ADVANCED MATHEMATICS FORMULA SHEET

The formulae below may assist you. This is not a comprehensive list. You are of course free to use formulas that do not appear on this list.

MODULE 1 INTEGRATION

Approximate area = $\frac{1}{2}[f(x_0)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-1})+f(x_n)]\Delta x$

$U=[f(x_1)+f(x_2)+\ldots+f(x_n)]\Delta x$

$L=[f(x_0)+f(x_1)+\ldots+f(x_{n-1})]\Delta x$

(U and L: for f increasing)

$$\int_a^b cx^k\,dx = c\left(\frac{b^{k+1}}{k+1}-\frac{a^{k+1}}{k+1}\right)$$

$$Vol=\pi\int_a^b y^2\,dx \quad \text{or} \quad \pi\int_a^b x^2\,dy$$

MODULE 2 DIFFERENTIATION

Function	**Derivative**
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x.\tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x.\cot x$
$f(g(x))$	$f'(g(x)).g'(x)$
$f(x).g(x)$	$g(x).f'(x)+f(x).g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{g(x).f'(x)-f(x).g'(x)}{[g(x)]^2}$

$$\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}$$

$$\pi\ rad = 180^\circ$$

$$S=r\theta$$

Area of sector = $\frac{1}{2}r^2\theta=\frac{1}{2}rS$

$$\lim_{h\to 0}\frac{\sin h}{h}=1$$

$$\lim_{h\to 0}\frac{1-\cos h}{h}=0$$

MODULE 3 VECTORS

$$|\underline{a}| = \sqrt{x^2 + y^2 + z^2}$$

$$\cos\theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}||\underline{b}|}$$

$$d(P;\pi) = \left| \frac{ax_P + by_P + cz_P + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

$$\underline{OP} = \underline{OA} + t\,\underline{AB}$$

$$\underline{OP} = \underline{OA} + t\,\underline{AB} + s\,\underline{AC}$$

$$ax + by + cx + d = 0$$

MODULE 4 MATRICES

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $A^{-1} = \frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ where $\Delta = ad - bc$

$$\begin{pmatrix} cos\,\theta & -sin\,\theta \\ sin\,\theta & cos\,\theta \end{pmatrix}$$

$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ – (Shear parallel to the x axis.)

MODULE 5 NUMERICAL METHODS

$$x_1 = \frac{a\ f(b) - b\ f(a)}{f(b) - f(a)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = \frac{x_n f(x_{n-1}) - x_{n-1}\ f(x_n)}{f(x_{n-1}) - f(x_n)}$$

$$y = f(x_0) + (x - x_0) f[x_0, x_1]$$

$$y = f(x_0) + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2]$$

$$f(x_0 + sp) = f(x_0) + s\Delta f_0 + \frac{s(s-1)\Delta^2 f_0}{2!} + \frac{s(s-1)(s-2)\Delta^3 f_0}{3!}$$

MODULE 6 ELEMENTARY LOGIC AND BOOLEAN ALGEBRA

A system $(B, +, \bullet, 0, 1)$ is a Boolean algebra if

I $(\forall a, b \in B)$ $(a + b \in B$ and $a \bullet b \in B)$

II $(\forall a \in B)$ $(a + b = b + a$ and $a \bullet b = b \bullet a)$

III $(\forall a \in B)$ $(a + 0 = a$ *and* $a \bullet 1 = a)$

IV $(\forall a \in B)$ $(\exists \overline{a} \in B)$ $(a + \overline{a} = 1$ and $a \bullet \overline{a} = 0)$

V $(\forall a, b, c \in B)$ $[a + (b \bullet c) = (a + b) \bullet (a + c)$ and $a \bullet (b + c) = (a \bullet b) + (a \bullet c)]$

On the basis of I to V the following theorems can be proved:

1. $a + a = a$; $a \bullet a = a$
2. 0 and 1 are unique
3. $a + 1 = 1$; $a \bullet 0 = 0$
4. $(\forall a \in B)(\overline{a}$ is unique$)$
5. $a + a \bullet b = a$ and $a \bullet (a + b) = a$
6. $(\forall a \in B)$ $(\overline{\overline{a}} = a)$
7. $a + (b + c) = (a + b) + c$ and $a \bullet (b \bullet c) = (a \bullet b) \bullet c$
8. $\overline{a + b} = \overline{a} \bullet \overline{b}$ and $\overline{a \bullet b} = \overline{a} + \overline{b}$
9. $a + \overline{a} \bullet b = a + b$ and $a \bullet (\overline{a} + b) = a \bullet b$

Implications

$p \Rightarrow q$

Contrapositive $\overline{q} \Rightarrow \overline{p}$ Law of Implication $\overline{p} \cup q$

Opposite $\overline{p} \Rightarrow \overline{q}$ Converse $q \Rightarrow p$

ARISTOTELIAN SYLLOGISM

Modus Ponens

$$\begin{array}{l} p \Rightarrow q \\ \underline{p \qquad} \\ \therefore q \end{array}$$

Modus Tollens

$$\begin{array}{l} p \Rightarrow q \\ \underline{\bar{q} \qquad} \\ \therefore \bar{p} \end{array}$$

Disjunctive Syllogism

$$\begin{array}{l} p \cup q \\ \underline{\bar{p} \qquad} \\ \therefore q \end{array}$$

Hypothetical Syllogism

$$\begin{array}{l} \underline{(p \Rightarrow q) \cap (q \Rightarrow r)} \\ \therefore p \Rightarrow r \end{array}$$

MODULE 7 PROBABILITY, PERMUTATIONS AND COMBINATIONS

Axioms

Let S be a sample space. Then

I $0 \leq P(E) \leq 1$ for every event E of S

II $P(S) = 1$

III $P(E \cup F) = P(E) + P(F)$ for events E and F if $E \cap F = \phi$

Definitions and Theorems

$P(\phi) = 0$ $P(\bar{A}) = 1 - P(A)$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ $P(A \cap B) = P(A).P(B)$ iff A and B are indepedent

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

<u>Miscellaneous</u>

${}^nP_r = \dfrac{n!}{(n-r)!}$ ${}^nC_r = \binom{n}{r} = \dfrac{n!}{(n-r)!r!}$

$P(E) = \dfrac{n(E)}{n(S)}$ (events equiprobable)

$\overline{A \cap B} = \bar{A} \cup \bar{B}$ $\overline{A \cup B} = \bar{A} \cap \bar{B}$