be given the number for the letter $p = 4$. This means we can replace p by 4 in an expression.

Therefore $p + 2 = 4 + 2 = 6$

$p - 1 = 4 - 1 = 3$

$3p = 3 \times 4 = 12$

$\frac{p}{2} = \frac{4}{2} = 2$

$p^3 = 4 \times 4 \times 4 = 64$

We say that we have **substituted** 4 for p in the expression.

Example: Given $t = 3$ and $m = 5$, find the value of the following:

$7t = 7 \times 3 = 21$

$2t + 4m = 2 \times 3 + 4 \times 5$

$= 6 + 20$

$= 26$

$5t^2 = 5 \times 3$ squared

$= 5 \times 9$

$= 45$

$\underline{4m} + 2t = \underline{4 \times 5} + 2 \times 3$

10 10

$= 8$

Formula

An example of a formula is $P = 2L + 2W$.

If we are given $L = 5$ and $W = 7$, we can work out the value of P by substituting into the formula as follows:

$P = 2 \times 5 + 2 \times 7$

$P = 10 + 14$

$P = 24$

Solving Simple Equations

Methods of Solving Equations

An equation has a letter, an ‘equals’ sign and has two sides, as in this example:

$y + 5 = 7$

To solve an equation we find the value of the letter.

Method

Remove all the terms from one side of the equation, leaving the letter by itself.

$y + 5 = 7$ (– 5 from both sides)

To remove the + 5 we subtract 5. To keep the two sides equal, we must take 5 from both sides.

So $y = 7 – 5$
$y = 2$ is the solution to the equation.

Example 1: Solve $y + 8 = 11$. (– 8 from both sides)
$y = 11 – 8$
$y = 3$

Example 2: Solve $y + 2 = 7$. (– 2 from both sides)
$y = 7 – 2$
$y = 5$

Note: a quick way of solving this equation is to move the + 2 to the other side and change its sign to – 2.

Example 3: $y + 10 = 14$ (– 10 from both sides)
$y = 14 – 10$
$y = 4$

Example 4: $y – 6 = 2$ (+ 6 to both sides)
$y = 2 + 6$ Note: we add 6 in this case
$y = 8$

Example 5: $3y = 15$ (÷ both sides by 3)
$y = 15 \div 3$ We change the multiply by 3 to divide by 3.
$y = 5$

Example 6: $\frac{y}{7} = 2$ (x both sides by 7)
$y = 2 \times 7$ We change the divide by 7 to multiply by 7.
$y = 14$

General Rule

An important general rule is that if we move a term to the other side of the equation we change its sign.

- + changes to –
- – changes to +
- x changes to ÷
- ÷ changes to x

If there is more than one term to move, always move the + or – term first.

Example 1: Solve $2x + 5 = 15$. (– 5 from both sides)

$$2x = 15 - 5$$

$$2x = 10 \quad \text{(then ÷ 2)}$$

$$x = 10 \div 2$$

$$x = 5$$

Example 2: Solve $\frac{x}{3} - 4 = 2$. (+ 4 to both sides)

$$\frac{x}{3} = 2 + 4$$

$$\frac{x}{3} = 6 \quad \text{(x both sides by 3)}$$

$$x = 6 \times 3$$

$$x = 18$$

Brackets

Brackets are used to group terms together.
If we want to remove brackets, then everything inside the bracket must be multiplied by the term on the outside.

$3\ (y + 2) = 3 \times y + 3 \times 2 = 3y + 6$

Both *y* and the + 2 must be multiplied by 3.

$5\ (y - 3) = 5 \times y - 5 \times 3$
$= 5y - 15$

Equations with Brackets

Example 1: Solve $5\,(y - 3) = 20$.
Remove the brackets.

$$5y - 15 = 20$$
$$5y = 20 + 15$$
$$5y = 35$$
$$y = 35 \div 5$$
$$y = 7$$

Example 2: Solve $\frac{p + 4}{3} = 5$.

Note: This line brackets $p + 4$ together. So we cannot $(- 4)$ first.

$$\frac{p + 4}{3} = 5 \qquad (\text{x } 3)$$

$$p + 4 = 5 \times 3$$
$$p + 4 = 15 \qquad (- 4 \text{ from both sides})$$
$$p = 15 - 4$$
$$p = 11$$

Equations with Letters on Both Sides

Example 1: Solve $3\,(y - 1) = y + 7$.

$$3y - 3 = y + 7 \qquad (+ 3 \text{ to both sides})$$
$$3y = y + 10 \qquad (- y \text{ from both sides})$$

The *y* terms must be brought to the same side and simplified.

$$3y - y = 10$$
$$2y = 10 \qquad (\div 2)$$
$$y = 5$$

Example 2: Solve $4\,(p + 2) = 18 - p$.

$$4p + 8 = 18 - p \qquad (\text{add } p \text{ to both sides})$$
$$5p + 8 = 18 \qquad (- 8 \text{ from both sides})$$
$$5p = 10 \qquad (\div 5)$$
$$p = 2$$

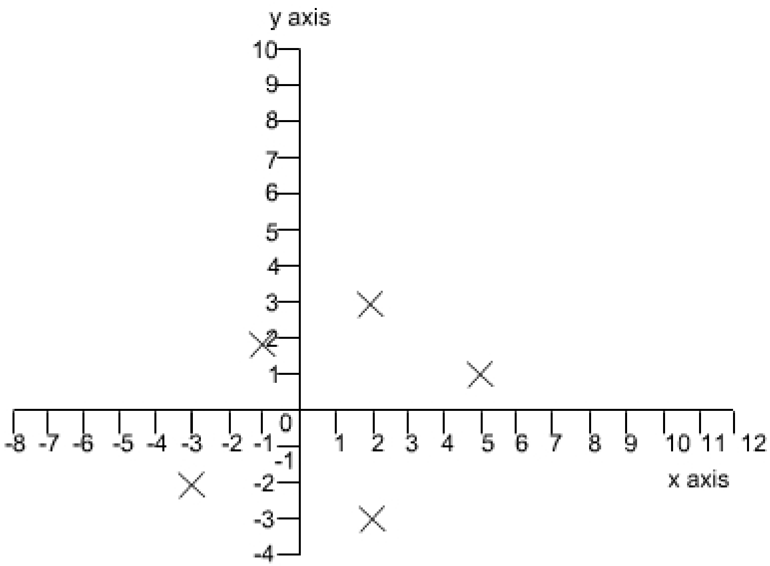
Coordinates and Graphs (Simple)

Coordinates

This is a way of plotting a point on a graph.

To plot the point A with coordinates (2,3) we start at 0 and move 2 places to the right and 3 places up on the *x* and *y* axes.

Example of plotting points: Plot the points A (2,3), B (5,1), C (–3,–2), D (2,–3) and E (–1,2) on the graph below.



Note: The first number in the bracket indicates the movement right or left.

The second number indicates the movement up or down.

Graphs (Simple)

The graph of a line can be given in the form of an equation.

Graphs of straight lines

An example of an equation of a straight line is $y = 2x$.

To plot the graph of the equation

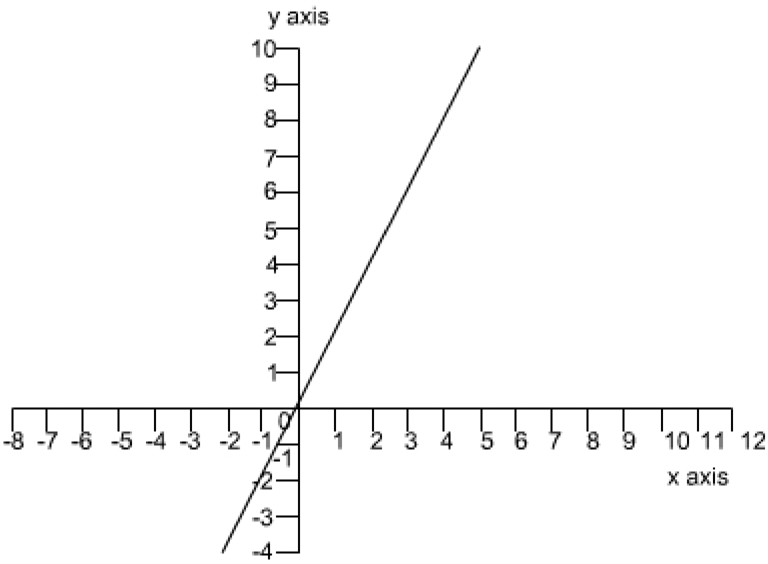
We can plot this graph by working out the coordinates. To do this we make up values for *x* and substitute them into the equation to find the values of *y*. The results are put in a table, as below.

Table of coordinates for *y* = 2*x*

x	*y*	
–1	–2	*y* = 2 x –1 = –2
0	0	*y* = 2 x 0 = 0
1	2	*y* = 2 x 1 = 2
3	6	y = 2 x 3= 6

Plotting these coordinates and drawing a line through them gives the graph of *y* = 2*x* below.

y = 2*x*



Some simple equations whose lines should be known.

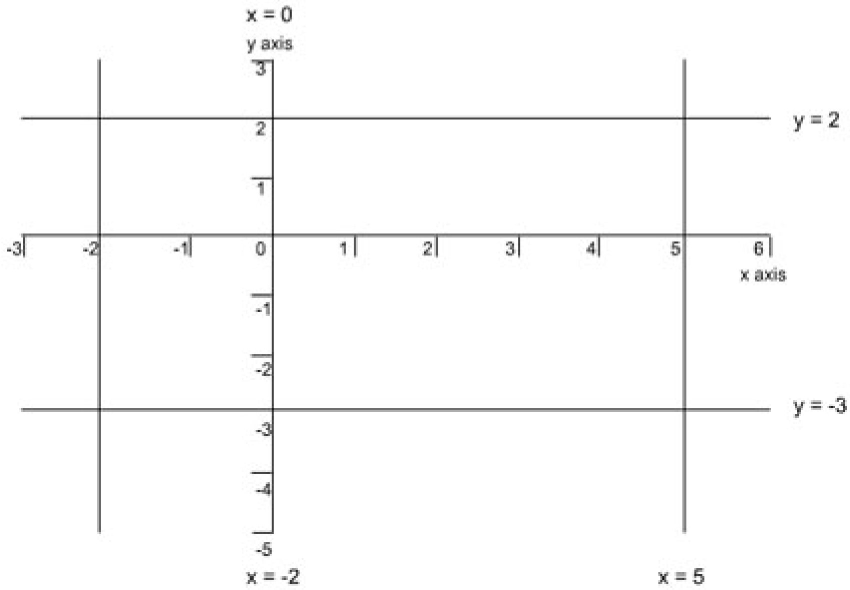
1) *y* lines are horizontal lines. For example, *y* = 2, *y* = –3, *y* = 0.

2) *x* lines are vertical lines. For example, *x* = 5, *x* =10, *x* = 0.

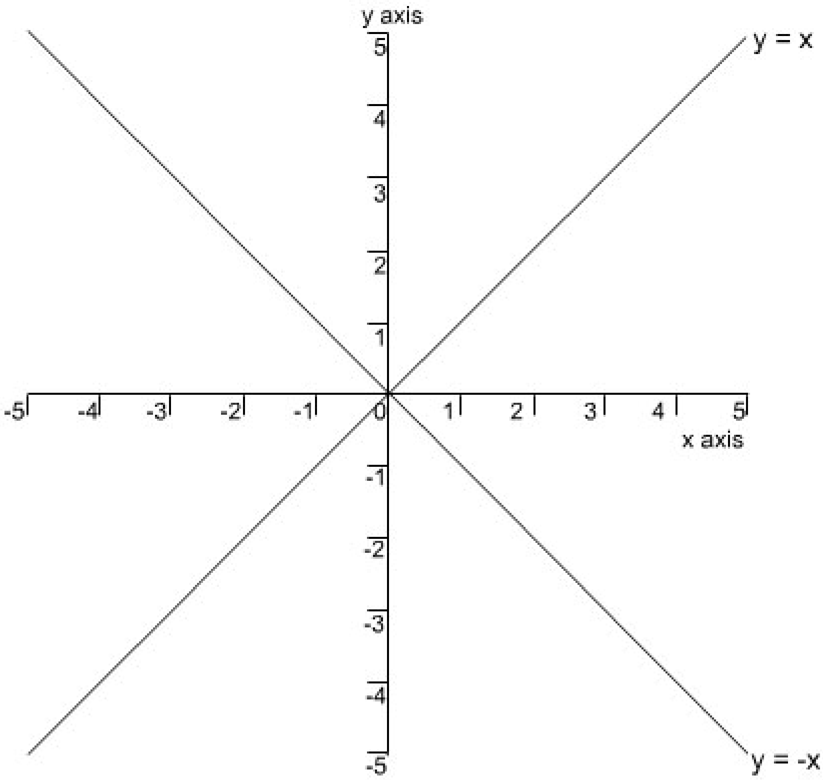
3) *y* = *x* and *y* = –*x* are diagonal lines.

4) *x* + *y* lines, for example *x* + *y* = 1 passes through 1 on the *x* axis and 1 on the *y* axis.

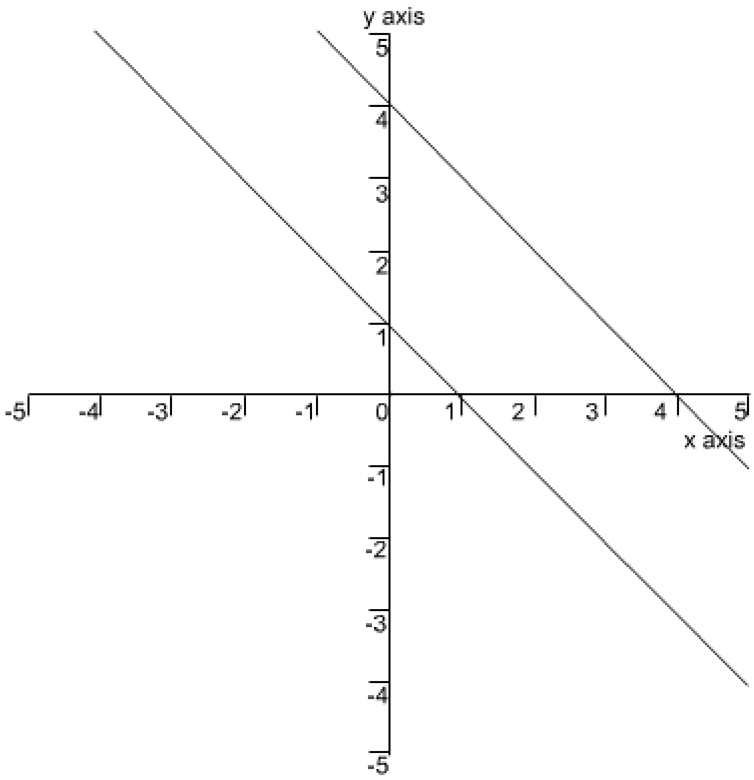
Graphs of *x* and *y* lines



Graph of $y = x$ and $y = -x$



Graph of $x + y = 4$ and $x + y = 1$



Quadratic Graphs

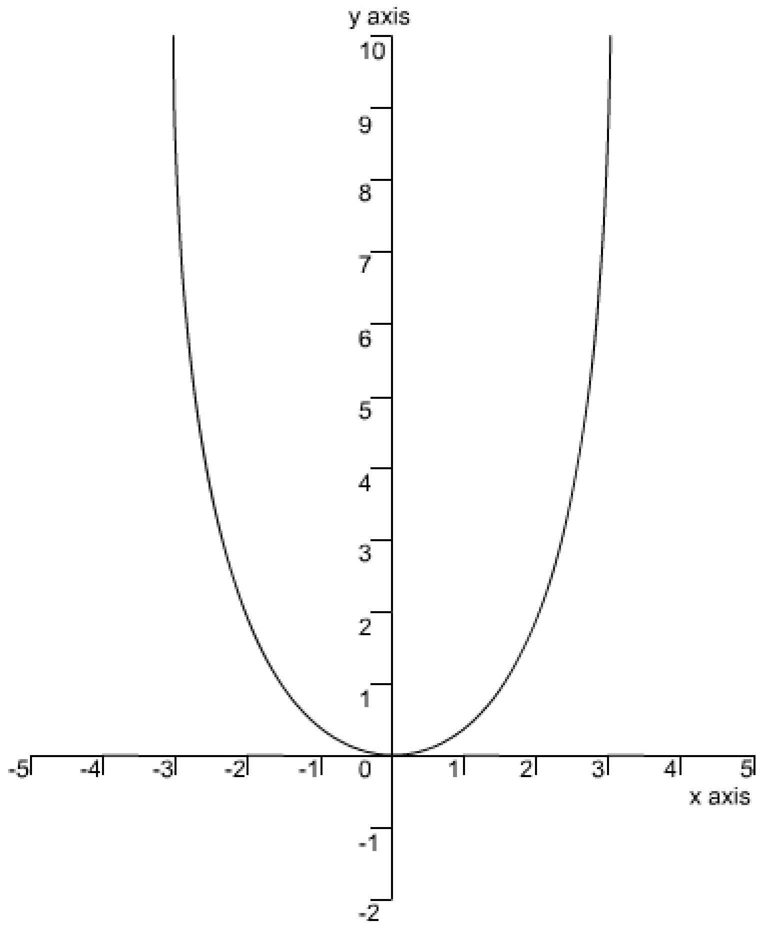
These are 'u' shaped and are derived from plotting the graph of an equation whose highest power is x^2 (a quadratic equation). The simplest example is $y = x^{2.}$

To plot the graph of $y = x^2$:

$$y = x^2$$

x	***y***
– 3	9
– 2	4
– 1	1
0	0
1	1
2	4
3	9

These coordinates are plotted giving the graph of $y = x^2$ as shown below.



y axis
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-5
-4
-3
-2
-1
1
2
3
4
5
x axis

Brackets

Brackets are used to group terms together.
If we want to remove brackets, then everything inside the bracket must be multiplied by the term on the outside.

$3\ (y + 2) = 3 \times y + 3 \times 2 = 3y + 6$

Both y and the + 2 must be multiplied by 3.

$5\ (y - 3) = 5 \times y - 5 \times 3$
$= 5y - 15$

Equations with Brackets

Example 1: Solve $5\ (y - 3) = 20$.
Remove the brackets:

$5y - 15 = 20$
$5y = 20 + 15$
$5y = 35$
$y = 35 \div 5$
$y = 7$

Example 2: Solve $\frac{p + 4}{3} = 5$.

Note: This line brackets $p + 4$ together. So we cannot (– 4) first.

$\frac{p + 4}{3} = 5$ (x 3)

$p + 4 = 5 \times 3$
$p + 4 = 15$ (– 4 from both sides)
$p = 15 - 4$
$p = 11$

Double Brackets

Example 1

$(y + 3)(y + 2) = y(y + 2) + 3(y + 2)$
$= y^2 + 2y + 3y + 6$
$= y^2 + 5y + 6$

Note: We multiply the second bracket by y and then by 3.

Example 2

$(y + 5)(y - 2) = y(y - 2) + 5(y - 2)$
$= y^2 - 2y + 5y - 10$
$= y^2 + 3y - 10$

Example 3

$(x - 3)(x - 4) = x(x - 4) - 3(x - 4)$
$= x^2 - 4x - 3x + 12$
$= x^2 - 7x + 12$

Note: This time we multiply by –3. This changes the signs in the second bracket.

Factorising

In algebra, we factorise by putting brackets into the expression.

There are three methods of factorising.

Method 1

Find a common factor.

Example 1: Factorise $3y + 6$.

In this case, 3 is a common factor because it will divide into $3y$ and 6.

Therefore the answer is written as:

$3(y + 2)$

In the next example there are two common factors.

Example 2: Factorise $5y^2 – 10y$.

5 and y are factors, so the answer is:

$5y\,(y – 2)$

When asked to factorise, we must take out all the common factors.

Method 2

A quadratic expression contains a squared term as its highest power.

Example 1: Factorise $y^2 + 5y + 6$.

In this method we use two brackets. Inside these, we put the factors of the first and last terms as follows:

y^2	+	$5y$	+	6
(y y)				(6 1) (3 2)
Factors of the first term				Factors of the last term

Note: There are two pairs of factors of the last term in this case. We choose the pair that adds up to the middle term (in this case, 5).

So the answer is $(y + 3)(y + 2)$.

The signs of the terms can be different, as in the following case.

Example 2: Factorise $y^2 + 3y – 10$.

(y, y) $(–5, 2)$

(5, –2)
(–10, 1)
(10, –1)

In this case, only 5 + –2 = 3 (which is the middle term).

Answer = (y + 5)(y – 2)

Example 3: Factorise $x^2 - 7x + 12$.
(x, x)   (– 3, – 4)
– 3 + – 4 = – 7 (which is the middle term)

Answer: (x – 3)(x – 4)

Method 3

This method is for expressions containing two squared terms and the sign between them must be minus.

Example 1: Factorise $x^2 - 9$.

Both x^2 and 9 are squared terms and a minus is between them so we can factorise in this way.

$x^2 - 9$
(x, x)   (3, 3)
(x + 3)   (x - 3)

Example 2: Factorise $y^2 - 25$.
(y, y) (5, 5)
(y + 5)(y – 5)
Note: There is a + sign is in one bracket and a – sign in the other.
Combinations of these methods can be used on some expressions.

Example 3: Factorise $2y^2 - 8$.

Common factor: $2(y^2 - 4)$
Difference between two squares: $2(y + 2)(y - 2)$

Note: We can check any answer by removing the brackets. This is called **'expanding brackets'**.

For example, 3(y + 2) = 3 x y + 3 x 2 = 3y + 6.

Note: We multiply everything inside the bracket by 3.

5y(y – 2) = 5y x y + 5y x – 2 = $5y^2 - 10y$

Double Brackets

Example 1

$(y + 3)(y + 2) = y(y + 2) + 3(y + 2)$
$= y^2 + 2y + 3y + 6$
$= y^2 + 5y + 6$

Note: We multiply the second bracket by *y* and then by 3.

Example 2

$(y + 5)(y - 2) = y(y - 2) + 5(y - 2)$
$= y^2 - 2y + 5y - 10$
$= y^2 + 3y - 10$

Example 3

$(x - 3)(x - 4) = x(x - 4) - 3(x - 4)$
$= x^2 - 4x - 3x + 12$
$= x^2 - 7x + 12$

Note: This time we multiply by –3. This changes the signs in the second bracket.

Indices

Indices are commonly referred to as 'powers'.

For example, x^3 is read as '*x* to the index 3' or '*x* to the power of 3'.

Indices tell us how many times the term is to be multiplied by itself.

For example: $y^3 = y \times y \times y$

Other indices:

$y^{-2} = \frac{1}{y^2}$ (Negative index)

$y^{1/3} = \sqrt[3]{y}$ (Fractional index)

$y^{1/4} = \sqrt[4]{y}$

Note: The bottom number of the fraction gives the 'root' of the term.

Fractional Indices

The **general rule** for fractional indices is $y^{b/a} = \sqrt[b]{y^a}$.

$y^0 = 1$ (Zero index)

When the index is zero, the result is always 1.

Rules for Indices

Rule 1: When **multiplying like terms,** for example, $y^3 \times y^5 = y^8$, we ADD the powers of *y*.

Again, $2y^2 \times 7y^4 = 14\,y^6$ (note, the 2 and the 7 are multiplied).

Rule 2: When **dividing like terms**, for example, $y^8 / y^2 = y^6$, we SUBTRACT the powers of *y*.

Again, $8y^6 / 4y^2 = 2y^4$ (note, the 8 and the 4 are divided).

Rule 3: Brackets and indices, for example, $(y^2)^4 = y^8$, the powers are MULTIPLIED in this case.

Solving Quadractic Equations

A quadratic equation is one in which the highest power is 2. For example, $x^2 + 5x + 6 = 0$.

Method When Equation is Equal to Zero

The right hand side must be 0 in order to use the following method.

Example 1: Solve $x^2 + 5x + 6 = 0$

Step 1: Factorise the quadratic.

$$(x + 3)(x + 2) = 0$$

Step 2: Put each factor equal to 0.

$$x + 3 = 0 \text{ or } x + 2 = 0$$

(Note: if two brackets are multiplied to give 0, then one of them must be 0).

Step 3: Solve these two simple equations.

$$\begin{aligned} x + 3 &= 0 \qquad & \text{or } x + 2 &= 0 \\ x &= -3 & x &= -2 \end{aligned}$$

Therefore –3 and –2 are the solutions of the equation.

There are two solutions because of the shape of the graph (refer to study note on graphs).

Example 2: Solve $x^2 + 7x - 18 = 0$.

$$\begin{aligned} (x + 9)(x - 2) &= 0 \\ x + 9 &= 0 \qquad \text{or} \qquad & x - 2 &= 0 \\ x &= -9 & x &= 2 \end{aligned}$$

Example 3: Solve $x^2 - 8x + 12 = 0$.

$$\begin{aligned} (x - 6)(x - 2) \qquad &= 0 \\ x - 2 &= 0 \qquad \text{or} \qquad & x - 6 &= 0 \\ x &= 2 & x &= 6 \end{aligned}$$

Method When Equation is Not Equal to Zero

If the equation given is not equal to zero, then follow this procedure.

Example: Solve $x^2 + 5x + 3 = 17$.

Make the right side equal to zero by subtracting 17 from both sides.

$$x^2 + 5x - 14 = 0$$
$$(x + 7)(x - 2) = 0$$
$x = -7$ or $x = 2$

Solving Simultaneous Equations

Method for Solving Simultaneous Equations

Dealing with simultaneous equations requires the solving of two equations at the same time, as in the example below.

Solve: $2x + y = 1$
$6x - 2y = 13$

1. Make the number in front of *x* or *y* the same by multiplying.

$2x + y = 1$ (x 2)
$4x + 2y = 2$

We still have $6x - 2y = 13$.

2. Get rid of the chosen term by adding or subtracting the two equations.

$4x + 2y = 2$
$\underline{6x - 2y = 13}$
$10x = 15$ (in this case, the two are added – see **Important Note**)

3. Now solve for *x*.

$$x = \frac{15}{10}$$
$$\boldsymbol{x = 1{,}5}$$

4. Substitute $x = 1{,}5$ into either of the equations.

$2x + y = 1$
$(2 \times 1{,}5) + y = 1$
$3 + y = 1$
$\boldsymbol{y = -2}$

Answer: $x = 1{,}5$
$y = -2$

Checking Your Answer

This may be done by substituting $x = 1{,}5$ and $y = -2$ into the other equation, as below, and see that it works:
$6x - 2y = 13$
$(6 \times 1{,}5) - (2x - 2) = 13$
$9 - (-4) = 13$
$9 + 4 = 13$

Important Note

We subtract the two equations to get rid of the chosen term when the signs of the chosen terms are the same.

You can remember this by thinking of SSS (Same Signs Subtract).

$3x + 2y = 16$
$2x + 2y = 14$
$x = 2$ (subtract the equations)

Where signs are different, we must add the equations in order to get rid of the chosen term.

You can solve these equations by plotting the two graphs. The solution is the coordinates of the point where the two lines meet (refer to later study note on graphs).

Subject of a Formula

Changing the Subject of a Formula

$v = u + at$ is a formula with v as the subject. Sometimes we want to change the subject. Another way of saying this is 'write t in terms of v, a, and u'.

In this case we are changing the subject to t.

We use the same rules as equations:

$$v = u + at \qquad (-u)$$

$$v - u = at \qquad (\div a)$$

$$\frac{v - u}{a} = t$$

Normally we put the subject on the left, so: $t = \dfrac{v - u}{a}$

Many formulae can be rearranged in this way using the rules of equations.

However there are two special situations, as described below.

Special Situations

1. When the new subject is inside a square root sign.

For example:

$$p = t\sqrt{q} \qquad (\div t)$$

$$p/t = \sqrt{q} \qquad \text{(square both sides)}$$

$$\frac{p^2}{t^2} = q$$

$$q = \frac{p^2}{t^2}$$

Note: We can remove the square root sign by making it the subject, then squaring both sides of the formula.

2. When the new subject is squared.

For example:

$$y = \frac{p^2 - 3}{2} \qquad (\text{x } 2)$$

$$2y = p^2 - 3 \qquad (+\ 3)$$

$$2y + 3 = p^2 \qquad \text{(square root)}$$

$$p = \sqrt{2y + 3}$$

Note: We make the squared term the new subject, then square root.

Inequalities

Symbols

The symbols for inequalities are as follows:

Greater than	›
Less than	‹
Greater than or equal to	≥
Less than or equal to	≤

Solving Inequalities

We can solve inequalities in the same way as we solved equations.

Example 1

Solve $2y + 3 > 15$ (– 3)
$2y > 12$ (÷ 2)
$y > 6$

$y > 6$ is the solution, showing that the following are possible integer values of y: 7, 8, 9, 10 ....

Example 2

Solve $3y - 6 \leq 9$ (+ 6)
$3y \leq 15$ (÷ 3)
$y \leq 5$

In this case, we include 5 in the solution because of the **≤** sign.

The possible integer values of y are: 5, 6, 7, 8, 9..........

If the term in y is negative, always move it to the other side and make it positive, as in the following example.

Example 3

Solve $5 - 2y > 3$ (+ 2y)
$5 > 3 + 2y$
$2 > 2y$ (÷ 2)
$1 > y$

We read an inequality from the letter side so this reads 'y is less than 1' ($y < 1$).

The possible values of y are 0, –1, –2, –3...........

Note what to do if two inequality signs are used, as in the following case.

Example 4

Solve $3x - 1 > 2x < x + 5$.
In this instance, we split them up.

$3x - 1 > 2x$ and $2x < x + 5$
$3x - 2x > 1$ $2x - x < 5$

$x > 1$ $x < 5$

The possible values of x are 2, 3, 4.

The solution to an inequality is called '**the range of values**'.

Inequalities can be plotted graphically (See the study note on graphs).

Solving Equations by Trial and Improvement

Worked Example

The following is an illustration of how equations may be solved by means of trial and improvement.

Solve $x^2 + 3x = 9$.

In this method, we try values of x in the equation, for example $x = 2$.

$2^2 + 3 \times 2 = 10$ (too large)

Next we could try $x = 1$.

$1^2 + 3 \times 1 = 4$ (too small)

Then we could try a value halfway between 1 and 2 such as $x = 1{,}5$.

Using a calculator: $1{,}5^2 + 3 \times 1{,}5 = 6{,}75$ (too small)

Try $x = 1{,}8$.

$1{,}8^2 + 3 \times 1{,}8 = 8{,}64$ (too small)

Try $x = 1{,}9$.

$1{,}9^2 + 3 \times 1{,}9 = 9{,}31$ (too large)

If we are asked for an answer to one decimal place we must try $x = 1{,}85$.

$1{,}85^2 + 3 \times 1{,}85 = 8{,}9725$ (too small)

So $x = 1{,}9$ (one decimal place)

Nth Term and Sequences

Sequences

A number sequence is a set of numbers (terms) in which a pattern can be seen and a rule can be used to find every term in the sequence.

For example:

5, 10, 20, 40… double the last term each time…..80, 160

3, 5, 7, 9,…..Add two each time…..11, 13

25, 21, 17, 13,…minus four each time…..9, 5

The above examples have simple patterns, with harder sequences we need to look for a pattern and then establish the rule in order to calculate any term in the sequence.

---

nth Term

The rule for finding any term is called the nth term.

For example: Given the sequence 6, 10, 14, 18,……

a) Find the nth term; b) Find the 20th term; c) If the nth term is 42, what is the value of *n*?

We look at the differences between each term:

```
6 10 14 18
\ _/\_ /\_ /
  4 4 4
```

The difference is four.

The general formula for the nth term is:

nth term = $a + (n – 1)d$

where a = the first term = 6,
n = the number of the term, and
d = the difference = 4.

a) For this sequence:

nth term = $6 + (n – 1) 4$
$= 6 + 4n – 4$
$= 2 + 4n$

We can now use this formula to work out the value of any term in the sequence.

b) 20th term =
$2 + 4 \times 20$
$= 82$

because $n = 20$

c) nth term = 42
$42 = 2 + 4n$
$40 = 4n$
$n = 10$

So the 10th term is 42.

This formula will work for any linear sequence. In a linear sequence, the difference is constant. 4 in the sequence above.

Quadratic Sequences

In this type, the first difference is not constant. The second difference gives a constant.

For example: 3, 8, 15, 24, 36……… is a sequence.

```
3 8 15 24 35
\_/\_ /\_ /\_ /        (first difference)
  5 7 9 11
 \_ /\_ /\_ /          (second difference)
    2 2 2
```

This is a quadratic sequence as the second difference is a constant (in this case, 2).

The general formula for a quadratic is:
nth term = $a + (n - 1)d_1 + \frac{1}{2}(n - 1)(n - 2)d_2$

Where a = first term = 3; d_1= first difference = 5; d_2= second difference = 2

nth term = $3 + (n - 1)5 + \frac{1}{2}(n - 1)(n - 2)2$
$= 3 + 5n - 5 + n^2 - 3n + 2$
$= 2n + n^2$

We can use this to find the 100th term:

100th term = 1 002 + 200
= 10 200

Provided that the second difference is a constant we can use this method for any quadratic sequence.

Algebraic Direct and Inverse Proportion

Direct Proportion (Algebraic)

If two variables *y* and *x* are in direct proportion, we can write down an equation connecting them.

Equation for direct proportion

$y = kx$ where *k* is a constant.

Examples of problems

1) Given that *y* is directly proportional to *x* and that $y = 56$ when $x = 8$. Calculate *y* when $x = 12$.

a) Find the value of *k*: $y = kx$ $x = 8$ $y = 56$

$56 = k \times 8$

$k = 56/8$

$k = 7$

b) Substitute $k = 7$ into the equation: $y = 7x$.

'when $x = 12$'

$y = 7 \times 12$

$y = 84$

2) Given that *p* is directly proportional to t^2 and that $p = 16$ when $t = 2$.

a) Calculate *p* when $t = 5$. b) Calculate *t* when $p = 81$.

a) $p = kt^2$

$16 = k \times 4$

$k = 4$

$p = 4t^2$ ($t = 5$)

$p = 4 \times 25$

$p = 100$

b) $p = 4t^2$ ($p = 81$)

$81 = 4 \times t^2$

$t^2 = 81/4$

$t = 9/2$

$t = 4,5$

Inverse Proportion

If two variables y and x are in inverse proportion, the equation is: $y = k/x$

Examples of problems

1) If y is inversely proportional to x and $y = 5$ when $x = 4$,

a) Calculate y when $x = 10$. b) Calculate x when $y = 40$.

a) $y = k/x$

$5 = k/4$

$k = 20$

$y = 20/x$ $\qquad$ $x = 10$

$y = 20/10$

$y = 2$

b) $40 = 20/x$

$x = 0{,}5$

If f is inversely proportional $\sqrt{w}$ and $f = 10$ when $w = 16$,

a) Calculate f when $w = 9$. b) Calculate w when $f = 4$.

a) $f = \mathrm{k}/\sqrt{w}$

$10 = k/\sqrt{16}$

$10 = k/4$

$k = 40$

$f = 40/\sqrt{9}$ $\qquad$ $w = 9$

$f = 40/3$

$f = 13{,}3$ (one decimal point)

b) $4 = 40/\sqrt{w}$

$vw = 40/4$

$\sqrt{w} = 10$

$w = 100$

Simplifying Fractions

Fractions

1/8	1/8	1/8	1/8
1/8	1/8	1/8	1/8

This rectangle has been divided into eight equal parts.
Each part is one eighth of the rectangle (1/8).

Shading fractions of diagrams

Shade three quarters of the rectangle

1 ////////////////////////////	1 ////////////////////////////	1 ////////////////////////////	1
4 ////////////////////////////	4 ////////////////////////////	4 ////////////////////////////	4

Note each quarter is made up of two eighths. So ¾ is the same as 6/8.

These are called **equivalent fractions**. We say ¾ = 6/8.
If we multiply the top and the bottom of a fraction by the same number we get an equivalent fraction.

½ = 3/6 (x 3 top and bottom)

2/3 = 8/12 (x 4 top and bottom)

Other Types of Fractions

1) Improper or top heavy fractions:

The top number is bigger than the bottom. For example, 3/2 is a top heavy fraction.

2) Mixed numbers:

A mixture of whole numbers and fractions. For example, 1½.

This can be changed to a top heavy fraction:

$1\frac{1}{2} = \frac{2x1+1}{2} = \frac{3}{2}$

We multiply the whole number by the bottom number of the fraction and add the top of the fraction:
2 x 1 + 1 = 3 , over the bottom , gives 3/2.

For example:

$5\frac{2}{3} = \frac{3 \times 5 + 2}{3} = \frac{17}{3}$

Changing a top heavy to a mixed:

$\frac{17}{3}$ = 17 ÷ 3 = 5 remainder 2

This is written as 5 and 2/3 ($5\frac{2}{3}$)

Cancelling a Fraction

This is converting a fraction to its simplest form (lowest terms).
We do this by dividing the top and bottom by the same number.

For example:

5/10 = ½ (÷ top and bottom by 5)
6/8 = ¾ (÷ 2)
12/20 = 3/5 (÷ 4)

Note: ½ , ¾ and 3/5 cannot be simplified further.

Writing as a fraction

For example: I have 20 sweets and I eat 15, what fraction have I eaten?
Fraction eaten is 15 out of 20, this is written as a fraction 15/20.
We must always simplify: 15/20 = ¾ (÷ 5)

Finding 'a fraction of':
For example: What is ¾ of 8 ?
Rule: Divide by the bottom and times by the top.

8 ÷ 4 x 3 = 6
So, ¾ of 8 = 6

For example: 2/5 of 15 = 15 ÷ 5 x 2 = 6

Adding and Subtracting

For example:

3/5 +1/5 = 4/5 4/5 – 1/5 = 3/5
2/7 + 3/7 = 5/7 7/8 – 3/8 = 4/8 =1/2

When the bottom numbers are the same: We add or subtract the top numbers.

When the bottom numbers are different. For example: ½ + 1/3 = ?
We must make them the same by multiplying: 2 x 3 = 6. By making the bottom numbers 6, we must multiply the top numbers by the same amount:
In other words, ½ = 3/6 and 1/3 = 2/6 and these can now be added to get 5/6.

½ + 1/3 = $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

For example:

4/5 – 2/3 = ?
(5 x 3 =15)
4/5 = 12/15 2/3 = 10/15
So, 12/15 – 10/15 = 2/15

Mixed numbers

We can add or subtract the whole numbers and then the fractions.
For example:

$3\frac{1}{4} + 2\frac{1}{3} + 5\frac{7}{12}$

$5\frac{7}{12} - 2\frac{1}{3} = 3\frac{3}{12} = 3\frac{1}{4}$

Multiplication and Division

$\frac{3}{4} \times 2/3 = \frac{3 \times 2}{4 \times 3} = 6/12 = \frac{1}{2}$

We multiply the top numbers and multiply the bottom. Then simplify the answer, if possible.

If the numbers are large:
We can cancel diagonally:

15/16 x 24/35 = ?

15 and 35 cancel by 5 giving
3/16 x 24/7
16 and 24 cancel by 8 giving
3/2 and 3/7

So, 15/16 x 24/35 = 3/2 x 3/7 = 9/14

Mixed numbers

$3\frac{1}{4} \times 2\frac{1}{3} = ?$

Change to top heavy:

13/4 x 7/3 = 91/12 = 7