GRADE 12 EXAMINATION
NOVEMBER 2009

ADVANCED PROGRAMME MATHEMATICS

Time: 3 hours | 300 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 19 pages, an answer sheet for question 4.1 and an Information Booklet of 4 pages (i – iv). Please check that your question paper is complete. Please remove the insert and answer sheet from the middle of the question paper.

2. This question paper consists of FOUR Modules:

   MODULE 1: CALCULUS AND ALGEBRA (200 marks) is compulsory.

   Choose ONE of the THREE Optional Modules:

   MODULE 2: STATISTICS (100 marks) OR
   MODULE 3: FINANCE AND MODELLING (100 marks) OR
   MODULE 4: MATRICES AND GRAPH THEORY (100 marks)

3. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.

4. All necessary calculations must be clearly shown and writing should be legible.

5. Diagrams have not been drawn to scale.

6. Write all your answers in the separate Answer Book provided.

7. Round off your answers to two decimal digits, unless otherwise indicated.

MODULE 1 CALCULUS AND ALGEBRA

QUESTION 1

Use Mathematical induction and prove that $3^n + 3^{n+1} + 3^{n+2}$ is divisible by 13, for all $n \in \mathbb{N}$. (13)

13 marks

QUESTION 2

2.1 Given $f(x) = e^{x+2} - 1$

(a) Determine the equation of $f^{-1}(x)$, the inverse of $f(x)$. (4)

(b) Hence sketch the graph of $f^{-1}(x)$, clearly indicating the values of the intercepts with the axes correct to 1 decimal place, and asymptotes. (9)

2.2

(a) If $\sqrt{x + iy} = a + ib$, show that $x = a^2 - b^2$ and $y = 2ab$. (5)

(b) Hence find x and y if $\sqrt{x + iy} = 5i - 12$. (4)

2.3 If $f(x) = \dfrac{1}{4 - x^2}$, decompose $f(x)$ into partial fractions. (9)

31 marks

QUESTION 3

3.1 It is given that $x = 1 - 2i$ and $x = 1 + 2i$ are both zeros of $g(x)$. Use this information to show that $x^2 - 2x + 5$ is a factor of $g(x)$. (4)

3.2 Given: $g(x) = x^4 - 6x^3 + 18x^2 - 30x + 25$

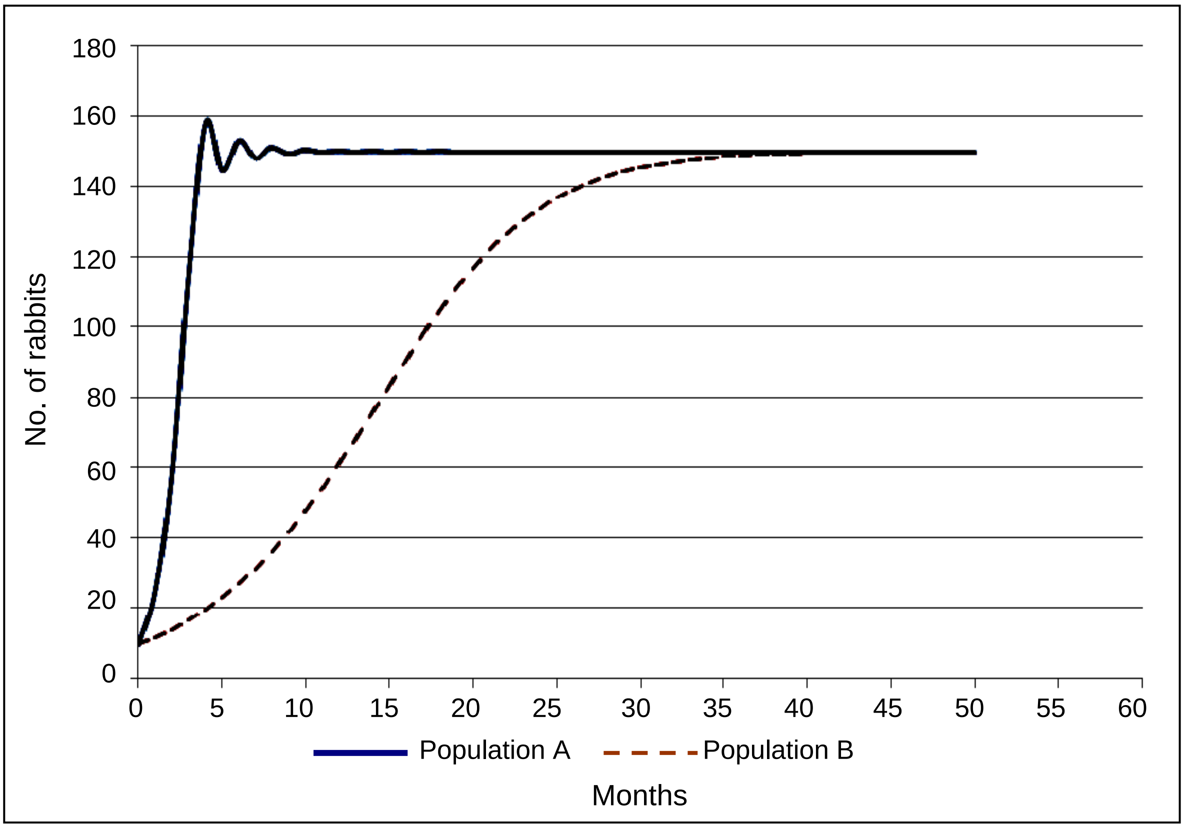
According to Gauss, a polynomial of degree n has n zeros. Therefore $g(x)$ has four zeros. Determine the other two zeros of $g(x)$ if $x = 1 - 2i$ and $x = 1 + 2i$ are both zeros of $g(x)$.

(11)

15 marks

QUESTION 5

The following graphs show the growth of two independent populations of rabbits. Both have an initial population of 10. One population is fed a special diet to increase their fertility. The carrying capacity of the fields in which they are kept is the same for both groups and there are no predators.



5.1 What is the carrying capacity of the fields? (1)

5.2 Which population received the special diet to increase its fertility? Give a reason for your answer. (2)

5.3 Estimate the intrinsic growth rate of population B. Show calculations to support your answer. (7)

5.4 Explain what is happening to population A between the 4^{th} and the 10^{th} month. (2)

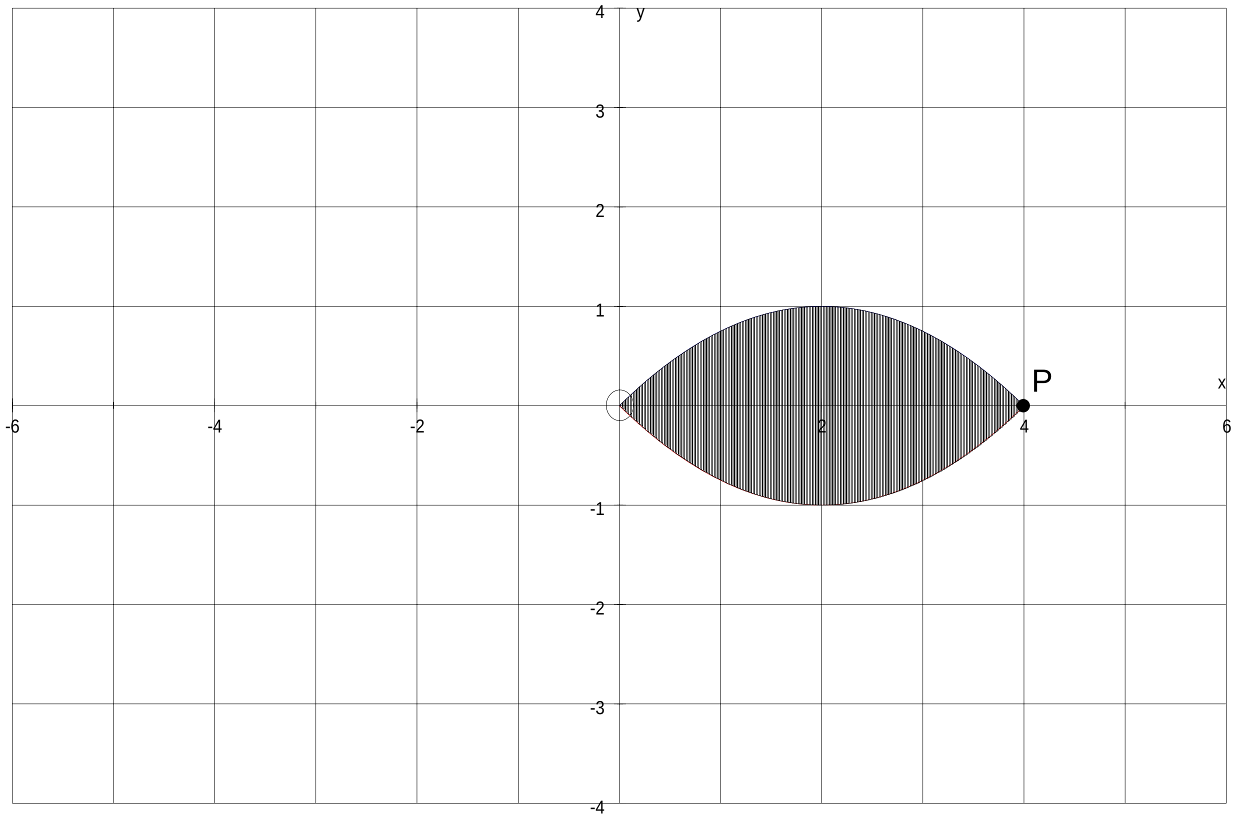
12 marks

Total for Module 3: 100 marks

MODULE 4 GRAPH THEORY AND MATRICES

QUESTION 1

The flowers of several plants have rotational symmetry of degree 5. To make a picture of a flower showing this symmetry a teacher gives the following picture of a petal, then asks the pupils to draw another four petals with the origin as the centre so that the flower has rotational symmetry of degree 5. (It is acceptable for the petals to overlap)



1.1 The first petal that Sam draws is the one next to this petal but rotated in an anti-clockwise direction. Determine the coordinates of the point on this petal that is furthest from the origin. (Answer correct to 2 decimal places) (6)

1.2 Peter thinks the flower will look prettier if he alters the given petal using the following transformation $\begin{pmatrix} 2{,}5 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) How will this change the shape of the original petal? (2)

(b) Find the coordinates of the point P on Peter's petal under this transformation. (3)

1.3 Paul claims that he can make Peter's flower by first finding all the petals using Sam's anti-clockwise rotation, then applying Peter's transformation to all the points in the flower.

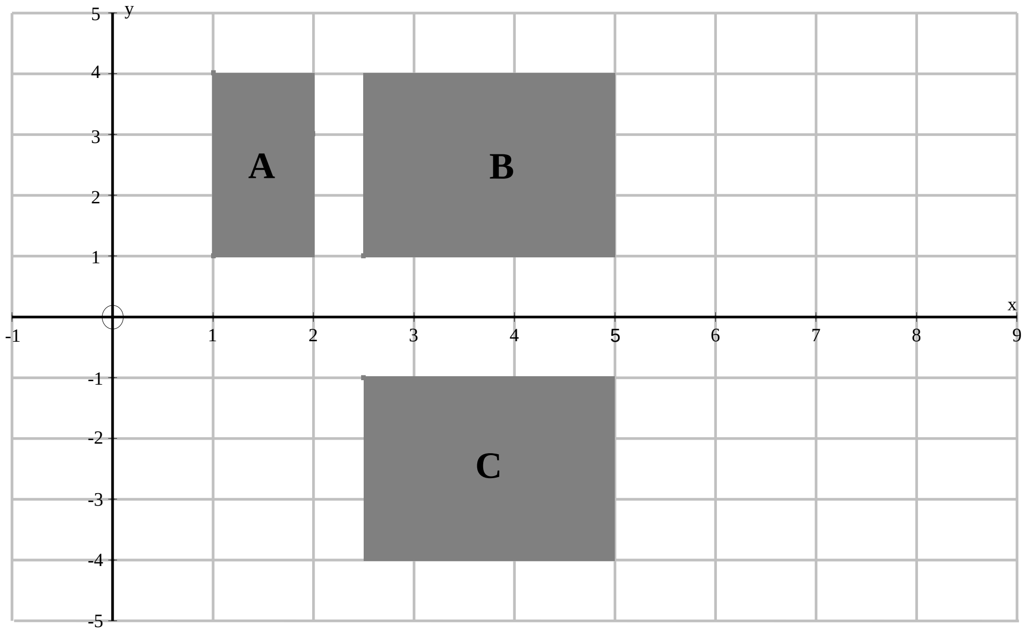
Will this work? Explain. (4)

15 marks

QUESTION 2

Describe the transformation and write down the single matrix which maps:

2.1 shape A onto shape B. (4)
2.2 shape B onto shape C. (2)
2.3 shape C onto shape A. (8)



14 marks

QUESTION 3

The determinant of the matrix, P, below is 20. Find the value of a given $a \in Z$

$$P = \begin{pmatrix} 1 & a & 1 \\ a & 4 & 3a \\ 3 & 1 & 2 \end{pmatrix}$$ (10)

10 marks

QUESTION 4

An AP Maths class is given the following system of equations:

$$2x + y - z = -7$$
$$x - y + z = 10$$
$$3x + 5y + 2z = 7$$

4.1 Mpho starts to find the solution by correctly applying row reduction but gives up. Complete her working and determine the solution to this system of equations.

Mpho's working:

Augmented matrix:

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & -7 \\ 1 & -1 & 1 & 10 \\ 3 & 5 & 2 & 7 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 10 \\ 0 & 3 & -3 & -27 \\ 0 & 8 & -1 & -23 \end{array}\right)$$

(8)

4.2 Sandra decides to solve the equations by using the inverse of Matrix A where $A.X = Y$

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 5 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \text{and} \quad Y = \begin{pmatrix} -7 \\ 10 \\ 7 \end{pmatrix}$$

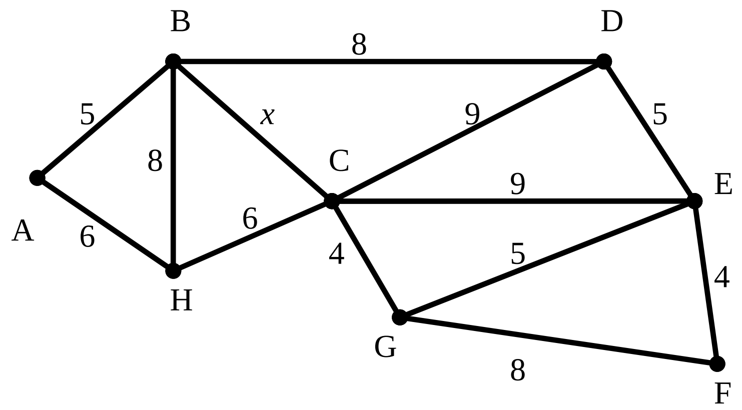
and $\det A = -21$.

(a) Find A^{-1}. (7)

(b) Express X in terms of A^{-1} and Y. (2)

17 marks

QUESTION 5



Referring to the above network answer the following questions:

5.1 Find the maximum value of x, where $x \in N$, that would ensure that edge BC is included in the unique shortest path between A and F. (10)

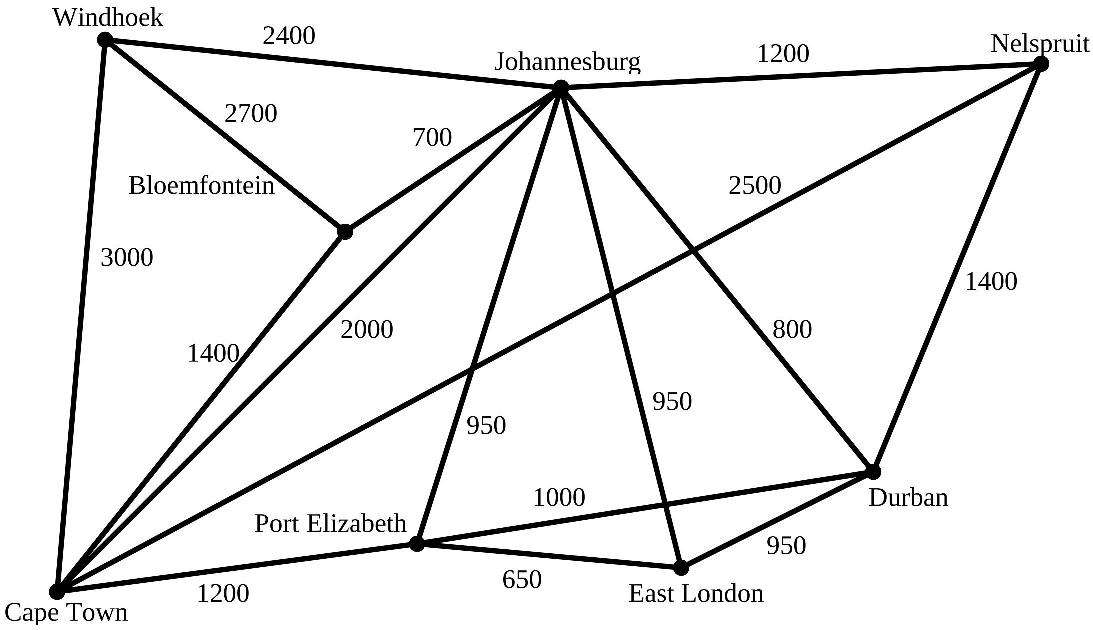
5.2 (a) List all the vertices of odd degree. (3)

(b) Which edges need to be duplicated in order to produce an Eulerian circuit of minimum weight? (7)

20 marks

QUESTION 6

Below is a diagram representing the cost of airfares between the major centres in South Africa and Namibia. A local businessman is based in Johannesburg and is required to visit each of the centres once a month.



6.1 Determine a lower bound for the cost of the airfares, by removing Johannesburg and using Kruskal's algorithm. Clearly state the order in which you choose the vertices. (9)

6.2 Giving reasons for your answers determine whether the lower bound found in 6.1 would be affected if:

(a) Prim's algorithm is used instead of Kruskal's algorithm. (2)

(b) Durban is used as the initial vertex instead of Johannesburg. (4)

6.3 Find an upper bound for the airfare cost using the nearest-neighbour algorithm and Johannesburg as the starting point. (9)

24 marks

Total for Module 4: 100 marks

Total: 300 marks